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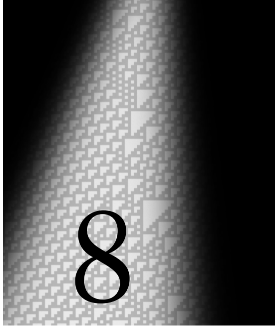
A NEW KIND OF SCIENCE

SECTION 8.1

Issues of Modelling



8

Implications for Everyday Systems

Issues of Modelling

In the previous chapter I showed how various general forms of behavior that are common in nature can be understood by thinking in terms of simple programs. In this chapter what I will do is to take what we have learned, and look at a sequence of fairly specific kinds of systems in nature and elsewhere, and in each case discuss how the most obvious features of their behavior arise.

The majority of the systems I consider are quite familiar from everyday life, and at first one might assume that the origins of their behavior would long ago have been discovered. But in fact, in almost all cases, rather little turns out to be known, and indeed at any fundamental level the behavior that is observed has often in the past seemed quite mysterious. But what we will discover in this chapter is that by thinking in terms of simple programs, the fundamental origins of this behavior become much less mysterious.

It should be said at the outset that it is not my purpose to explain every detail of all the various kinds of systems that I discuss. And in fact, to do this for even just one kind of system would most likely take at least another whole book, if not much more.

But what I do want to do is to identify the basic mechanisms that are responsible for the most obvious features of the behavior of each kind of system. I want to understand, for example, how in general snowflakes come to have the intricate shapes they do. But I am not concerned, for example, with details such as what the precise curvature of the tips of the arms of the snowflake will be.

In most cases the basic approach I take is to try to construct the very simplest possible model for each system. From the intuition of traditional science we might think that if the behavior of a system is complex, then any model for the system must also somehow be correspondingly complex.

But one of the central discoveries of this book is that this is not in fact the case, and that at least if one thinks in terms of programs rather than traditional mathematical equations, then even models that are based on extremely simple underlying rules can yield behavior of great complexity. And in fact in the course of this chapter, I will construct a whole sequence of remarkably simple models that do rather well at reproducing the main features of complex behavior in a wide range of everyday natural and other systems.

Any model is ultimately an idealization in which only certain aspects of a system are captured, and others are ignored. And certainly in each kind of system that I consider here there are many details that the models I discuss do not address. But in most cases there have in the past never really been models that can even reproduce the most obvious features of the behavior we see. So it is already major progress that the models I discuss yield pictures that look even roughly right.

In many traditional fields of science any model which could yield such pictures would immediately be considered highly successful. But in some fields—especially those where traditional mathematics has been used the most extensively—it has come to be believed that in a sense the only truly objective or scientific way to test a model is to look at certain rather specific details.

Most often what is done is to extract a small set of numbers from the observed behavior of a system, and then to see how accurately these numbers can be reproduced by the model. And for systems whose overall behavior is fairly simple, this approach indeed often works quite well. But when the overall behavior is complex, it becomes impossible to characterize it in any complete way by just a few numbers.

And indeed in the literature of traditional science I have quite often seen models which were taken very seriously because they could be made to reproduce a few specific numbers, but which are shown up as completely wrong if one works out the overall behavior that they imply. And in my experience by far the best first step in assessing a model is not to look at numbers or other details, but rather just to use one's eyes, and to compare overall pictures of a system with pictures from the model.

If there are almost no similarities then one can reasonably conclude that the model is wrong. But if there are some similarities and some differences, then one must decide whether or not the differences are crucial. Quite often this will depend, at least in part, on how one intends to use the model. But with appropriate judgement it is usually not too difficult from looking at overall behavior to get at least some sense of whether a particular model is on the right track.

Typically it is not a good sign if the model ends up being almost as complicated as the phenomenon it purports to describe. And it is an even worse sign if when new observations are made the model constantly needs to be patched in order to account for them.

It is usually a good sign on the other hand if a model is simple, yet still manages to reproduce, even quite roughly, a large number of features of a particular system. And it is an even better sign if a fair fraction of these features are ones that were not known, or at least not explicitly considered, when the model was first constructed.

One might perhaps think that in the end one could always tell whether a model was correct by explicitly looking at sufficiently low-level underlying elements in a system and comparing them with elements in the model. But one must realize that a model is only ever supposed to provide an abstract representation of a system—and there is nothing to say that the various elements in this representation need have any direct correspondence with the elements of the system itself.

Thus, for example, a traditional mathematical model might say that the motion of a planet is governed by a set of differential equations. But one does not imagine that this means that the planet itself contains a device that explicitly solves such equations. Rather, the idea is that the equations provide some kind of abstract representation for the physical effects that actually determine the motion of the planet.

When I have discussed models like the ones in this chapter with other scientists I have however often encountered great confusion about such issues. Perhaps it is because in a simple program it is so easy to see the underlying elements and the rules that govern them. But countless times I have been asked how models based on simple programs can possibly be correct, since even though they may successfully reproduce the behavior of some system, one can plainly see that the system itself does not, for example, actually consist of discrete cells that, say, follow the rules of a cellular automaton.

But the whole point is that all any model is supposed to do—whether it is a cellular automaton, a differential equation, or anything else—is to provide an abstract representation of effects that are important in determining the behavior of a system. And below the level of these effects there is no reason that the model should actually operate like the system itself.

Thus, for example, a cellular automaton can readily be set up to represent the effect of an inhibition on growth at points on the surface of a snowflake where new material has recently been added. But in the cellular automaton this effect is just implemented by some rule for certain configurations of cells—and there is no need for the rule to correspond in any way to the detailed dynamics of water molecules.

So even though there need not be any correspondence between elements in a system and in a model, one might imagine that there must still be some kind of complete correspondence between effects. But the whole point of a model is to have a simplified representation of a system, from which those features in which one is interested can readily be deduced or understood. And the only way to achieve this is to pick out only certain effects that are important, and to ignore all others.

Indeed, in practice, the main challenge in constructing models is precisely to identify which effects are important enough that they have to be kept, and which are not. In some simple situations, it is sometimes possible to set up experiments in which one can essentially isolate each individual effect and explicitly measure its importance. But in the majority of cases the best evidence that some particular set of effects are in fact the important ones ultimately comes just from the success of models that are based on these effects.

The systems that I discuss in this chapter are mostly complicated enough that there are at least tens of quite different effects that could contribute to their overall behavior. But in trying to construct the simplest possible models, I have always picked out just a few effects that I believe will be the most important. Inevitably there will be phenomena that depend on other effects, and which are therefore not correctly reproduced by the models I consider. So if these phenomena are crucial to some particular application, then there will be no choice but to extend the model for that application.

But insofar as the goal is to understand the basic mechanisms that are responsible for the most obvious features of overall behavior, it is important to keep the underlying model as simple as possible. For even with just a few extensions models usually become so complicated that it is almost impossible to tell where any particular feature of behavior really comes from.

Over the years I have been able to watch the progress of perhaps a dozen significant models that I have constructed—though in most cases never published—for a variety of kinds of systems with complex behavior. My original models have typically been extremely simple. And the initial response to them has usually been great surprise that such simple models could ever yield behavior that has even roughly the right features. But experts in the particular types of systems involved have usually been quick to point out that there are many details that my models do not correctly reproduce.

Then after an initial period where the models are often said to be too simplistic to be worth considering, there begin to be all sorts of extensions added that attempt to capture more effects and more details. The result of this is that after a few years my original models have evolved into models that are almost unrecognizably complex. But these models have often then been used with great success for many practical purposes. And at that point, with their success established, it sometimes happens that the models are examined more carefully—and it is then discovered that many of the extensions that were added were in fact quite unnecessary, so that in the end, after perhaps a decade has passed, it becomes recognized that models equivalent to the simple ones I originally proposed do indeed work quite well.

One might have thought that in the literature of traditional science new models would be proposed all the time. But in fact the vast majority of what is done in practically every field of science involves not developing new models but rather accumulating experimental data or working out consequences of existing models.

And among the models that have been used, almost all those that have gone beyond the level of being purely descriptive have ended up being formulated in very much the same kind of way: typically as collections of mathematical equations. Yet as I emphasized at the very beginning of this book, this is, I believe, the main reason that in the past it has been so difficult to find workable models for systems whose behavior is complex. And indeed it is one of the central ideas of this book to go beyond mathematical equations, and to consider models that are based on programs which can effectively involve rules of any kind.

It is in many respects easier to work with programs than with equations. For once one has a program, one can always find out what its behavior will be just by running it. Yet with an equation one may need to do elaborate mathematical analysis in order to find out what behavior it can lead to. It does not help that models based on equations are often stated in a purely implicit form, so that rather than giving an actual procedure for determining how a system will behave—as a program does—they just give constraints on what the behavior must be, and provide no particular guidance about finding out what, if any, behavior will in fact satisfy these constraints.

And even when models based on equations can be written in an explicit form, they still typically involve continuous variables which cannot for example be handled directly by a practical computer. When their overall behavior is sufficiently simple, complete mathematical formulas to describe this behavior can sometimes be found. But as soon as the behavior is more complex there is usually no choice but to use some form of approximation. And despite many attempts over the past fifty or so years, it has almost never been possible to demonstrate that results obtained from such approximations even correctly reproduce what the original mathematical equations would imply.

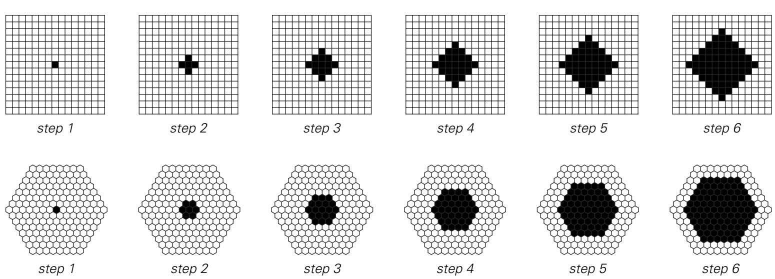
Models based on simple programs, however, suffer from no such problems. For essentially all of them involve only discrete elements which can be handled quite directly on a practical computer. And this means that it becomes straightforward in principle—and often highly efficient in practice—to work out at least the basic consequences of such models.

Many of the models that I discuss in this chapter are actually based on some of the very simplest kinds of programs that I consider anywhere in this book. But as we shall see, even these models appear quite sufficient to capture the behavior of a remarkably wide range of systems from nature and elsewhere—establishing beyond any doubt, I believe, the practical value of thinking in terms of simple programs.

The Growth of Crystals

At a microscopic level crystals consist of regular arrays of atoms laid out much like the cells in a cellular automaton. A crystal forms when a liquid or gas is cooled below its freezing point. Crystals always start from a seed—often a foreign object such as a grain of dust—and then grow by progressively adding more atoms to their surface.

As an idealization of this process, one can consider a cellular automaton in which black cells represent regions of solid and white cells represent regions of liquid or gas. If one assumes that any cell which is adjacent to a black cell will itself become black on the next step, then one gets the patterns of growth shown below.



Cellular automata with rules that specify that a cell should become black if any of its neighbors are already black. The patterns produced have a simple faceted form that reflects directly the structure of the underlying lattice of cells.