A note on long-term smoothing of sunspot numbers

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ABSTRACT

In this short contribution, we applied the method of long-term adaptive smoothing developed by Mann to several series of the sunspot number. Our results are supporting the idea that we are facing a new Gleissberg Minimum because the decrease in sunspots number is not similar to the observed decrease in the beginning of the Maunder Minimum.

Keywords: smoothing, sunspot number, Maunder Minimum

1. Introduction

A suitable smoothing of sunspot number (SN) series is essential to study the long-term behavior of solar activity, especially because SN often exhibit non-stationary statistical behavior. The process to smooth time series is particularly problematic when we are studying time series with trends near the boundaries. In this case, the smoothing is not uniquely defined (Park, 1992; Ghil et al., 2002; Mann, 2004; Arguez et al., 2008).

The anomalous behavior of recent solar activity, significantly weaker than expected, has been widely reported (Russell et al., 2010; de Toma, 2011; Solanki and Krivova, 2011). There is a discrepancy between those who think that it shows the beginning of a new minimum of the Gleissberg cycle (Feynman and Ruzmaikin, 2011) and those others who think that it is the beginning of a new Maunder minimum (Miyahara et al., 2010). This discrepancy highlights the importance of the study on the boundaries of the smoothed solar time series.

In this paper, we highlight the importance of boundary conditions that we impose in the smoothing of SN series. We will apply to Group Sunspot Number (Hoyt and Schatten, 1998) and International Sunspot Number (Clette et al., 2007) series the smoothing method based on Mann (2008) that was previously applied to climate series. Both SNs have been studied extensively (Hoyt and Schatten, 1998; Clette et al., 2007; Vaquero, 2007; Svalgaard, 2012). We will also use a series composed of both of this, which has been implemented with a number of corrections in some data and we will called CSN (Composite sunspot number).

2. Method

The smoothing method that we use for our three SNs series is inspired on the method developed by Mann (2004, 2008) that has been applied to climate series. Mann (2004) used a Butterworth filter to smooth the signal of the series, but the novelty is to smooth the boundaries. Several ways to do this are suggested, depending on whether the criteria to follow in the boundaries is to obtain the slightest deviation in the average, the minimum dispersion or the minimum slope for the series values. He advocates the following conditions on the boundaries following Mann (2004):
MINIMUM NORM: Add to the edges one and a half cycle with the value corresponding to the last half cycle mean of the series. This ensures that the values at the boundaries will not deviate much from this average.

MINIMUM SLOPE: also adds to the edges one and a half cycle specularly reflected values, that means V(b-x) = V(b+x), where b is the edge of the series. This ensures that the deviation values in the series are minimal.
MINIMUM ROUGHNESS: Add to the edges one and a half cycle with values reflected in the y and x axis, that is, V(b-x) = -V(b+x), which ensures that there is minimal deviation in slope from the series.

Once we have implemented these conditions at the edges, we compute the MSE (mean square error) committed by smoothing made with each of these constraints. Then, we define another function that tests all linear combinations of the previous three and choose the one that minimizes the MSE (Mann, 2008). This function is called LOWPASSADAPTIVE and constitutes the best smoothing for all previous series, with the minimum MSE.

In this work, we have used these features but with a Butterworth filter of 6 order and with a cutoff frequency of $f_0 = 0.023$. Therefore, using the simple transformation $\Delta\tau = 2/f_0$, we can see that variations of a cycle length in the order of 88 years (approximately the Gleissberg cycle length) are allowed.

3. Results and Discussion

We have applied this smoothing, with different boundary conditions, for annual series of sunspot numbers (ISN, GSN, and a composite of the others that we have called CSN). We have computed the error made with each of them. The names given to the different smoothing are:

SMOOTH 0.0: takes no restriction on the edges
SMOOTH 1.0: restriction takes edges as the "minimum norm"
SMOOTH 1.1: the restriction on the edges is "minimum slope"
SMOOTH 1.2: the restriction is "minimum roughness"
SMOOTH ADAPT: smoothing is a linear combination of the foregoing which minimizes the MSE.

3.1. Group Sunspot Number

The GSN series cover the period from 1610 to 1995. We take the annual values of the series as well as the others that we will try, because we are interested in the secular behavior. Therefore, it makes little sense to take monthly or daily data. The results can be seen in the Figure 1.

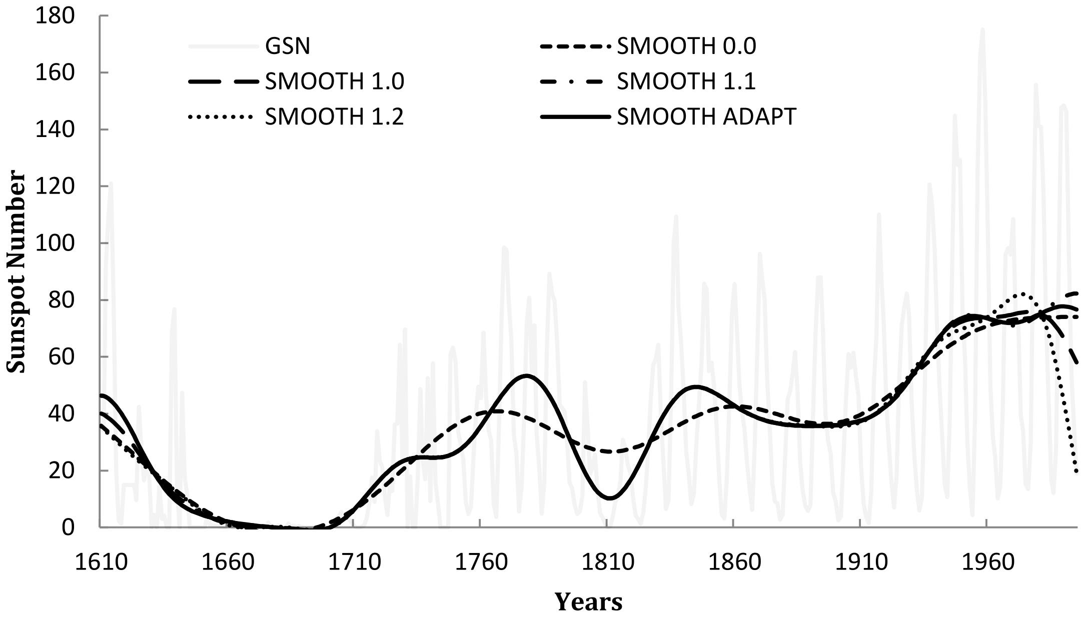

Fig. 1. Long-term smoothed Group Sunspot Number from 1610 to 1995.

Table 1 list the MSE for each different smoothing. Obviously, as it is seen in Figure 1, the restriction on the edges (except the smoothing that does not have any restriction) makes little changes far from them, but it is crucial when it comes to defining the trend of series in the coming years. Furthermore, trend of series clearly depends of constraints on the edges, and since more reliable smoothing is the last one (smooth adapt), as tells us the value of MSE, we conclude that the decrease of the amplitude of cycles is not as pronounced as expected.

Smoothing type	GSN	ISN	CSN
Smooth 0.0	1.9086	4.9112	1.6543
Smooth 1.0	0.6080	0.7979	0.6074
Smooth 1.1	0.6040	0.7948	0.6061
Smooth 1.2	0.6364	0.8073	0.6209
Smooth Adapt	0.6037	0.7948	0.6059

TABLE 1. Values of MSE according to the smoothing type and the sunspot number series.

3.2. International Sunspot Number

The annual number of ISN ranges from 1700-2012. As in the previous series we apply the smoothing with various constraints on the edges. Results are shown in Figure 2.

Data for each smoothing (MSE) are listed in Table 1. The adaptive smoothing coincides with that which uses the minimum slope constraint (Figure 2). It also can be seen that the adaptive smoothing sets a trend indicating that although the amplitude of the solar cycle continues to drop, it becomes less steep.

3.3. Composite Sunspot Number

Finally, we want to analyze a composition of the previous series. In order to take the widest possible range of annual data, we have taken the GSN series from 1610, which included the data in the Maunder Minimum, until 1995, and from here, as this series brings no value, the data we have used are the ISN from then until 2012.

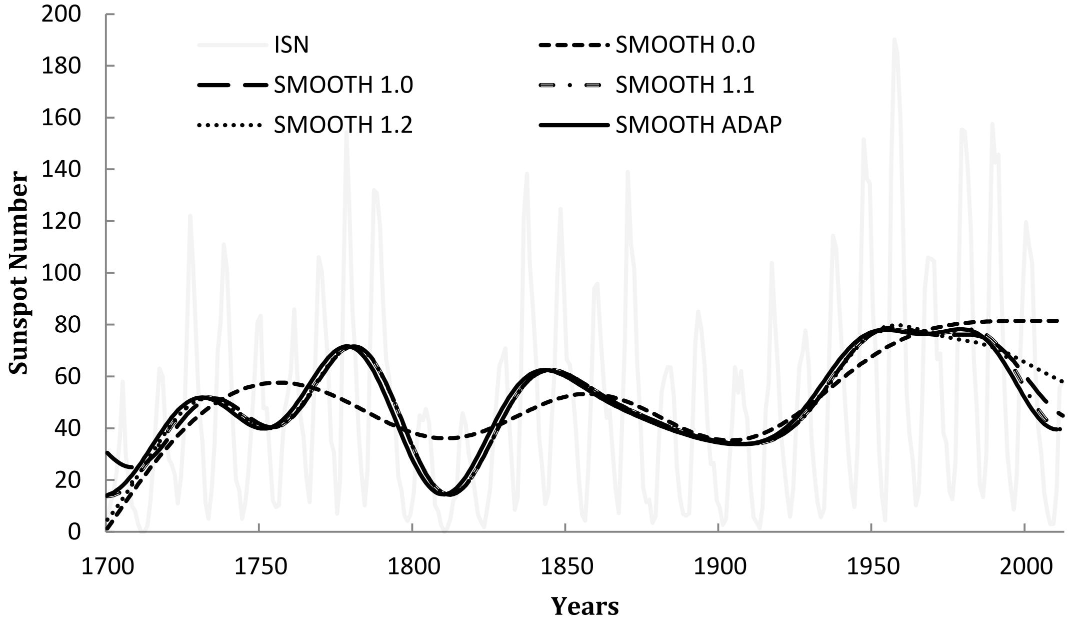

Fig. 2. Long-term smoothed International Sunspot Number from 1700 to 2012.

We have also made a number of corrections in the data series for values near the Maunder Minimum, according to Vaquero et al. (2011).We also vary some erroneous data for the eighteenth century, as Vaquero and Vázquez (2009) explained. The results obtained with different restriction limits were as shown in Figure 3.

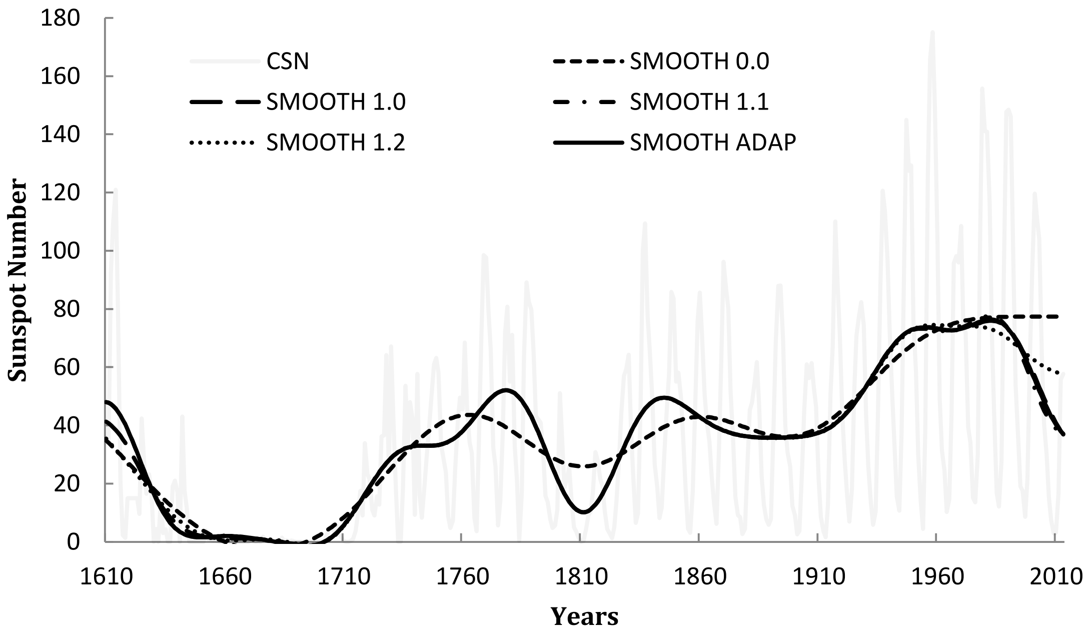

Fig. 3. Long-term smoothed Composite Sunspot Number from 1610 to 2012.

Data for the MSE of each smoothed are listed in Table 1. In this case, the smoothing of the CSN series, which includes both data from the Maunder Minimum and the most actual ones, we can see that the Smooth 1.0, the Smooth 1.1 and the adaptive smoothing nearly coincide and mark a striking resemblance in their behavior at the top with what happened in the Gleissberg Minimum in the early nineteenth century.

4. Conclusions

We have applied several smoothing method to the long-term SNs series. Our results are supporting the work by Feymann and Ruzmaikin (2011) who claimed that we are facing a new Gleissberg Minimum because the decrease in sunspots number is not similar to the observed decrease in the beginning of the Maunder Minimum (Vaquero et al, 2011). Note that revised versions of the Sunspot Number have been recently published (Clette et al., 2014; Clette et al., 2015).

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