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EUROPEAN COMMISSION

Guidance on sampling methods for audit authorities

Programming periods 2007-2013 and 2014-2020

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List of Acronyms

AA – Audit Authority

ACR – Annual Control Report

AE – Anticipated Error

AR – Audit Risk

BP – Basic Precision

BV – Book Value (expenditure declared to the Commission in reference period)

COCOF – Committee of the Coordination of Funds

CR – Control Risk

DR – Detection Risk

E_i – Individual errors in the sample

$\bar{E}$ – Mean error of the sample

EC – European Community

EE – Projected Error

EDR – Extrapolated Deviation Rate

EF – Expansion Factor

ETC – European Territorial Cooperation

IA – Incremental Allowance

IR – Inherent Risk

IT – Information Technologies

MCS – Managing and Control System

MUS – Monetary Unit Sampling

PPS – Probability Proportional to Size

RF – Reliability Factor

SE – (Effective, i.e., after performing audit work) Sampling Error (precision)

SI – Sampling Interval

TE – Maximum Tolerable Error

TPE – Total Projected Error (corresponds also to the TPER, acronym used for programming period 2007-2013)

ULD – Upper Limit of Deviation

ULE – Upper Limit of Error

1 Introduction

The present guide to sampling for auditing purposes has been prepared with the objective of providing audit authorities in the Member States with an updated overview of the most commonly used and suitable sampling methods, thus providing support for the implementation of the regulatory framework for the 2007-2013 programming period and, where applicable, the 2014-2020 programming period.

International auditing standards and updated sampling theory provide guidance on the use of audit sampling and other means of selecting items for testing when designing audit procedures.

The present guidance replaces the previous guidance on the same subject (ref. COCOF 08/0021/03-EN of 04/04/2013). The present document is without prejudice of other complementary Commission guidelines, namely the:

- Programming period 2007-2013:
    - "Guidance note on annual control reports and opinions" of 18/02/2009, ref. COCOF 09/0004/01-EN and EFFC/0037/2009-EN of 23/02/2009;
    - "Guidance on treatment of errors disclosed in the annual control reports" ref. EGESIF_15-0007-01 of 09/10/2015;
    - "Guidance on a common methodology for the assessment of management and control systems [MCS] in the Member States" ref. COCOF 08/0019/01- EN and EFFC/27/2008 of 12/09/2008.
- Programming period 2014-2020:
    - Guidance for Member States on the Annual Control Report and Audit Opinion (Programming period 2014-2020), ref. EGESIF_15-0002-02 final of 9/10/2015;
    - Guidance for the Commission and Member States on a common methodology for the assessment of management and control systems in the Member States (EGESIF_14-0010-final of 18/12/2014).

Thus, complementary reading of these additional documents is advised in order to get a complete view of the guidelines related to the production of annual control reports.

2 Regulatory references

Regulation	Articles
Programming period 2007-2013	
Reg. (EC) No 1083/2006	Article 62 - Functions of the audit authority
Reg. (EC) No 1828/2006	Article 17 - Sampling Annex IV – Technical Parameters for Random Statistical Sampling Pursuant to Article 17
Reg. (EC) No 1198/2006	Article 61 – Functions of the audit authority
Reg. (EC) No 498/2007	Articles 43 – Sampling Annex IV – Technical parameters
Programming period 2014-2020	
Reg. (EU) No 1303/2013 Common Provisions Regulation *(hereafter CPR)*	Article 127 (5)- Functions of the audit authority Article 148(1) – Proportional control of operational programmes
Reg. (EU) No 480/2014 Commission Delegated Regulation (hereafter CDR)	Article 28 - Methodology for the selection of the sample of operations

3 Audit risk model and audit procedures

3.1 Risk model

Audit risk is the risk that the auditor issues an unqualified opinion, when the declaration of expenditure contains material errors.

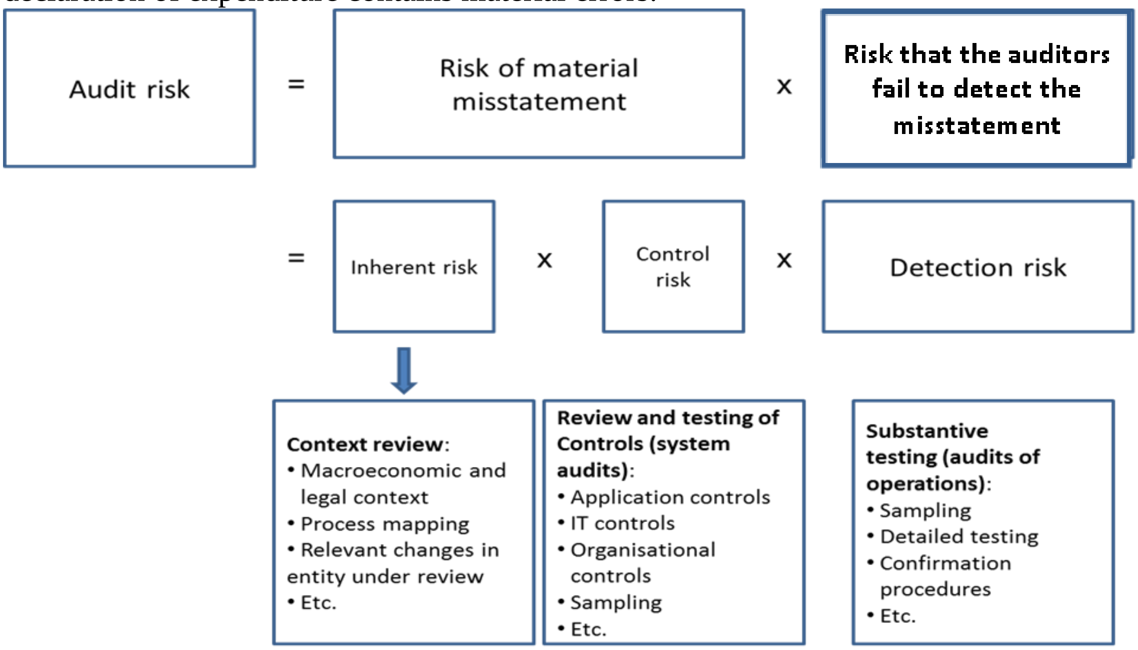

Fig 1. Audit risk model

The three components of audit risk are referred to respectively as inherent risk (IR), control risk (CR) and detection risk (DR). This gives rise to the audit risk model

$$AR = IR \times CR \times DR$$

where:

- IR, inherent risk, is the perceived level of risk that a material error may occur in the statements of expenditure submitted to the Commission, or underlying levels of aggregation, in the absence of internal control procedures. The inherent risk is linked to the kind of activities of the audited entity and will depend on external factors (cultural, political, economic, business activities, clients and suppliers, etc.) and internal factors (type of organisation, procedures, competence of staff, recent changes to processes or management positions, etc.). IR risk needs to be assessed before starting detailed audit procedures (interviews with management and key personnel, reviewing contextual information such as organisation charts, manuals and internal/external documents). For the Structural and Fisheries Funds, the inherent risk is usually set at a high percentage.
- CR, control risk, is the perceived level of risk that a material error in statements of expenditure submitted to the Commission, or underlying levels of aggregation, will not be prevented, detected and corrected by the management's internal control procedures. As such the control risks are related to how well inherent risks are managed (controlled) and will depend on the internal control system including application controls, IT controls and organisational controls, to name a few. Control risks can be evaluated by means of **system audits** - detailed tests of controls and reporting, which are intended to provide evidence about the effectiveness of the design and operation of a control system in preventing or detecting material errors and about the organisation's ability to record, process, summarize and report data.

The product of inherent and control risk (i.e. $IR \times CR$) is referred to as the **risk of material error**. The risk of material error is related to the result of the **system audits**.

- DR, detection risk, is the perceived level of risk that a material error in the statements of expenditure submitted to the Commission, or underlying levels of aggregation, will not be detected by the auditor. Detection risks are related to how adequately the audits are performed, including sampling methodology, competence of staff, audit techniques, audit tools, etc. Detection risks are related to performing audits of operations. This includes substantive tests of details or transactions relating to operations in a programme, usually based on sampling of operations.

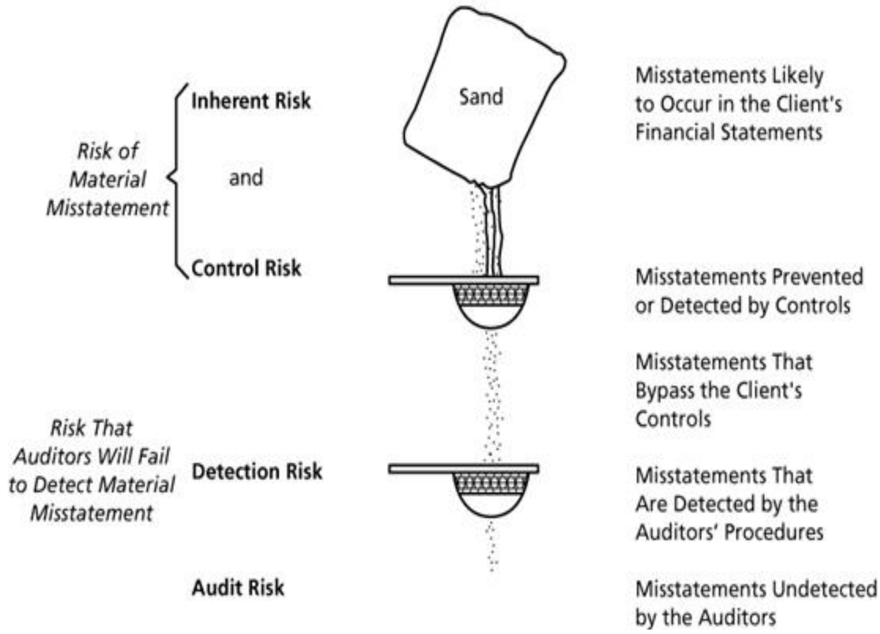

Fig. 2 Illustration of audit risk (adapted from an unknown source)

The assurance model is the opposite of the risk model. If the audit risk is considered to be 5%, the audit assurance is considered to be 95%.

The use of the audit risk/audit assurance model relates to the planning and the underlying resource allocation for a particular operational programme or several operational programmes and has two purposes:

- Providing a high level of assurance: assurance is provided at a certain level, e.g. for 95% assurance, audit risk is then 5%.
- Performing efficient audits: with a given assurance level of for example 95%, the auditor should develop audit procedures taking into consideration the *IR* and *CR*. This allows the audit team to reduce audit effort in some areas and to focus on the more risky areas to be audited.

Note that the setting of the detection, which in turn controls the sample size for the sampling of operations, is a straightforward result, provided that the *IR* and the *CR* have been previously assessed. In fact,

$$AR = IR \times CR \times DR \Longrightarrow DR = \frac{AR}{IR \times CR}$$

where the AR is usually set to 5%, IR and CR are assessed by the auditor.

Illustration

Low control assurance: Given a desired, and accepted audit risk of 5%, and if inherent risk (=100%) and control risk (= 50%) are high, meaning it is a high risk entity where internal control procedures are not adequate to manage risks, the auditor should strive for a very low detection risk at 10%. In order to obtain a low detection risk the amount of substantive testing and therefore sample size need to be large.

$$DR = \frac{AR}{IR \times CR} = \frac{0{,}05}{1 \times 0{,}5} = 0{,}1$$

High control assurance: In a different context, where inherent risk is high (100%) but where adequate controls are in place, one can assess the control risk as 12.5%. To achieve a 5% audit risk level, the detection risk level can be at 40%, the latter meaning that the auditor can take more risks by reducing the sample size. At the end, this will mean a less detailed and a less costly audit.

$$DR = \frac{AR}{IR \times CR} = \frac{0{,}05}{1 \times 0{,}125} = 0{,}4$$

Note that both examples result in the same achieved audit risk of 5% within different environments.

To plan the audit work, a sequence should be applied in which the different risk levels are assessed. First, the inherent risk needs to be assessed and, in relation to this, control risk needs to be reviewed. Based on these two factors, the detection risk can be set by the audit team and will involve the choice of audit procedures to be used during the detailed tests.

However, the audit risk model provides a framework for reflection on how to construct an audit plan and allocate resources, in practice it may be difficult to quantify precisely inherent risk and control risk.

Assurance/confidence levels for the audit of operations depend mainly on the quality of the system of internal controls. Auditors evaluate risk components based on knowledge and experience using terms such as LOW, MODERATE/AVERAGE or HIGH rather than using precise probabilities. If major weaknesses are identified during the systems audit, the control risk is high and the assurance level obtained from the system would be low. If no major weaknesses exist, the control risk is low and if the inherent risk is also low, the assurance level obtained from the system would be high.

As previously indicated, if major weaknesses are identified during the systems audit, one can say that the risk of material error is high (control risks in combination with inherent risks) and as such the assurance level given by the system would be low. Annex IV of the Regulations indicates that if the assurance level obtained from the system is low the confidence level to be applied for sampling of operation would be not less than 90%.

If no major weaknesses in the systems exist the risk of material errors is low, and the assurance level given by the system would be high meaning that the confidence level to be applied for sampling of operations would be not less than 60%.

Section 3.2 provides a detailed framework for choosing the assurance/confidence level for the audit of operations.

3.2 Assurance/confidence level for the audit of operations

3.2.1 Introduction

Substantive tests should be performed on samples, the size of which will depend on a confidence level determined according to the assurance level obtained from the system audit, i.e.

- not less than 60% if assurance is high;
- average assurance (no percentage corresponding to this assurance level is specified in the Commission Regulation although a 70% to 80% of assurance is advised);
- not less than 90% if assurance is low.

The audit authority should establish criteria used for system audits in order to determine the reliability of the management and control systems. These criteria should include a quantified assessment of all key elements of the systems (key requirements) and encompass the main authorities and intermediate bodies participating in the management and control of the operational programme.

The Commission has developed a guidance note on the methodology for the evaluation of the management and control systems[1]. It is applicable both to mainstream and ETC programmes. It is recommended that the AA takes account of this methodology.

In this methodology, four reliability levels are foreseen:
- Works well. No, or only minor improvements are needed;
- Works. Some improvement(s) needed;
- Works partially. Substantial improvements needed;
- Essentially does not work.

The confidence level for sampling is determined according to the reliability level obtained from the system audits.

[1] COCOF 08/0019/01-EN of 06/06/2008; EGESIF_14-0010 of 18/12/2014.

One could consider three levels of assurance on systems: high, average and low. The average level effectively corresponds to the second and third categories of the methodology for evaluation of the management and control systems, which provide a more refined differentiation between the two extremes of high/"works well" and low/"does not work".

The recommended relationship is shown in the table below:

Assurance level from the system audits	**Related reliability in the Regulation/assurance from the system**	**Confidence level**	**Detection Risk**
1. Works well. No, or only minor improvement(s) needed.	High	Not less than 60%	Less or equal to 40%
2. Works. Some improvement(s) are needed.	Average	70%	30%
3. Works partially. Substantial improvements needed.	Average	80%	20%
4. Essentially does not work.	Low	Not below 90%	Not greater than 10%

Table 1. Confidence level for the audit of operations according to the assurance from the system

It is expected that at the beginning of the programming period, the assurance level is low as no or only a limited number of system audits will have taken place. The confidence level to be used would therefore be not less than 90%. However, if the systems remain unchanged from the previous programming period and there is reliable audit evidence on the assurance they provide, the Member State could use another confidence level (between 60% and 90%). The confidence level can also be reduced during a programming period if no material errors are found or there is evidence that the systems have been improved over time. The methodology applied for determining this confidence level will have to be explained in the audit strategy and the audit evidence used to determine the confidence level will have to be mentioned.

Setting an appropriate confidence level is a critical issue for the auditing of operations, as sample size is strongly dependent on this level (the higher the confidence level the larger the sample size). Therefore the regulations offer the possibility of reducing the confidence level and consequently audit workload for systems with a low error rate (therefore high assurance), while maintaining the requirement of a high confidence level (consequently larger sample size) in the case of a systems that has a potentially high error rate (therefore low assurance).

The AA are encouraged to actively use sampling parameters that correspond to the reality of the functioning of systems, avoiding oversize audit samples and respective workload, provided adequate precision is ensured.

3.2.2 Determination of the applicable assurance level when grouping programmes

The audit authority should apply **one** assurance level in the case of grouping of programmes.

In case the system audits reveal that within the group of programmes there are differences in the conclusions on the functioning of the various programmes, the following options are available:

- to create two (or more) groups, for example the first for programmes with a low level of assurance (confidence level of 90%), the second group for programmes with a high level of assurance (a confidence level of 60%), etc. The two groups are treated as two different populations. Consequently the number of controls to be performed will be higher, as a sample from each separate group will have to be taken;
- to apply the lowest assurance level obtained at the individual programme level for the whole group of programmes. The group of programmes is treated as one single population. In this case, audit conclusions will be drawn to the whole group of programmes. Consequently, conclusions about each individual program will not usually be possible.

In the latter case, it is possible to use a sampling design stratified by programme, which will usually allow a smaller sample size. Nevertheless, even when using stratification a single assurance level have to be used and conclusions are still only possible for the whole group of programmes. See Section 7.8 for a more detailed presentation of strategies for auditing groups of programmes and multi-fund programmes.

4 Statistical concepts related to audits of operations

4.1 Sampling method

The sampling method encompasses two elements: the sampling design (e.g. equal probability, probability proportional to size) and the projection (estimation) procedure. Together, these two elements provide the framework to calculate sample size.

The most well know sampling methods suitable for the audit of operations are presented in Section 5.1. Please note that the first distinction between sampling methods is made between statistical and non-statistical sampling.

A statistical sampling method has the following characteristics:

- each item in the population has a known and positive selection probability;
- randomness should be ensured by using proper random number generating software, specialised or not (e.g. MS Excel provides random numbers);
- sample size is calculated in such a way that allow to achieve a certain level of desirable precision.

In a similar way, Article 28(4) of Regulation (EU) No 480/2014 refers that, "for the purpose of application of Article 127(1) of Regulation (EU) No 1303/2013, a sampling method is statistical when it ensures: (i) a random selection of the sample items; (ii) the use of probability theory to evaluate sample results, including measurement and control of the sampling risk and of the planned and achieved precision.

Statistical sampling methods allow the selection of a sample that is “representing” the population (reason why statistical selection is so important). The final goal is to project (extrapolate or estimate) to the population, the value of a parameter (the “variable”) observed in a sample, allowing to conclude whether a population is materially misstated or not and, if so, by how much (an error amount).

Non-statistical sampling does not allow the calculation of precision, consequently there is no control of the audit risk and it is impossible to ensure that the sample is representing the population. Therefore, the error has to be assessed empirically.

In the programming period 2007-2013 statistical sampling is required by Council Regulations (EC) No 1083/2006 and No 1198/2006 and Commission Regulations (EC) No 1828/2006 and No 498/2007 for substantive tests (audit of operations). In the programming period 2014-2020 the relevant requirement concerning statistical sampling methods is included in Article 127(1) CPR and in Article 28 CDR. Non-statistical selection is considered appropriate for cases where statistical selection is impossible, e.g. associated to very small populations or sample sizes (cf. section 6.4).

4.2 Selection method

The selection method can belong to one of two broad categories:

- Statistical selection, or
- Non-statistical selection.

Statistical selection includes two possible techniques:

- Random selection;
- Systematic selection.

In random selection random, numbers are generated for each population unit in order to select the units constituting the sample.

Systematic sampling uses a random starting point and then applies a systematic rule to select the additional items (e.g. each 20th item after the random starting point).

Usually the equal probability methods are based on random selection and MUS is based on systematic selection.

Non-statistical selection covers the following possibilities (among others):

- Haphazard selection
- Block selection
- Judgement selection
- Risk based sampling combining elements of the three possibilities above

Haphazard selection is “false random” selection, in the sense of an individual “randomly” selecting the items, implying an unmeasured bias in the selection (e.g. items easier to analyse, items easily assessed, items picked from a list displayed particularly on the screen, etc…).

Block selection is similar to cluster sampling (as of groups of population units), where the cluster is picked non-randomly.

Judgment selection is purely based on the auditor’s discretion, whatever the rationale (e.g. items with similar names, all operations related to a specific domain of research, etc…).

Risk-based sampling is a non-statistical selection of items based on various intentional elements, often taking from all three non-statistical selection methods.

4.3 Projection (estimation)

As stated before the final goal when applying a sampling method is to project (extrapolate or estimate) the level of error (misstatement) observed in the sample to the whole population. This process will allow to conclude whether a population is materially misstated or not and, if so, by how much (an error amount). Therefore, the level of error found in the sample is not of interest by itself[2], being merely instrumental, i.e. a mean through which the error is projected to the population.

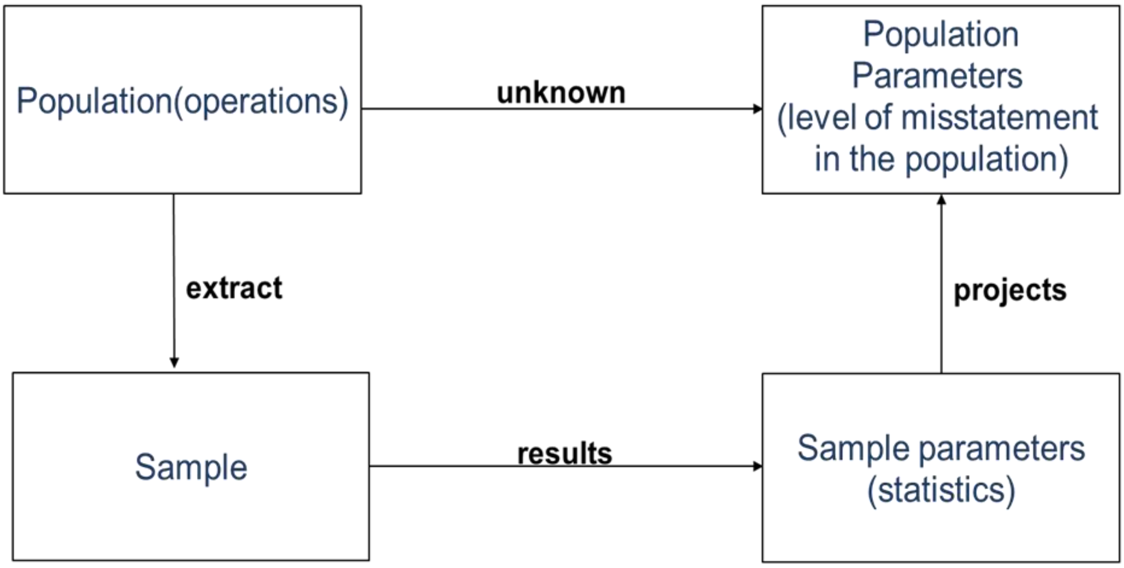

Fig. 3 Sample selection and projection

Sample statistics used to project the error to the population are called estimators. The act of projection is called estimation and the value calculated from the sample (projected value) is called the estimate. Clearly, this estimate, only based on a fraction of the population, is affected by an error called the sampling error.

4.4 Precision (sampling error)

This is the error that arises because we are not observing the whole population. In fact, sampling always implies an estimation (extrapolation) error as we rely on sample data to extrapolate to the whole population. Sampling error is an indication of the difference between the sample projection (estimate) and the true (unknown) population parameter (value of error). It represents, in fact, the uncertainty in the projection of results to the population. A measure of this error is usually called **precision** or accuracy of the estimation. It depends mainly on **sample size**, **population variability** and in smaller degree **population size**.

[2] Even though individual errors found in the sample need to be appropriately corrected.

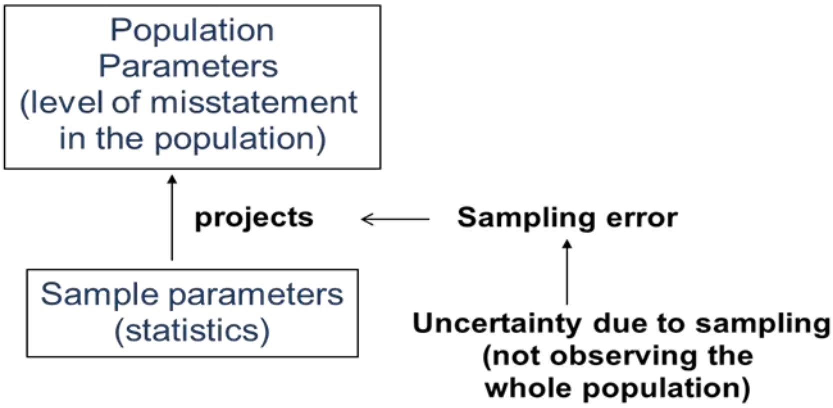

Fig. 4 Sampling error

A distinction should be made between planned precision and effective precision (SE in the formulas presented in Section 6). While planned precision is the maximum planned sampling error for sample size determination (usually is the difference between maximum tolerable error and the anticipated error and it should be set to a value lower than the materiality level), the effective precision is an indication of the difference between the sample projection (estimate) and the true (unknown) population parameter (value of error) and represents the uncertainty in the projection of results to the population.

4.5 Population

The population for sampling purposes includes the expenditure declared to the Commission for operations within a programme or group of programmes in the reference period, except for negative sampling units as explained below in section 4.6. All operations included in that expenditure should be comprised in the sampled population, except where the proportional control arrangements set out by Article 148(1) CPR and Article 28(8) of the Delegated Regulation (EU) No 480/2014 apply in the context of the sampling carried out for the programming period 2014-2020. The exclusion of operations from the population to be sampled is not possible under the 2007-2013 legal framework[3], except in cases of "force majeure"[4].

The AA may decide to widen the audit to other related expenditure declared by the selected operations and concerning the previous reference period, in order to increase

[3] This means that the following expenditure items should indeed be included in the population from which the random sample is drawn and should not be excluded at the stage of sampling: (i) operations related with financial engineering instruments (FEI); (ii) projects considered "too small"; (iii) projects audited in previous years or projects with a beneficiary audited in previous years; (iv) projects subject to flat rate corrections.

[4] Cf. section 7.6 of the updated Guidance on Treatment of Errors (EGESIF_15-0007-01 of 09/10/2015), relating to the approach the AA should adopt in case supporting documentation of the sampled operations is lost or damaged due to "force majeure" (e.g. natural disasters).

the efficiency of the audits. The results from checking additional expenditure outside the reference period should not be taken into account for determining the total error rate.

In general, all the expenditure declared to the Commission for all the selected operations in the sample should be subject to audit. Nevertheless, whenever the selected operations include a large number of payment claims or invoices, **the AA may apply two-stage sampling**, as explained below in section 7.6.

As a rule, the AA should select its sample from **the total expenditure declared (i.e. public and private expenditure)**, as results from Article 17(3) of Regulation (EC) No 1828/2006[5] and Article 127(1) CPR. In any case, the audits of operations should verify the total expenditure declared, as follows from Articles 16(2) and 17(4) of the Regulation (EC) No 1828/2006[6] and Article 27(2) CDR. However, it has occurred that an AA selects the sample from public expenditure declared, under the argument that the Fund contribution is paid on this basis. This practice may result from an erroneous interpretation by the Certifying Authority, leading to the fact that expenditure claims submitted to the Commission only include the public expenditure, while the correct approach is that the CA should declare always the total expenditure even where the co-financing is calculated on the basis of the public expenditure[7].

In this situation and when the AA uses Probability Proportional to Size sampling method (i.e. the MUS for statistical sampling), this may result in two sorts of issues:

a) This process may result in a bias in the sampling results because some sampling units with a comparatively high private contribution had less chance of being selected.

b) The fact that the AA audits the total expenditure based on a sample drawn only from the public expenditure may result in the effective precision being too large.

Concerning point (a) above, where the AA selects the sample based on public expenditure, the AA may consider the need to select a complementary sample from that subpopulation:
- if there are high value sampling units[8] that were not sampled (because of the problem identified above) and
- if there are risks associated with the expenditure declared for those sampling units.

[5] Article 43(3) of Regulation (EC) No. 498/2007

[6] Articles 42(2) and 43(4) of Regulation (EC) No. 498/2007.

[7] This is also required for audit trail purposes, since the expenditure to be audited on the spot at beneficiary's level is the total expenditure declared and not only the public expenditure; usually, the expenditure items are co-financed by public and private funds and in practice the whole expenditure is audited.

[8] A rule of thumb to define what is a "high value item" is when the respective total expenditure declared is higher than the threshold of 2% of total expenditure for the programme.

As for the point (b) above, when the AA projects the errors to the total expenditure and the upper error limit is higher than materiality where the most likely error is below 2%, this indicates a loose precision. This may imply that the sampling results are inconclusive and

- recalculation of the confidence level[9] is necessary or, if not feasible,
- additional sampling is required[10], namely where the effective precision is above two percentage points[11].

Attention is drawn for the fact that, **as general approach, if the effective precision (UEL-MLE) is less than two percentage points, we consider that, in principle and taking into account all elements of information for the programme at stake, there is no need to consider additional work.**

4.6 Negative sampling units

It can happen that there are sampling units (operations or payment claims) that are negative, in particular due to financial corrections applied by national authorities.

In this case, the negative sampling unit should be included in a separate population and should be audited separately[12] with the objective of verifying if the amount corrected corresponds to what has been decided by the Member State or the Commission. If the AA concludes that the amount corrected is less than what was decided, then this matter should be disclosed in the Annual Control Report, in particular when this non-compliance constitutes an indication of weaknesses in the Member State's corrective capacity.

In this context, when calculating the total error rate, the AA only considers the errors found in the population of positive amounts and this is the book value to be considered in both the projection of random errors and in the total error rate. Before calculating the projected error rate, the AA should verify that the errors found are not already corrected in the reference period (i.e. included in the population of negative amounts, as described above). If this is the case, these errors should not be included in the projected error rate.[13]

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[9] Cf. section 7.7 of the present guidance.

[10] Cf. section 7.2.2 of the present guidance.

[11] Cf. the last paragraph of section 7.1 of the present guidance.

[12] Of course, the AA may also draw a sample from such a separate population if it contains too numerous units, leading to a heavy workload.

[13] See also guidance on treatment of errors, which presents other cases justifying that some errors are not included in the total error rate.

Concretely, the AA has to identify, in the total population of sampling units (i.e. operations or payment claims) to be sampled, the ones with a negative balance and audit them as a separate population. Using operation as sampling unit, the process is illustrated as follows (the same reasoning applies to payment claims if they are used as sampling unit):

- Operation X: 100 000 € (no corrections were applied during the reference period);

- Operation Y: 20 000 € => if this amount results from 25 000 € less 5 000 € (due to corrections/deductions applied during the reference period), the AA does not have to consider the 5 000 € in the separate population of negative amounts;

- Operation Z: - 5 000 € (resulting from 10 000 € of new expenditure in the reference period less 15 000 € of correction) => to be included in the separate population of negative amounts;

- Total expenditure declared for the programme (net amount): 115 000 € (= 120 000 – 5 000);

- Population from which the random sample is to be selected: all the operations with positive amounts = X + Y (in the case above, this would be 120 000 €, considering for simplification reasons, that the programme would be constituted by the three operations above-mentioned). Operation Z is to be audited separately.

The approach explained above implies that the AA is not required to identify, as a separate population, the negative amounts within the sampling unit. In most cases, this would not be cost-effective[14]. Thus, in the case of operation Y the AA could include the amount of 5 000 € in the negative population (leading to inclusion of 25 000 € in the positive population) or, as in the example above, include 20 000 € in the positive population. Another approach would be to deduct financial corrections/other negative amounts which refer to current sampling period from the positive population in order to produce the net amount and to include the amount of corrections/other negative amounts related to preceding sampling periods in the population of negative amounts.

In particular, if the operation Y represents a sampling unit in the current sampling period, and the negative amount of 5 000 € deducted in the current sampling period from the expenditure declared includes:
- 4 000 € constituting financial corrections related to expenditure declared in the previous sampling periods,
- 700 € constituting financial correction related to expenditure declared in the current sampling period,

[14] The identification the negative amounts within the sampling unit is even less recommended when applying sub-sampling (or two-stage sampling) as this would imply to identify all negative amounts within all the sampling units of each sub-sample.

- 300 € which corrects a clerical error in view of over-declaration of expenditure in the previous sampling periods,
the AA could include 24 300 € (= 25 000 € – 700 €) in the positive population, whereas the amount of 4 300 € (representing financial corrections/artificial negative sampling units which relate to the previous sampling periods) in the negative population.

In summary, there are three approaches concerning separation between positive and negative sampling units:

1) Negative amounts are included in the positive population if the sum of negative and positive amounts within the sampling unit is positive.
2) All positive amounts are included in the positive population and all negative amounts are included in the negative population.
3) Negative amounts related to the previous sampling periods (such as corrections of amounts declared in previous years) are included in the negative population, whereas negative amounts correcting/adjusting the positive amounts in the positive population of the current sampling period are included in the positive population.

In the Comission's view, options 2 and 3 are recommended. Option 1 is acceptable but may involve the risk that operations or payment claims subject to corrections in the reference period concerning the expenditure declared in previous years have less chances of being sampled/selected.

Where the IT systems in the MS are set-up in such a way that provide the data on negative amounts within the sampling unit, it is up to the AA to consider whether applying this level of detail to the sampling approach is necessary, in order to mitigate the risk identified above.

If the AA considers that, due to the above methodology, the risk mentioned above **should be disclosed in the ACR**. This risk can be assessed when auditing the negative amounts and the conclusion is that there are a significant number of items with positive expenditure included in the negative sampling units. Based on its professional judgement, the AA should assess if a complementary sample (of that positive expenditure) is necessary in order to mitigate such risk.

For the purposes of the "Table for declared expenditure and sample audits" included in the ACR, the AA should present in the column "Expenditure declared in reference period" the population of positive amounts. The AA should present in the ACR a reconciliation of the expenditure declared (net amount) with the population from which the random sample of positive amounts was drawn.

The artificial negative sampling units (clerical errors, reversal entries in the accounts not corresponding to financial corrections, revenues of revenue-generating projects and transfer of operations from one programme to another (or within a programme) unrelated with irregularities detected in that operation) should not be excluded from the sampling procedures. The AA could opt to give them similar treatment as in the case of financial corrections and include them in the negative population. Alternatively, a sample of such units could be selected from a specific population of artificial negative sampling units. The CA should record the nature of the negative sampling units (in particular, allowing the distinction between financial corrections resulting from irregularities and artificial negative sampling units) on a regular basis for the purposes of ensuring that only financial corrections are included in the annual reporting on withdrawals and recoveries under Article 20 of Regulation (EC) No 1828/2006 (for 2014-2020, this reporting is included in the accounts). Therefore, the audit of the negative sampling units should include verification of correctness of such recording for the selected units.

It should be noted that it is not expected that the AA calculates an error rate based on results of the audit of negative sampling units. However, it is recommended that the negative sampling units are selected at random. Financial corrections derived from irregularities detected by the AA or the EC that are constantly monitored by the AA could be excluded from the random sample on negative units. If the AA considers that in view of specific problems it would prefer to opt for a risk-based approach, it is recommended to apply a mixed approach with at least a part of negative sampling units selected at random.

The audit of negative sampling units can be included in the audit of accounts for the programming period 2014-2020.

4.7 Stratification

Stratification is when the population is divided in sub-populations called strata and independent samples are drawn from each stratum.

The main goal of stratification is two-folded: on one hand usually allows an improvement of precision (for the same sample size) or a reduction of sample size (for the same level of precision); on the other hand ensures that the subpopulations corresponding to each stratum are represented in the sample.

Whenever we expect that the level of error (misstatement) will be different for different groups in the population (e.g. by programme, region, intermediate body, risk of the operation) this classification is a good candidate to implement stratification.

Different sampling methods can be applied to different strata. For example, it is common to apply a 100% audit of the high-value items and apply a statistical sampling method to audit a sample of the remaining lower-value items that are included in the additional stratum or strata. This is useful in the event that the population include a few quite high-value items, as it lowers the variability in each stratum and therefore allows an improvement of precision (or reduction of sample size).

4.8 Sampling unit

In the programming period 2014-2020 determination of the sampling unit is regulated by Commission Delegated Regulation No 480/2013. In particular, Article 28 of this Regulation stipulates:
"The sampling unit shall be determined by the audit authority, based on professional judgement. The sampling unit may be an operation, a project within an operation or a payment claim by a beneficiary…"

Where the AA decided to use an operation as a sampling unit and the number of operations for a reference period is insufficient to allow the use of a statistical method (this threshold is between 50 and 150 population units), application of payment claim as the sampling units could help by increasing the population size to the threshold enabling the use of a statistical sampling method.

In view of the legal framework foreseen for the programming period 2014-2020, the AA may also opt to use either operations (projects) or the beneficiary's payment claims as the sampling unit in the programming period 2007-2013.

4.9 Materiality

A materiality level of 2% maximum is applicable to the expenditure declared to the Commission in the reference period (positive population). The AA can consider reducing the materiality for planning purposes (tolerable error). The materiality is used:

- As a threshold to compare the projected error in expenditure
- To define the tolerable/acceptable error that is used for determining sample size

4.10 Tolerable error and planned precision

The tolerable error is the maximum acceptable error rate that can be found in the population for a certain reference period. With a 2% materiality level this maximum tolerable error is therefore 2% of the expenditure declared to the Commission for that reference period.

The planned precision is the maximum sampling error accepted for the projection of errors in a certain reference period, i.e. the maximum deviation between the true population error and the projection produced from sample data. It should be set by the auditor to a value lower the tolerable error, because otherwise the results of sampling of operations will have a high risk of being inconclusive and a complementary or additional sample may be needed.

For example, for a population with total book value of 10,000,000 € the corresponding tolerable error is 200,000 € (2% of the total book value). If the projected error is 5,000 € and the auditor sets the precision exactly to 200,000 € (this error arises because the auditor is only observing a small part of the population, i.e. the sample), then the upper error limit (upper limit of the confidence interval) will be about 205,000€. This is an inconclusive result as we have a very small projected error but an upper limit that exceeds the materiality threshold.

The most adequate way to settle the planned precision is to calculate it equal to the difference between the tolerable error and the anticipated error (the projected error that the auditor expects to obtain at the end of the audit). This anticipated error will of course be based on the auditor professional judgment, supported by the evidence gathered in the auditing activities in previous years for the same of similar population or in preliminary/pilot sample.

Note that the choice of a realistic anticipated error is important, since the sample size is highly dependent on the value chosen for this error. See also section 7.1.

Section 6 presents detailed formulas to use in the sample size determination process.

4.11 Variability

The variability of the population is a very influential parameter on sample size. Variability is usually measured by a parameter known as standard-deviation[15] and usually represented by σ. For example, for a population of 100 operations where all operations have the same level of error of € 1,000,000 € (average error of μ = 1,000,000 €) there is no variability (indeed, the standard-deviation of errors is zero). On the other hand, for a population of 100 operation in which 50 share an error of 0€ and the remaining 50 share an error of 2,000,000 € (the same average error of μ = 1,000,000 €) the standard-deviation of errors is high (1,000,000€).

The sample size needed to audit a population of low variability is smaller than the one needed for a population of high variability. In the extreme case of the first example (with a variance of 0), a sample size of one operation would be sufficient to project the population error accurately.

[15] The standard deviation is a measure of the variability of the population around its mean. It can be calculated using errors or book-values. When calculated over the population is usually represented by σ and when calculated over the sample is represented by s. The larger the standard deviation the more heterogeneous is the population (or the sample). The variance is the square of the standard deviation.

The standard-deviation (s) is the most common measure of variability as it is more easily understandable than variance (s^2). Indeed the standard-deviation is expressed in the units of the variable for which we seek to measure variability. On the contrary, the variance is expressed in the square of the units of the variable for which variability we measure and it is a simple average of the squares of the variable deviance values around the mean[16]:

$$Variance: s^2 = \frac{1}{\#\,of\,units} \sum_{i=1}^{\#\,of\,units} (V_i - \bar{V})^2$$

where V_i represents the individual values of the variable V and $\bar{V} = \frac{\sum_{i=1}^{\#\,of\,units} V_i}{\#\,of\,units}$ represents the mean error.

The standard deviation is simply the square-root of the variance:

$$s = \sqrt{s^2}$$

The standard deviation of the errors of the examples mentioned at the beginning of this section can be calculated as:

a) Case 1
   a. N=100
   b. All the operation have the same level of error of € 1,000,000 €
   c. Mean error
   $$\frac{\sum_{i=1}^{100} 1{,}000{,}000}{100} = \frac{100 \times 1{,}000{,}000}{100} = 1{,}000{,}000$$
   d. Standard deviation of errors
   $$s = \sqrt{\frac{1}{100}\sum_{i=1}^{100}(1{,}000{,}000 - 1{,}000{,}000)^2} = 0$$
b) Case 2
   a. N=100
   b. 50 operations have 0 of error and 50 operations have 2,000,000 € of error
   c. Mean error
   $$\frac{\sum_{i=1}^{50} 0 + \sum_{i=1}^{50} 2{,}000{,}000}{100} = \frac{50 \times 2{,}000{,}000}{100} = 1{,}000{,}000$$
   d. Standard deviation of errors

[16] Whenever the variance is calculated with sample data it should include the alternative formula $s^2 = \frac{1}{\#\,of\,units-1}\sum_{i=1}^{\#\,of\,units}(V_i - \bar{V})^2$ which should be used in order to compensate for the degree of freedom lost in the estimation.

$$s=\sqrt{\frac{1}{100}\left(\sum_{i=1}^{50}(0-1{,}000{,}000)^2+\sum_{i=1}^{50}(2{,}000{,}000-1{,}000{,}000)^2\right)}$$

$$=\sqrt{\frac{50\times 1{,}000{,}000^2+50\times 1{,}000{,}000^2}{100}}$$

$$=\sqrt{1{,}000{,}000^2}=1{,}000{,}000$$

4.12 Confidence interval and Upper Limit of Error

The confidence interval is the interval that contains the true (unknown) population value (error) with a certain probability (called confidence level). The confidence interval's general formula is as follows:

$$[EE-SE;EE+SE]$$

where

- EE represents the projected or extrapolated error; also corresponds to the Most Likely Error (MLE) in the MUS terminology;
- SE represents the precision (sampling error);

The projected/extrapolated error (EE) and the Upper Limit of Error (EE+SE) are the two most important instruments to conclude whether a population of operations is materially misstated or not[17]. Of course, the ULE can only be calculated when statistical sampling is used; hence, for non-statistical sampling the EE is always the best estimate of the error in the population.

When statistical sampling is used, the following situations can arise:

- If EE is larger than the materiality threshold (hereafter 2%, for simplification) , then the AA concludes that there is material error;
- If EE is lower than 2% and the ULE is lower than 2%, the AA concludes that the population is not misstated by more than 2% at the specified level of sampling risk.
- If EE is lower than 2% but the ULE is larger than 2%, the AA concludes that additional work is needed. Accordingly to the INTOSAI guideline n° 23[18], the additional work can include:

[17] Statistical methods allow also to calculate the lower limit of error, which is of less importance for evaluation of results. That is why other statistical models may focus more specifically on the projected (most likely error) and on the upper limit of error.
[18] See http://www.eca.europa.eu/Lists/ECADocuments/GUIDELINES/GUIDELINES_EN.PDF

- *"requesting the audited entity to investigate the errors/exceptions found and the potential for further errors/exceptions. This may lead to agreed adjustments in the financial statements;*
- *carrying out further testing with a view to reducing the sampling risk and thus the allowance that has to be built into the evaluation of results;*
- *using alternative audit procedures to obtain additional assurance."*

The AA should use its professional judgment to select one of the options indicated above and report accordingly in the ACR.

Attention is drawn for the fact that, in most cases where an ULE is well above 2% this could be prevented or minimized if the AA considers a realistic anticipated error when calculating the original sample size (see sections 7.1 and 7.2.2 below, for more details).

When following the third option (projected error is lower than 2% but the ULE is higher than 2%), in some cases, the AA may find that the results are still conclusive for a smaller confidence level than the planned one. **When this recalculated confidence level is still compatible with an assessment of the quality of the management and control systems, it would be safe to conclude that the population is not materially misstated even without carrying out additional audit work.** See Section 7.7 for an explanation of the recalculation of confidence levels.

4.13 Confidence level

The confidence level is set by the Regulation for the purpose of defining the sample size for substantive tests.

As the sample size is directly affected by the confidence level, the objective of the Regulation is clearly to offer the possibility of reducing audit workload for systems with an established low error rate (and therefore high assurance), while maintaining the requirement to check a high number of items in the case a system has a potentially high error rate (and therefore low assurance).

The easiest way to interpret the meaning of confidence level is the probability that a confidence interval produced by sample data contains the true population error (unknown). For example, if the error in the population is projected to be 6,000,000€ and the 90% confidence level interval is

$$[5{,}000{,}000€; 7{,}000{,}000€],$$

it means that there is 90% probability of the true (but unknown) population error is between these two bounds. The implications of these strategic choices for the audit planning and sampling of operations are explained in the following chapters.

4.14 Error rate

The **sample error rate** is computed as the ratio between total error in the sample and total book value of the sampled items, the **projected error rate** is computed as the ratio between **projected population error** and total book value. Again, note that the sample error is of no interest by itself as it should be considered a mere instrument to calculate the projected error[19].

5 Sampling techniques for the audit of operations

5.1 Overview

Within the audit of operations, the purpose of sampling is to select the operations to be audited through substantive tests; the population comprises the expenditure declared to the Commission for operations within a programme/group of programmes in the reference period.

Figure 5 shows a summary of the most used sampling methods for audit.

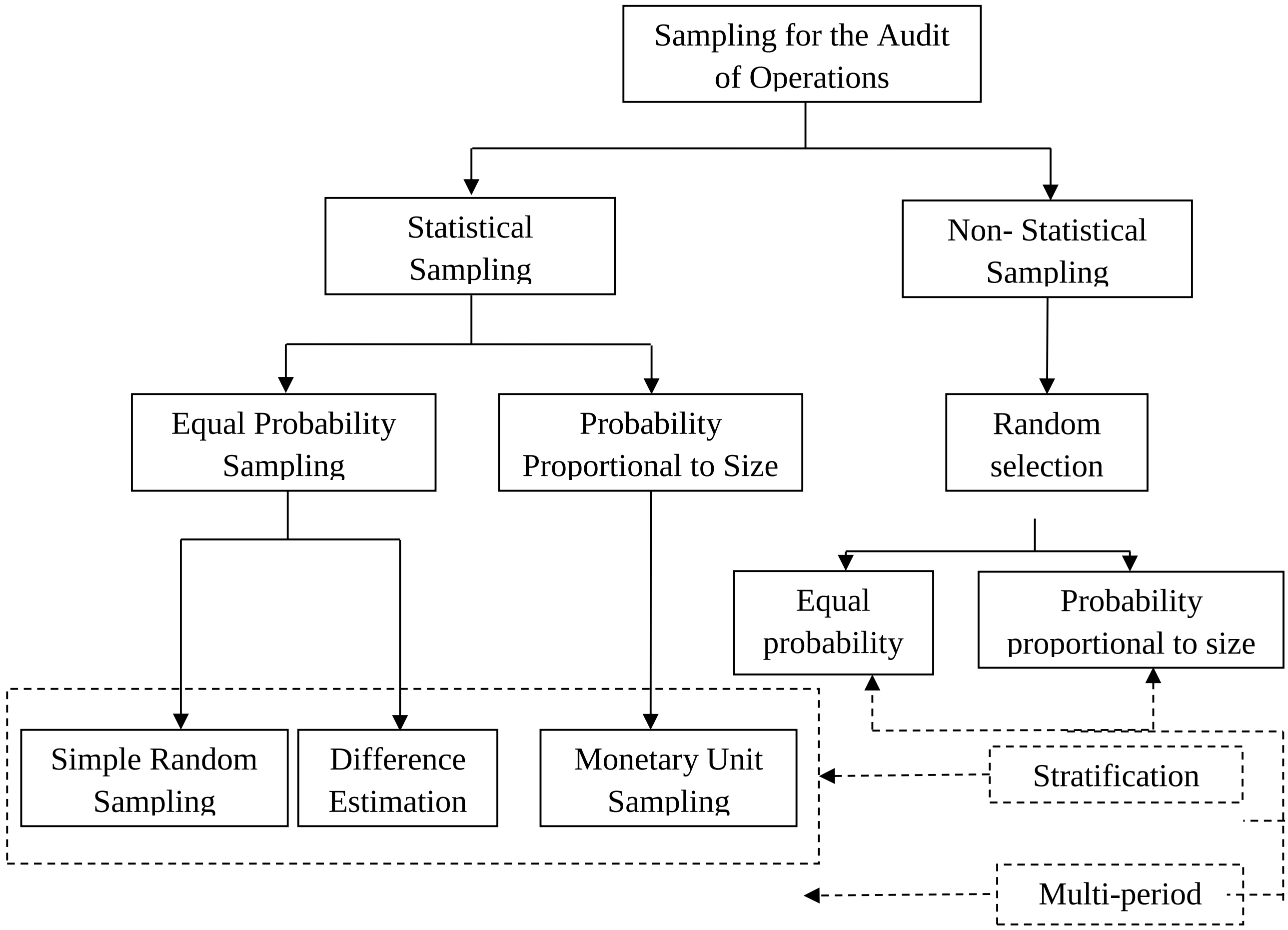


Fig. 5 Sampling methods for the audit of operations

[19] In some sampling methods, namely the ones based on equal probability selection, the sample error rate can be used to project the population error rate.

As stated before, please note that the first distinction between sampling methods is made between statistical and non-statistical sampling.

Section 5.2 presents the conditions of applicability of the different sampling designs and refers the unique extreme situations where non-statistical sampling is admissible.

Within statistical sampling, the major distinction between methods is based on the selection probabilities: equal-selection probabilities methods (including simple random sampling and difference estimation) and probability proportional to size methods where the well-known monetary unit sampling (MUS) method stands out.

Monetary unit sampling (MUS) is in fact a probability-proportional-to-size (PPS). The name comes from the fact that operations are selected with probabilities proportional to their monetary value. The higher the monetary value the higher the probability of selection. Again, favourable conditions for the application of each specific method are discussed in the following section.

Despite the specific sampling method that is selected, auditing the operations through sampling should always follow a basic common structure:

1. **Define the objectives of the substantive tests**: usually the determination of the level of error in the expenditure declared to the Commission for a given year for a programme (or group of programmes) based on a projection from a sample.
2. **Define the population**: expenditure declared to the Commission for a given year for a programme or for a group of programmes, and the **sampling unit**, which is the item to be selected to the sample (usually the operation, but other possibilities are available as the payment claim).
3. **Define population parameters**: this included defining the tolerable error (2% of the expenditure declared to the Commission), the anticipated error (expected by the auditor), the confidence level (taking into account the audit risk model) and (usually) a measure of population variability.
4. **Determine the sample size**, according to the sampling method used. It is important to note that the final sample size is always rounded up to the nearest integer.[20]
5. **Select the sample and perform the audit**.
6. **Project results, calculate precision and draw conclusion**: this step covers the computation of the precision and projected error and comparing these results with the materiality threshold.

[20] In case the sample size is calculated for different strata and periods, it is acceptable that the sample sizes for some strata/periods are not rounded up provided that the general sample size is rounded up.

The choice of a particular sampling method refines this archetypal structure, by providing a formula to compute the sample size and a framework for projecting results.

Also note that the specific formulas for sample size determination vary with the chosen sampling method. Nevertheless, despite the chosen method, the sample size will depend on three parameters:

- The confidence level (the higher the confidence level the larger the sample size)
- The variability of the population[21] (i.e. how variable are the values of the population; if all the operations in the population have similar values of error the population is said to be less variable than a population where all the operations show extremely different values of error). The higher the variability of the population the larger the sample size.
- The planned precision set by the auditor; this planned precision is typically the difference between the tolerable error of 2% of the expenditure and the anticipated error. Assuming an anticipated error below 2%, the larger the anticipated error (or the smaller the planned precision) the larger the sample size.

Specific formulas for determining sample size are offered in Section 6. Nevertheless, one important rule of the thumb is never to use a sample size smaller than 30 units (in order that the distributional assumptions used to create confidence intervals will hold).

5.2 Conditions of applicability of sampling designs

As a preliminary remark on the choice of a method to select the operations to be audited, whilst the criteria that should lead to this decision are numerous, from a statistical point of view the choice is mainly based on the expectation regarding the variability of errors and their relationship with the expenditure.

The table below gives some indications on the most appropriate methods depending on the criteria.

[21] The calculation of the sample size.in MUS conservative does not depend on any parameters related to the variability for the population..

Sampling Method	Favourable conditions
Standard MUS	Errors have high variability[22] and are approximately proportional to the level of expenditure (i.e. error rates are of low variability) The values of expenditure per operation show high variability
Conservative MUS	Errors have high variability and are approximately proportional to the level of expenditure The values of expenditure per operation show high variability Proportion of errors is expected to be low[23] Anticipated error rate has to be smaller than 2%
Difference estimation	Errors are relatively constant or of low variability An estimate of the total corrected expenditure in the population is needed
Simple random sampling	General proposed method that can be applied when the previous conditions do not hold Can be applied using mean-per-unit estimation or ratio estimation (see Section 6.1.1.3 for guidelines for choosing between these two estimation techniques)
Non-statistical methods	If the application of statistical method is impossible (see discussion below)
Stratification	Can be used in combination with any of the above methods It is particularly useful whenever the level of error is expected to vary significantly among population groups (subpopulations)

Table 2. Favourable conditions for the choice of sampling methods

Although the previous advices should be followed, actually no method can be universally classified as the only suited method or even the “best method”. In general, all methods can be applied. The consequence of choosing a method that is not the most suitable for a certain situation is that the sample size will have to be larger than the one obtained when using a more appropriate method. Nevertheless, it will always be possible to select a representative sample trough any of the methods, provided that an adequate sample size is considered.

[22] High variability means the errors across operations are not similar, that is, there are small and large errors in contrast with the case where all the errors are more or less of similar values (cf. section 4.11).

[23] As the MUS conservative approach is based on a distribution for rare events, is it particularly suited when the ratio of number of errors to the total number of operations in the population (proportion of errors) is expected to be low.

Also note that stratification can be used in combination with any sampling method. The reasoning underlying stratification is the partition of the population in groups (strata) more homogeneous (with less variability) than the whole population. Instead of having a population with high variability it is possible to have two or more subpopulations with lower variability. Stratification should be used to either **minimise variability or isolate error-generating subsets of the population**. In both cases stratification will reduce the needed sample size.

As stated before, statistical sampling should be used to draw conclusions about the amount of error in a population. However, there are special justified cases where a non-statistical sampling method may be used on the professional judgement of the audit authority, in accordance with internationally accepted audit standards.

In practice, the specific situations that may justify the use of non-statistical sampling are related to the population size. In fact, it may happen to work with a very small population, whose size is insufficient to allow the use of statistical methods (the population is smaller or very close to the recommended sample size) [24].

The audit authority must use all possible means to achieve a sufficiently large population: by grouping programs, when part of a common system; and/or by using as the unit the beneficiaries' periodic payment claims. AA should also consider that even in an extreme situation where the statistical approach is not possible in the beginning of the program period, it should be applied as soon as it is feasible.

5.3 Notation

Before presenting the main sampling methods for audit of operations it is useful to define a set of concepts related to sampling that are common to all the methods. Thus:

- *z* is a parameter from the normal distribution related to the confidence level determined from system audits. The possible values of z are presented in the following table. A complete table with values of the normal distribution can be found in appendix 3.

Confidence level	**60%**	**70%**	**80%**	**90%**	**95%**
System assurance level	High	Moderate	Moderate	Low	No assurance
z	0.842	1.036	1.282	1.645	1.960

Table 3. Values of *z* by confidence level

[24] Cf. section 6.4.1.

- N is the population size (e.g. number of operations in a programme or payment claims); if the population is stratified, an index h is used to denote the respective stratum, $N_h, h = 1,2, \dots, H$ and H is the number of strata;
- n is the sample size; if the population is stratified, an index h is used to denote the respective stratum, $n_h, h = 1,2, \dots, H$ and H is the number of strata;
- TE be the maximum tolerable error admissible by the regulation, that is, 2% of the total expenditure declared to the Commission (the Book Value, BV);
- $BV_i, i = 1,2, \dots, N$ is the book value (the expenditure declared to the Commission) of an item (operation/payment claim);
- $CBV_i, i = 1,2, \dots, N$ is the corrected book value, the expenditure determined after auditing procedures of an item (operation/payment claim);
- $E_i = BV_i - CBV_i, i = 1,2, \dots, N$, is the amount of error of an item and is defined as the difference between the book value of the i-th item included in sample and the respective corrected book value; if the population is stratified an index h is used to denote the respective stratum, $E_{hi} = BV_{hi} - CBV_{hi}, i = 1,2, \dots, N_h, h = 1,2, \dots, H$ and H is the number of strata;
- AE is the anticipated error defined by the auditor based on the expected level of error at the level of the operations (e.g. an anticipated error rate times the Total expenditure at the level of the population). AE can be obtained from historical data (projected error in past period) or from a preliminary/pilot sample of low sample size (the same used to determine the standard deviation).

The above mentioned parameters are often accompanied in the guidance by specific subscripts which could relate to the character of the parameter or a stratum that the parameter refers to. In particular:

- *r* is used with standard deviation when it refers to standard deviation of error rates;
- *e* refers to exhaustive stratum/high value stratum; if used with standard deviation this notation could also refer to standard deviation of errors (as opposed to standard deviation of error rates);
- *w* is used with standard deviation when a weighted value is used;
- *s* refers to a non-exhaustive stratum;
- t is used with stratified two- or multi-period sampling formulas to refer to particular periods;
- q is used with standard deviation to refer to the variable q in simple random sampling (ratio estimation)
- h refers to a stratum.

If a parameter is accompanied by several subscripts, they could be used in different order without changing the meaning of the notation.

6 Sampling methods

6.1 Simple random sampling

6.1.1 Standard approach

6.1.1.1 Introduction

Simple random sampling is a statistical sampling method. It is the most well-known among the equal probability selection methods. Aims to project to the level of error observed in the sample to the whole population.

The statistical unit to be sampled is the operation (or payment claim). Units in the sample are selected randomly with equal probabilities. Simple random sampling is a generic method that fits different types of populations, although, as it does not use auxiliary information, usually requires larger sample sizes than MUS (whenever the level of expenditure varies significantly among operations and there is positive association between expenditure and errors). The projection of errors can be based on two sub-methods: mean-per-unit estimation or ratio estimation (see Section 6.1.1.3).

As all other methods, this method can be combined with stratification (favourable conditions for stratification are discussed in Section 5.2)

6.1.1.2 Sample size

Computing sample size n within the framework of simple random sampling relies on the following information:

- Population size N
- Confidence level determined from systems audit and the related coefficient z from a normal distribution (see Section 5.3)
- Maximum tolerable error TE (usually 2% of the total expenditure)
- Anticipated error AE chosen by the auditor according to professional judgment and previous information
- The standard deviation σ_e of the errors.

The sample size is computed as follows[25]:

[25] When dealing with a small population size, i.e. if the final sample size represents a large proportion of the population (as a rule of thumb more than 10% of the population) a more exact formula can be used leading to $n = \left(\frac{N \times z \times \sigma_e}{TE-AE}\right)^2 \Big/ \left(1 + \left(\frac{\sqrt{N} \times z \times \sigma_e}{TE-AE}\right)^2\right)$. This correction is valid for simple random sampling and for difference estimation. It can also be introduced in two steps by calculating the sample size n with the usual formula and sequentially correct it using $n' = \frac{n \times N}{n+N-1}$.

$$n = \left(\frac{N \times z \times \sigma_e}{TE - AE}\right)^2$$

where σ_e is the standard-deviation of errors in the population. Note that this standard-deviation of the errors for the total population is assumed to be known in the above calculation. In practice, this will almost never be the case and audit authorities will have to rely either on historical data (standard-deviation of the errors for the population in the past period) or on a preliminary/pilot sample of low sample size (sample size is recommended to be not smaller than 20 to 30 units). In the latter case a preliminary sample of size n^p is selected and a preliminary estimate of the variance of errors (square of the standard-deviation) is obtained though

$$\sigma_e^2 = \frac{1}{n^p - 1}\sum_{i=1}^{n^p}(E_i - \bar{E})^2,$$

where E_i represent the individual errors for units in the sample and $\bar{E} = \frac{\sum_{i=1}^{n^p} E_i}{n^p}$ represents the mean error of the sample.

Note that the pilot sample can subsequently be used as a part of the sample chosen for audit.

6.1.1.3 Projected error

There are two possible ways to project the sampling error to the population. The first is based on mean-per-unit estimation (absolute errors) and the second on ratio estimation (error rates).

Mean-per-unit estimation (absolute errors)

Multiply the average error per operation observed in the sample by the number of operations in the population, yielding the projected error:

$$EE_1 = N \times \frac{\sum_{i=1}^{n} E_i}{n}.$$

Ratio estimation (error rates)

Multiply the average error rate observed in the sample by the book value at the level of the population:

$$EE_2 = BV \times \frac{\sum_{i=1}^{n} E_i}{\sum_{i=1}^{n} BV_i}$$

The sample error rate in the above formula is just the division of the total amount of error in the sample by the total amount of expenditure of units in the sample (expenditure audited).

It is not possible to know *a priori* which is the best extrapolation method as their relative merits depend on the level of association between errors and expenditure. As a basic rule of thumb, the second method should just be used when there is the expectation of high association between errors and expenditure (higher value items tend to exhibit higher errors) and the first method (mean-per-unit) when there is an expectation that errors are relatively independent from the level of expenditure (higher errors can be found either in units of high or low level of expenditure). In practice this assessment can be made using sample data as the decision about the extrapolation method can be taken after the sample is selected and audited. To select the most adequate extrapolation method one should use the sample data to calculate the variance of the book values of the sample units ($\mathrm{VAR_{BV}}$) and the covariance between the errors and the book values over the same units ($\mathrm{COV_{E,BV}}$). Formally, the ratio estimation should be selected whenever $\frac{\mathrm{COV_{E,BV}}}{\mathrm{VAR_{BV}}} > \mathrm{E}R/2$, where ER represents the sample error rate, i.e. the ratio between the sum of errors in the sample and the audited expenditure. Whenever the previous condition is not verified the mean-per-unit estimation should be used to project the errors to the population.

6.1.1.4 Precision

Remember that precision (sampling error) is a measure of the uncertainty associated with the projection (extrapolation). It is calculated differently according to the method that has been used for extrapolation.

Mean-per-unit estimation (absolute errors)
The precision is given by the following formula

$$SE_1 = N \times z \times \frac{s_e}{\sqrt{n}}$$

where s_e is the standard-deviation of errors in the sample (now calculated from the same sample used to project the errors to the population)

$$s_e^2 = \frac{1}{n-1}\sum_{i=1}^{n}(E_i - \bar{E})^2$$

Ratio estimation (error rates)
The precision is given by the following formula

$$SE_2 = N \times z \times \frac{s_q}{\sqrt{n}}$$

where s_q is the sample standard deviation of the variable q:

$$q_i = E_i - \frac{\sum_{i=1}^{n} E_i}{\sum_{i=1}^{n} BV_i} \times BV_i.$$

This variable is for each unit in the sample computed as the difference between its error and the product between its book value and the error rate in the sample.

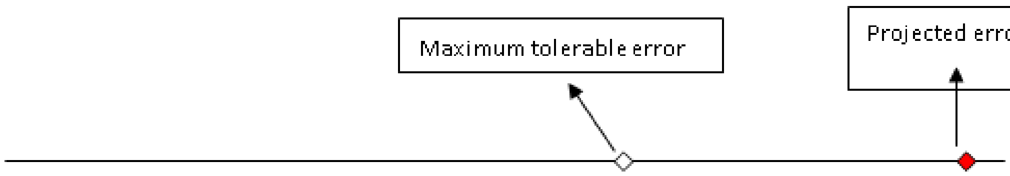
6.1.1.5 Evaluation

To draw a conclusion about the materiality of the errors the upper limit of error (ULE) should be calculated. This upper limit is equal to the summation of the projected error EE itself and the precision of the extrapolation

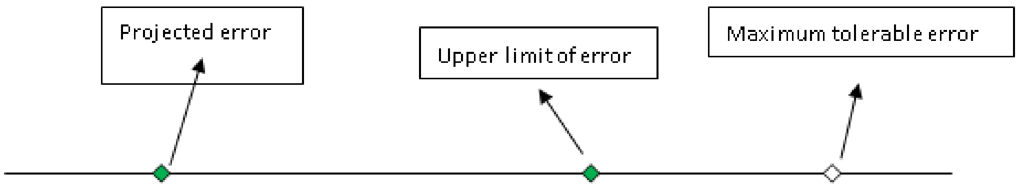
$$ULE = EE + SE$$

Then the projected error and the upper limit should both be compared to the maximum tolerable error to draw audit conclusions:

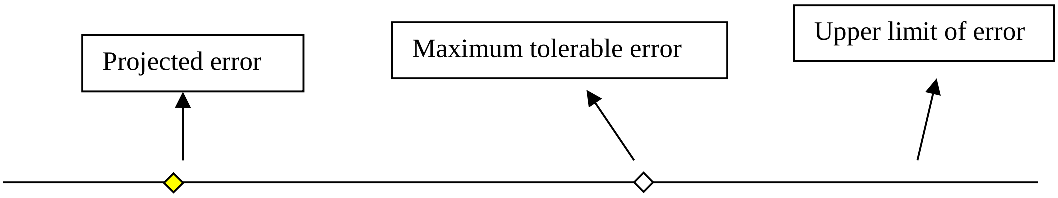
- If projected error is larger than maximum tolerable error, it means that the auditor would conclude that there is enough evidence to support that errors in the population are larger than materiality threshold:



- If the upper limit of error is lower than maximum tolerable error, then the auditor should conclude that errors in the population are lower than materiality threshold.



- If the projected error is lower than maximum tolerable error but the upper limit of error is larger than the maximum tolerable error, this means that the sampling results may be inconclusive. See further explanations in Section 4.12



6.1.1.6 Example

Let us assume a population of expenditure declared to the Commission in a given year for operations in a programme or group of programmes. The system audits carried out by the audit authority have yielded a moderate assurance level. Therefore, a confidence level of 80% seems to be adequate for audit of operations. The following table shows the main population characteristics.

Population size (number of operations)	3,852
Book value (sum of the expenditure in the reference period)	46,501,186 €

A preliminary sample of 20 operations yielded a preliminary estimate for the standard deviation of errors of 518 € (computed in MS Excel as **":=STDEV.S(D2:D21)**"):

	A	B	C	D
1	**Operation**	**Book Value (BV)**	**Correct Value (AV)**	**Error**
2	98	13,054 €	13,054 €	- €
3	120	10,758 €	10,758 €	- €
4	542	8,714 €	8,264 €	450 €
5	554	8,645 €	8,645 €	- €
6	587	9,297 €	9,297 €	- €
7	1156	7,908 €	7,908 €	- €
8	1325	6,717 €	6,717 €	- €
9	1453	16,535 €	16,535 €	- €
10	1840	15,718 €	15,718 €	- €
11	1904	13,175 €	13,175 €	- €
12	2028	6,486 €	6,486 €	- €
13	2338	13,072 €	13,072 €	- €
14	2428	8,753 €	8,753 €	- €
15	2735	17,507 €	17,507 €	- €
16	3054	8,875 €	8,875 €	- €
17	3196	6,568 €	6,568 €	- €
18	3276	6,478 €	6,478 €	- €
19	3321	12,448 €	12,448 €	- €
20	3366	17,894 €	15,598 €	2,296 €
21	3666	13,558 €	13,558 €	- €
22	**Total**	**222,160 €**	**219,413 €**	**2,747 €**
23	**Sample error rate:=D22/B22 --->**			1.24%
24	**Sample standard deviation of errors:= STDEV.S(D2:D21) ---------->**			518 €

The first step is to compute the required sample size, using the formula:

$$n = \left(\frac{N \times z \times \sigma_e}{TE - AE}\right)^2$$

where z is 1.282 (coefficient corresponding to a 80% confidence level), σ_e is 518 € and TE, the tolerable error, is 2% (maximum materiality level set by the Regulation) of the book value, i.e. 2% x 46,501,186 € = 930,024 €. This preliminary sample yields a sample error rate of 1.24%. Further, based either on previous year experience and on the conclusions of the report on managing and control systems the audit authority expects an error rate not larger than 1.24%, Thus AE, the anticipated error, is 1.24% of the total expenditure, i.e., 1,24% x 46,501,186 € = 576,615 €:

$$n = \left(\frac{3{,}852 \times 1.282 \times 518}{930{,}024 - 576{,}615}\right)^2 \approx 53$$

The minimum sample size is therefore 53 operations.

The previous preliminary sample of 20 is used as part of the main sample. Therefore, the auditor only has to randomly select 33 further operations. The following table shows the results for the whole sample of 53 operations:

	A	B	C	D	E	F
1	Operation	Book Value (BV)	Correct Value (AV)	Error	Error rate	q_i
2	(1)	(2)	(3)	(4)	(4)/(2)	(4)-SUM(4)/SUM(2)*(2)
3	74	9,093 €	9,093 €	- €	0.00%	- 107.17 €
4	98	13,054 €	13,054 €	- €	0.00%	- 153.85 €
5	120	10,758 €	10,758 €	- €	0.00%	- 126.79 €
6	153	16,194 €	16,194 €	- €	0.00%	- 190.86 €
7	223	11,662 €	11,662 €	- €	0.00%	- 137.45 €
8	246	16,331 €	16,331 €	- €	0.00%	- 192.48 €
9	542	8,714 €	8,264 €	450 €	5.17%	347.61 €
10	554	8,645 €	8,645 €	- €	0.00%	- 101.89 €
11	587	9,297 €	9,297 €	- €	0.00%	- 109.57 €
12	915	7,999 €	7,999 €	- €	0.00%	- 94.28 €
13	1014	11,906 €	11,906 €	- €	0.00%	- 140.32 €
14	1114	15,505 €	15,505 €	- €	0.00%	- 182.74 €
15	1156	7,908 €	7,908 €	- €	0.00%	- 93.20 €
16	1325	6,717 €	6,717 €	- €	0.00%	- 79.17 €
17	1403	9,730 €	9,730 €	- €	0.00%	- 114.68 €
18	1453	16,535 €	16,535 €	- €	0.00%	- 194.88 €
19	1577	17,723 €	17,723 €	- €	0.00%	- 208.88 €
20	1621	16,095 €	16,095 €	- €	0.00%	- 189.69 €
21	1624	15,171 €	15,171 €	- €	0.00%	- 178.80 €
54	(...)	(...)	(...)	(...)	(...)	(...)
55	3749	9971	9971	0	0.00%	- 117.52 €
56	Total	661,580 €	653,783 €	7,797 €		
57	Sample standard deviation of errors:= STDEV.S(D3:D55)----->			758 €		755 €

The total book value of the 53 sampled operations is 661,580 € (computed in MS Excel as **":=SUM(B3:B55)**"). The total error amount in the sample is 7,797 € (computed in MS Excel as **":=SUM(D3:D55)**"). This amount, divided by the sample size, is the sample average operation error.

In order to identify whether the mean-per-unit or ration estimation is the best estimation method, the AA calculates the ratio of covariance between the errors and the book values to the variance of the book values of the sampled operations, which is equal to 0.02078. As the ratio is larger than half of the sample error rate ((7,797 €/661,580)/2=0.0059), the audit authority can be sure than ratio estimation is the most reliable estimation method. For pedagogic purposes, both estimation methods are illustrated below.

Using mean-per-unit estimation, the projection of the error to the population is calculated by multiplying this average error by the population size (3,852 in this example). This figure is the projected error at the level of the programme:

$$EE_1 = N \times \frac{\sum_{i=1}^{53} E_i}{n} = 3{,}852 \times \frac{7{,}797}{53} = 566{,}703.$$

Using ratio estimation, the projection of the errors to the population can be achieved by multiplying the average error rate observed in the sample by the book value at the level of the population:

$$EE_2 = BV \times \frac{\sum_{i=1}^{53} E_i}{\sum_{i=1}^{53} BV_i} = 46{,}501{,}186 \ \times \frac{7{,}797}{661{,}580} = 548{,}058.$$

The sample error rate in the above formula is just the division of the total amount of error in the sample by the total audited expenditure.

The projected error rate is computed as the ratio between the projected error and the book value of the population (total expenditure). Using the mean-per-unit estimation the projected error rate is:

$$r_1 = \frac{566{,}703}{46{,}501{,}186} = 1.22\%$$

and using the ratio estimation is:

$$r_2 = \frac{548{,}058}{46{,}501{,}186} = 1.18\%$$

In both cases the projected error is smaller than the materiality level. However, final conclusions can only be drawn after taking into account the sampling error (precision).

The first step to obtain the precision is to calculate the standard deviation of errors in the sample (computed in MS Excel as **":=STDEV.S(D3:D55)**"):

$$s_e = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(E_i - \bar{E})^2} = \sqrt{\frac{1}{52}\sum_{i=1}^{53}(E_i - \bar{E})^2} = 758.$$

Thus, the precision of the mean-per-unit estimation is given by

$$SE_1 = N \times z \times \frac{s_e}{\sqrt{n}} = 3{,}852 \times 1.282 \times \frac{758}{\sqrt{53}} = 514{,}169.$$

For the ratio estimation it is necessary to create the variable

$$q_i = E_i - \frac{\sum_{i=1}^{53} E_i}{\sum_{i=1}^{53} BV_i} \times BV_i.$$

This variable is in the last column of the table (column F). For instance the value in cell F3 is given by the value of the error of the first operation (0 €) minus the sum of sample errors, in column D, 7,797 € ("**:=SUM(D3:D55)**") divided by the audited expenditure, in column B, 661,580 € ("**:=SUM(B3:B55)**") and multiplied by the book value of the operation (9,093 €):

$$q_1 = 0 - \frac{7{,}797}{661{,}580} \times 9{,}093 = -107.17.$$

Given the standard deviation of this variable, $s_q = 755$ (computed in MS Excel as **":=STDEV.S(F3:F55)**"), the precision of ratio estimation is given by the following formula

$$SE_2 = N \times z \times \frac{s_q}{\sqrt{n}} = 3{,}852 \times 1.282 \times \frac{755}{\sqrt{53}} = 512{,}134$$

To draw a conclusion about the materiality of the errors the upper limit of error (ULE) should be calculated. This upper limit is equal to the summation of the projected error EE itself and the precision of the projection

$$ULE = EE + SE$$

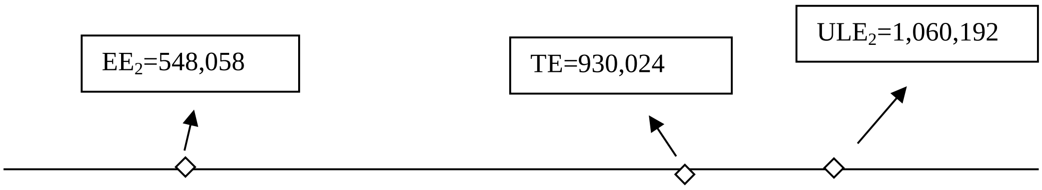
Then the projected error and the upper limit should both be compared to the maximum tolerable error to draw audit conclusions:

$$ULE_1 = EE_1 + SE_1 = 566{,}703 + 514{,}169 = 1{,}080{,}871$$

or

$$ULE_2 = EE_2 + SE_2 = 548{,}058 + 512{,}134 = 1{,}060{,}192$$

Finally, comparing to the materiality threshold of 2% of the total book value of the programme (2% x 46,501,186 € = 930,024 €) with the projected error and upper limit of error for ratio estimation (as this was the selected projection method), the conclusion is that the projected error is lower than the maximum tolerable error, but the upper limit of error is larger the maximum tolerable error. The auditor is able to conclude that additional work is needed, as there is not enough evidence to support that the population is not materially misstated. The specific additional work needed is discussed in Section 5.11.



6.1.2 Stratified simple random sampling

6.1.2.1 Introduction

In stratified simple random sampling, the population is divided in sub-populations called strata and independent samples are drawn from each stratum, using the standard simple random sampling approach.

Candidate criteria to implement stratification should take into account that in stratification we aim to find groups (strata) with less variability than the whole population. With simple random sampling, the stratification by level of expenditure per operation is usually a good approach, whenever it is expected that the level of error is associated with the level of expenditure. Other variables that we expect to explain the level of error in the operations are also good candidates for stratification. Some possible choices are programmes, regions, intermediate bodies, classes based on the risk of the operation, etc.

If stratification by level of expenditure is implemented, consider to identify a high-value stratum[26], apply a 100% audit of these items, and apply simple random sampling to audit samples of the remaining lower-value items that are included in the additional stratum or strata. This is useful in the event that the population included a few high-value items. In this case, the items belonging to the 100% stratum should be taken out of the population and all the steps considered in the remaining sections will apply only to the population of the low-value items. Please note that it is not mandatory to audit 100% of the high-value stratum units. The AA may develop a strategy based on several strata, corresponding to different levels of expenditure, and have all the strata audited through sampling. If a 100% audited stratum exists, it is to stress that the planned precision for sample size determination should be however based on the total book value of the population. Indeed, as the only source of error is the low-value items

[26] There is not a general rule to identify the cut-off value for the high value stratum. A rule of thumb would be to include all operations whose expenditure is larger than the materiality (2%) times the total population expenditure. More conservative approaches use a smaller cut-off usually dividing the materiality by 2 or 3, but the cut-off value depends on the characteristics of the population and should be based on professional judgment.

stratum, but the planned precision refers to the population level, the tolerable error and the anticipated error should be calculated at population level, as well.

6.1.2.2 Sample size

The sample size is computed as follows

$$n = \left(\frac{N \times z \times \sigma_w}{TE - AE}\right)^2$$

where σ_w^2 is the weighted mean of the variances of the errors for the whole set of strata:

$$\sigma_w^2 = \sum_{i=1}^{H} \frac{N_h}{N} \sigma_{eh}^2, h = 1,2, \ldots, H;$$

and σ_{eh}^2 is the variance of errors in each stratum. The variance of the errors is computed for each stratum as an independent population as

$$\sigma_{eh}^2 = \frac{1}{n_h^p - 1} \sum_{i=1}^{n_h^p} (E_{hi} - \bar{E}_h)^2, h = 1,2, \ldots, H$$

where E_{hi} represent the individual errors for units in the sample of stratum h and $\bar{E}_h$ represent the mean error of the sample in stratum h.

These values can be based on historical knowledge or on a preliminary/pilot sample of low sample size as previously presented for the standard simple random sampling method. In this later case the pilot sample can as usual subsequently be used as a part of the sample chosen for audit. If no historical information is available in the beginning of a programming period and it is not possible to access a pilot sample, the sample size may be calculated with the standard approach (for the first year of the period). The data collected in the audit sample of this first year can be used to refine sample size computation in the following years. The price to pay for this lack of information is that the sample size, for the first year, will probably be larger than the one that would be needed if auxiliary information about strata were available.

Once the total sample size, n, is computed the allocation of the sample by stratum is as follows:

$$n_h = \frac{N_h}{N} \times n.$$

This is a general allocation method, usually known as proportional allocation. Many other allocation methods are available. A more tailored allocation may in some cases bring additional precision gains or reduction of sample size. The adequacy of other allocation methods to each specific population requires some technical knowledge in sampling theory. Sometimes, it may happen that the allocation method produces a very small sample size for one or more strata. In practice it is advisable to use a minimum sample size of 3 units for every stratum in the population in order to allow the calculation of the standard-deviations that are necessary to calculate precision.

6.1.2.3 Projected error

Based on H randomly selected samples of operations, where the size of each one has been computed according to the above formula, the projected error at the level of the population can be computed through the two usual methods: mean-per-unit estimation and ratio estimation.

Mean-per-unit estimation
In each group of the population (stratum) multiply the average error per operation observed in the sample by the number of operations in the stratum (N_h); then sum all the results obtained for each stratum, yielding the projected error:

$$EE_1 = \sum_{h=1}^{H} N_h \times \frac{\sum_{i=1}^{n_h} E_i}{n_h}.$$

Ratio estimation

In each group of the population (stratum) multiply the average error rate observed in the sample by the population book value at the level of the stratum (BV_h):

$$EE_2 = \sum_{h=1}^{H} BV_h \times \frac{\sum_{i=1}^{n_h} E_i}{\sum_{i=1}^{n_h} BV_i}$$

The sample error rate in each stratum is just the division of the total amount of error in the sample of stratum by the total amount of expenditure in the same sample.

The choice between the two methods should be based upon the considerations presented for the standard simple random sampling method.

If a 100% stratum has been considered and previously taken from the population then the total amount of error observed in that exhaustive stratum should be added to the above estimate (EE_1 or EE_2) in order to produce the final projection of the amount of error in the whole population.

6.1.2.4 Precision

As for the standard method, precision (sampling error) is a measure of the uncertainty associated with the projection (extrapolation). It is calculated differently according to the method that has been used for extrapolation.

Mean-per-unit estimation (absolute errors)
The precision is given by the following formula

$$SE_1 = N \times z \times \frac{s_w}{\sqrt{n}},$$

where s_w^2 is the weighted mean of the variance of errors for the whole set of strata (now calculated from the same sample used to project the errors to the population):

$$s_w^2 = \sum_{i=1}^{H} \frac{N_h}{N} s_{eh}^2, h = 1,2, \dots, H;$$

and s_{eh}^2 is the estimated variance of errors for the sample of stratum h

$$s_{eh}^2 = \frac{1}{n_h - 1} \sum_{i=1}^{n_h} (E_{hi} - \bar{E}_h)^2 \, , h = 1,2, \dots, H$$

Ratio estimation (error rates)

The precision is given by the following formula

$$SE_2 = N \times z \times \frac{s_{qw}}{\sqrt{n}}$$

where

$$s_{qw}^2 = \sum_{h=1}^{H} \frac{N_h}{N} s_{qh}^2$$

is a weighted mean of the sample variances of the variable q_h, with

$$q_{ih} = E_{ih} - \frac{\sum_{i=1}^{n_h} E_{ih}}{\sum_{i=1}^{n_h} BV_{ih}} \times BV_{ih}.$$

This variable is for each unit in the sample computed as the difference between its error and the product between its book value and the error rate in the sample.

6.1.2.5 Evaluation

To draw a conclusion about the materiality of the errors the upper limit of error (ULE) should be calculated. This upper limit is equal to the summation of the projected error EE itself and the precision of the extrapolation

$$ULE = EE + SE$$

Then the projected error and the upper limit should both be compared to the maximum tolerable error to draw audit conclusions using exactly the same approach presented in Section 6.1.1.5.

6.1.2.6 Example

Let us assume a population of expenditure declared to the Commission in a given year for operations in a group of programmes. The management and control system is common to the group of programmes and the system audits carried out by the Audit Authority have yielded a moderate assurance level. Therefore, the audit authority decided to carry out audits of operation using a confidence level of 80%.

The AA has reasons to believe that there are substantial risks of error for high value operations, whatever the programme they belong to. Further, there are reasons to expect that there are different error rates across the programmes. Bearing in mind all this information, the AA decides to stratify the population by programme and by expenditure (isolating in a 100% sampling stratum all the operations with book value larger than the materiality).

The following table summarizes the available information.

Population size (number of operations)	4,807
Population size – stratum 1 (number of operations in programme 1)	3,582
Population size – stratum 2 (number of operations in programme 2)	1,225
Population size – stratum 3 (number of operations with BV > materiality level)	5
Book value (sum of the expenditure in the reference period)	1,396,535,319 €
Book value – stratum 1 (total expenditure in programme 1)	43,226,801 €
Book value – stratum 2 (total expenditure in programme 2)	1,348,417,361 €
Book value – stratum 3 (total expenditure of operations with BV > Materiality level)	4,891,156 €

The 100% sampling stratum containing the 5 high-value operation should be treated separately as stated in section 6.1.2.1. Therefore, hereafter, the value of N corresponds to the total number of operations in the population, deducted of the number of the operations included in the 100% sampling stratum, i.e. 4,802 (= 4,807 – 5) operations.

The first step is to compute the required sample size, using the formula:

$$n = \left(\frac{N \times z \times \sigma_w}{TE - AE}\right)^2$$

where z is 1.282 (coefficient corresponding to a 80% confidence level) and TE, the tolerable error, is 2% (maximum materiality level set by the Regulation) of the book value, i.e. 2% x 1,396,535,319 € = 27,930,706 €. Based either on previous year experience and on the conclusions of the report on managing and control systems the audit authority expects an error rate not larger than 1.8%, Thus, AE, the anticipated error, is 1.8% of the total expenditure, i.e., 1.8% x 1,396,535,319 € = 25,137,636 €.

Since the third stratum is a 100% sampling stratum, the sample size for this stratum is fixed and is equal to the size of the population, that is, the 5 high-value operations. The sample size for the remaining two strata is computed using the above formula, where σ_w^2 is the weighted average of the variances of the errors for the two remaining strata:

$$\sigma_w^2 = \sum_{i=1}^{2} \frac{N_h}{N} \sigma_{eh}^2, h = 1,2;$$

and σ_{eh}^2 is the variance of errors in each stratum. The variance of the errors is computed for each stratum as an independent population as

$$\sigma_{eh}^2 = \frac{1}{n_h^p - 1} \sum_{i=1}^{n_h^p} (E_{hi} - \bar{E}_h)^2 \text{ }, h = 1,2, \dots, H$$

where E_{hi} represents the individual errors for units in the sample of stratum h and $\bar{E}_h$ represents the mean error of the sample in stratum h.

A preliminary sample of 20 operations of stratum 1 yielded an estimate for the standard deviation of errors of 444 €:

	A	B	C	D
1	Operation	Book Value (BV)	Correct Value (AV)	Error
2	708	6,533 €	4,549 €	1,984 €
3	3084	7,009 €	7,009 €	- €
4	105	7,948 €	7,948 €	- €
5	878	8,910 €	8,910 €	- €
6	2101	8,937 €	8,937 €	- €
7	3117	9,708 €	9,708 €	- €
8	1856	9,728 €	9,728 €	- €
9	734	9,985 €	9,985 €	- €
10	1333	10,160 €	10,160 €	- €
11	668	11,008 €	11,008 €	- €
12	3394	12,116 €	12,116 €	- €
13	1307	12,515 €	12,515 €	- €
14	189	12,553 €	12,553 €	- €
15	15	12,798 €	12,798 €	- €
16	256	16,414 €	16,414 €	- €
17	2621	16,420 €	16,420 €	- €
18	2118	16,729 €	16,729 €	- €
19	3344	16,798 €	16,798 €	- €
20	1551	17,330 €	17,330 €	- €
21	1243	17,592 €	17,592 €	- €
22	**Total**	**241,191 €**	**239,207 €**	**1,984 €**
23	**Sample standard deviation of errors:= STDEV.S(D2:D21) ----------->**			444 €

The same procedure was followed for the population of stratum 2.

A preliminary sample of 20 operations of stratum 2 yielded an estimate for the standard deviation of errors of 9,818 €:

Stratum 1 – preliminary estimate of standard deviation of errors	444 €
Stratum 2 - preliminary estimate of standard deviation of errors	9,818 €

Therefore, the weighted average of the variances of the errors of these two strata is

$$\sigma_w^2 = \frac{3{,}582}{4{,}802} 444^2 + \frac{1{,}225}{4{,}802} 9{,}818^2 = 24{,}737{,}134$$

The sample size is given by

$$n = \left(\frac{4{,}802 \times 1.282 \times \sqrt{24{,}734{,}134}}{27{,}930{,}706 - 25{,}137{,}636} \right)^2 \approx 121$$

The total sample size is given by these 121 operations plus the 5 operation of the 100% sampling stratum, that is, 126 operations.

The allocation of the sample by stratum is as follows:

$$n_1 = \frac{N_1}{N_1 + N_2} \times n = \frac{3{,}582}{4{,}802} \times 121 \approx 90,$$

$$n_2 = n - n_1 = 31$$

and

$$n_3 = N_3 = 5$$

Auditing 90 operations in stratum 1, 31 operations in stratum 2 and 5 operations in stratum 3 will provide the auditor with a total error for the sampled operations. The previous preliminary samples of 20 in stratum 1 and 2 are used as part of the main sample. Therefore, the auditor has only to randomly select 70 further operations in stratum 1 and 11 in stratum 2. The following table shows the sample results the operations audited:

Sample results – stratum 1		
A	Sample book value	1,055,043 €
B	Sample total error	11,378 €
C	Sample average error (C=B/90)	126 €
D	Sample standard deviation of errors	698 €
Sample results – stratum 2		
E	Sample book value	35,377,240 €
F	Sample total error	102,899 €
G	Sample average error (G=F/31)	3,319 €
H	Sample standard deviation of errors	13,012 €
Sample results – stratum 3		
I	Sample book value	4,891,156 €
J	Sample total error	889 €
K	Sample average error (K=J/5)	178 €

The following figure illustrates the results for stratum 1:

	A	B	C	D	E	F
1	Operation	Book Value (BV)	Correct Value (AV)	Error	Error rate	q_i
2	(1)	(2)	(3)	(4)	(4)/(2)	(4)-SUM(4)/SUM(2)*(2)
3	559	6,106 €	6,106 €	- €	0.0%	- 65.85 €
4	1833	6,196 €	6,196 €	- €	0.0%	- 66.82 €
5	2759	6,441 €	6,441 €	- €	0.0%	- 69.46 €
6	708	6,533 €	4,549 €	1,984 €	30.4%	1,913.19 €
7	(...)	(...)	(...)	(...)	(...)	(...)
72	606	14,305 €	13,275 €	1,030 €	7.2%	875.98 €
73	341	14,448 €	12,626 €	1,822 €	12.6%	1,666.23 €
74	1701	14,501 €	14,501 €	- €	0.0%	- 156.38 €
75	416	14,715 €	14,715 €	- €	0.0%	- 158.69 €
76	672	15,237 €	15,237 €	- €	0.0%	- 164.32 €
77	2859	15,445 €	9,428 €	6,017 €	39.0%	5,850.57 €
78	854	15,929 €	15,929 €	- €	0.0%	- 171.78 €
79	2154	16,233 €	16,233 €	- €	0.0%	- 175.06 €
80	256	16,414 €	16,414 €	- €	0.0%	- 177.01 €
81	2621	16,420 €	16,420 €	- €	0.0%	- 177.08 €
82	1224	16,532 €	16,532 €	- €	0.0%	- 178.28 €
83	2118	16,729 €	16,729 €	- €	0.0%	- 180.41 €
84	3344	16,798 €	16,798 €	- €	0.0%	- 181.15 €
85	2250	17,063 €	17,063 €	- €	0.0%	- 184.01 €
86	1551	17,330 €	17,330 €	- €	0.0%	- 186.89 €
87	19	17,458 €	16,933 €	525 €	3.0%	336.44 €
88	654	17,505 €	17,505 €	- €	0.0%	- 188.78 €
89	1243	17,592 €	17,592 €	- €	0.0%	- 189.72 €
90	1869	17,595 €	17,595 €	- €	0.0%	- 189.75 €
91	2483	17,867 €	17,867 €	- €	0.0%	- 192.68 €
92	306	17,876 €	17,876 €	- €	0.0%	- 192.78 €
93	Total	1,055,043 €	1,043,665 €	11,378 €		
94	Sample standard deviation of errors:= STDEV.S(D3:D92)----->			698 €		695 €

In order to identify whether the mean-per-unit or ratio estimation is the best estimation method, the AA calculates the ratio of covariance between the errors and the book values to the variance of the book values of the sampled operations. As the ratio is larger than half of the sample error rate, the audit authority can be sure than ratio estimation is the most reliable estimation method. For pedagogic purposes, both estimation methods are illustrated below.

In the mean-per-unit estimation, extrapolating the error for the two sampling strata is done by multiplying the sample average error by the population size. The sum of these two figures has to be added to the error found in the 100% sampling strata, in order to project error to the population:

$$EE_1 = \sum_{h=1}^{3} N_h \times \frac{\sum_{i=1}^{n_h} E_i}{n_h} = 3{,}582 \times 126 + 1{,}225 \times 3{,}319 + 889 = 4{,}519{,}900$$

An alternative estimated result using ratio estimation is obtained by multiplying the average error rate observed in the stratum sample by the book value at the stratum level (for the two sampling strata). Then, the sum of these two figures has to be added to the error found in the 100% sampling strata, in order to project error to the population:

$$EE_2 = \sum_{h=1}^{3} BV_h \times \frac{\sum_{i=1}^{n_h} E_i}{\sum_{i=1}^{n_h} BV_i}$$
$$= 43{,}226{,}802 \times \frac{11{,}378}{1{,}055{,}043} + 1{,}348{,}417{,}361 \times \frac{102{,}899}{35{,}377{,}240} + 889$$
$$= 4{,}389{,}095.$$

The projected error rate is computed as the ratio between the projected error and the book value of the population (total expenditure). Using the mean-per-unit estimation the projected error rate is

$$r_1 = \frac{4{,}519{,}900}{1{,}396{,}535{,}319} = 0.32\%$$

and using the ratio estimation is:

$$r_2 = \frac{4{,}389{,}095.}{1{,}396{,}535{,}319} = 0.31\%$$

In both cases, the projected error is smaller than the materiality level. However, final conclusions can only be drawn after taking into account the sampling error (precision). Notice, that the only sources of sampling error are strata 1 and 2, since the high-value stratum is submitted to a 100% sampling. In what follows, only the two sampling strata are considered.

Given the standard deviations of errors in the sample of both strata (table with sample results), the weighted average of the variance of errors for the whole set of strata is:

$$s_w^2 = \sum_{i=1}^{2} \frac{N_h}{N} s_{eh}^2 = \frac{3{,}582}{4{,}802} \times 698^2 + \frac{1{,}225}{4{,}802} \times 13{,}012^2 = 43{,}507{,}225.$$

Therefore, the precision of the absolute error is given by the following formula:

$$SE_1 = N \times z \times \frac{s_w}{\sqrt{n}} = 4{,}802 \times 1.282 \times \frac{\sqrt{43{,}507{,}225}}{\sqrt{121}} = 3{,}695{,}304.$$

For the ratio estimation, it is necessary to create the variable

$$q_{ih} = E_{ih} - \frac{\sum_{i=1}^{n_h} E_{ih}}{\sum_{i=1}^{n_h} BV_{ih}} \times BV_{ih}.$$

The illustration for stratum 1 is in the last column of the previous table (column F). For instance the value in cell F3 is given by the value of the error of the first operation (0 €) minus the sum of sample errors, in column D, 11,378 € ("**:=SUM(D3:D92)**") divided by the sum of sample book values, in column B, 1,055,043 € ("**:=SUM(B3:B92)**"), multiplied by the book value of the operation (6,106 €):

$$q_{11} = 0 - \frac{11{,}378}{1{,}055{,}043} \times 6{,}106 = -65.85.$$

The standard deviation of this variable for stratum 1 is $s_{q1} = 695$ (computed in MS Excel as "**:=STDEV.S(F3:F92)**"). Using the methodology just described, the standard deviation for stratum 2 is $s_{q2} = 13{,}148$. Therefore the weighted sum of the variances of q_{ih}:

$$s_{qw}^2 = \sum_{h=1}^{3} \frac{N_h}{N} s_{qh}^2 = \frac{3{,}582}{4{,}802} \times 695^2 + \frac{1{,}225}{4{,}802} \times 13{,}148^2 = 44{,}412{,}784.$$

The precision for ratio estimation is given by

$$SE_2 = N \times z \times \frac{s_{qw}}{\sqrt{n}} = 4{,}802 \times 1.282 \times \frac{\sqrt{44{,}412{,}784}}{\sqrt{59}} = 3{,}733{,}563.$$

To draw a conclusion about the materiality of the errors the upper limit of error (ULE) should be calculated. This upper limit is equal to the summation of the projected error EE itself and the precision of the extrapolation

$$ULE = EE + SE$$

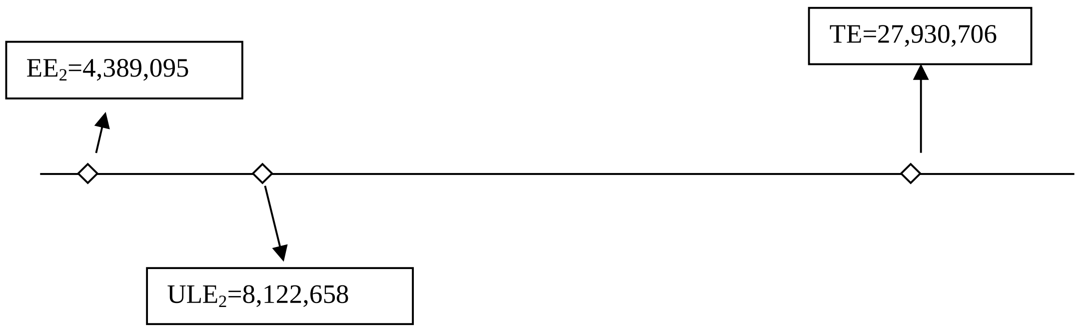
Then the projected error and the upper limit should both be compared to the maximum tolerable error to draw audit conclusions:

$$ULE_1 = EE_1 + SE_1 = 4{,}519{,}900 + 3{,}695{,}304 = 8{,}215{,}204$$

or

$$ULE_2 = EE_2 + SE_2 = 4{,}389{,}095 + 3{,}733{,}563 = 8{,}122{,}658$$

Finally, comparing to the materiality threshold of 2% of the total book value of the population (2% x 1,396,535,319 € = 27,930,706 €) with the projected results for the ratio estimation (the selected projection method) we observe that both the projected error and the upper error limit are smaller than the maximum tolerable error. Therefore, we conclude that there is sufficient evidence to support that the population is not materially misstated.



6.1.3 Simple random sampling – two periods

6.1.3.1 Introduction

The audit authority may decide to carry out the sampling process in several periods during the year (typically two semesters). The major advantage of this approach is not related with sample size reduction, but mainly allowing spreading the audit workload over the year, thus reducing the workload that would be done at the end of the year based on just one observation.

With this approach the year population is divided in two sub-populations, each one corresponding to the operations and expenditure of each semester. Independent samples are drawn for each semester, using the standard simple random sampling approach.

6.1.3.2 Sample size

First semester
At the first period of auditing (e.g. semester) the global sample size (for the set of two semesters) is computed as follows:

$$n = \left(\frac{N \times z \times \sigma_{ew}}{TE - AE}\right)^2$$

where σ_{ew}^2 is the weighted mean of the variances of the errors for in each semester:

$$\sigma_{ew}^2 = \frac{N_1}{N}\sigma_{e1}^2 + \frac{N_2}{N}\sigma_{e2}^2$$

and σ_{et}^2 is the variance of errors in each period t (semester). The variance of the errors for each semester is computed as an independent population as

$$\sigma_{et}^2 = \frac{1}{n_t^p - 1}\sum_{i=1}^{n_t^p}(E_{ti} - \bar{E}_t)^2\,, t = 1,2$$

where E_{ti} represent the individual errors for units in the sample of semester t and $\bar{E}_t$ represent the mean error of the sample in semester t.

Note that the values for the expected variances in both semesters values have to be set using professional judgments and must be based on historical knowledge. The option to implement a preliminary/pilot sample of low sample size as previously presented for the standard simple random sampling method is still available, but can only be performed for the first semester. In fact, at the first moment of observation expenditure for the second semester has not yet taken place and no objective data (besides historical) is available. If pilot samples are implemented, they can, as usual, subsequently be used as a part of the sample chosen for audit.

The auditor may consider that the expected variance of errors for the 2^{nd} semester is the same as for the 1^{st} semester. Hence, a simplified approach can be used for computing the global sample size as

$$n = \left(\frac{N \times z \times \sigma_{e1}}{TE - AE}\right)^2$$

Note that in this simplified approach only information about the variability of errors in the first period of observation is needed. The underlying assumption is that the variability of errors will be of similar magnitude in both semesters.

Also note that the formulas for sample size calculation require values for N_1 and N_2, i.e. number of operation in the population of the first and second semesters. When calculating sample size, the value for N_1 will be known, but the value of N_2 will be unknown and has to be imputed according to the expectations of the auditor (also based on historical information). Usually, this does not constitute a problem as all the operations active in the second semester already exist in the first semester and therefore $N_1 = N_2$.

Once the total sample size, n, is computed the allocation of the sample by semester is as follows:

$$n_1 = \frac{N_1}{N}n$$

and

$$n_2 = \frac{N_2}{N}n$$

Second semester

At the first observation period some assumptions were made relatively the following observation periods (typically the next semester). If characteristics of the population in the following periods differ significantly from the assumptions, sample size for the following period may have to be adjusted.

In fact, at the second period of auditing (e.g. semester) more information will be available:

- The number of operations active in the semester N_2 is correctly known;
- The sample standard-deviation of errors s_{e1} calculated from the sample of the first semester could be already available;
- The standard deviation of errors for the second semester σ_{e2} could now be more accurately assessed using real data.

If these parameters are not dramatically different from the ones estimated at the first semester using the expectations of the analyst, the originally planned sample size, for the second semester (n_2), won't require any adjustments. Nevertheless if the auditor finds that initial expectations significantly differ from the real population characteristics, the sample size may have to be adjusted in order to account for these inaccurate estimates. In this case, the sample size of the second semester should be recalculated using

$$n_2 = \frac{\left(z.N_2\ .\sigma_{e2}\right)^2}{(TE - AE)^2 - z^2.\frac{N_1^2}{n_1}.s_{e1}^2}$$

where s_{e1} is the standard-deviation of errors calculated from the sample of the first semester and σ_{e2} an estimate of the standard-deviation of errors in the second semester based on historical knowledge (eventually adjusted by information from the first semester) or a preliminary/pilot sample of the second semester.

6.1.3.3 Projected error

Based on the two sub-samples of each semester, the projected error at the level of the population can be computed through the two usual methods: mean-per-unit estimation and ratio estimation.

Mean-per-unit estimation

In each semester multiply the average error per operation observed in the sample by the number of operations in the population (N_t); then sum the results obtained for both semesters, yielding the projected error:

$$EE_1 = \frac{N_1}{n_1} \sum_{i=1}^{n_1} E_{1i} + \frac{N_2}{n_2} \sum_{i=1}^{n_2} E_{2i}$$

Ratio estimation

In each semester multiply the average error rate observed in the sample by the population book value of the respective semester (BV_t):

$$EE_2 = BV_1 \times \frac{\sum_{i=1}^{n_1} E_{1i}}{\sum_{i=1}^{n_1} BV_{1i}} + BV_2 \times \frac{\sum_{i=1}^{n_2} E_{2i}}{\sum_{i=1}^{n_2} BV_{2i}}$$

The sample error rate in each semester is just the division of the total amount of error in the sample of the semester by the total amount of expenditure in the same sample.

The choice between the two methods should be based upon the considerations presented for the standard simple random sampling method.

6.1.3.4 Precision

As for the standard method, precision (sampling error) is a measure of the uncertainty associated with the projection (extrapolation). It is calculated differently according to the method that has been used for extrapolation.

Mean-per-unit estimation (absolute errors)

The precision is given by the following formula

$$SE \quad = z \times \sqrt{\left(N_1^2 \times \frac{s_{e1}^2}{n_1} + N_2^2 \times \frac{s_{e2}^2}{n_2} \right)}$$

where s_{et} is the standard-deviation of errors in the sample of semester t, (now calculated from the same samples used to project the errors to the population)

$$s_{et}^2 = \frac{1}{n_t - 1} \sum_{i=1}^{n_t} (E_{ti} - \bar{E}_t)^2$$

Ratio estimation (error rates)

The precision is given by the following formula

$$SE = z \times \sqrt{\left(N_1^2 \times \frac{s_{q1}^2}{n_1} + N_2^2 \times \frac{s_{q2}^2}{n_2}\right)}$$

where s_{qt} is the standard deviation of the variable q in the sample of semester t, where

$$q_{ti} = E_{ti} - \frac{\sum_{i=1}^{n_t} E_{ti}}{\sum_{i=1}^{n_t} BV_{ti}} \times BV_{ti}.$$

6.1.3.5 Evaluation

To draw a conclusion about the materiality of the errors the upper limit of error (ULE) should be calculated. This upper limit is equal to the summation of the projected error EE itself and the precision of the extrapolation

$$ULE = EE + SE$$

Then the projected error and the upper limit should both be compared to the maximum tolerable error to draw audit conclusions using exactly the same approach presented in Section 6.1.1.5.

6.1.3.6 Example

An AA decided to spread the audit workload in two periods. At the end of the first semester AA considers the population divided into two groups corresponding to both semesters. At the end of the first semester, the characteristics of the population are the following:

Declared expenditure at the end of first semester	1,237,952,015 €
Size of population (operations - first semester)	3,852

Based on the experience, the AA knows that usually all the operations included in the programmes at the end of the reference period are already active in the population of the first semester. Furthermore, it is expected that the declared expenditure at the end of the first semester represents about 30% of the total declared expenditure at the end of the reference period. Based on these assumptions a summary of the population is described in the following table:

Declared expenditure of the first semester	1,237,952,015 €
Declared expenditure of the second semester (predicted)	2,888,554,702 €
Size of population (operations - period 1)	3,852

Size of population (operations – period 2, predicted)	3,852

The system audits carried out by the audit authority have yielded a high assurance level. Therefore, sampling this programme can be done with a confidence level of 60%.

At the first period, the global sample size (for the set of two semesters) is computed as follows:

$$n = \left(\frac{N \times z \times \sigma_w}{TE - AE}\right)^2$$

where σ_w^2 is the weighted mean of the variances of the errors in each semester:

$$\sigma_w^2 = \frac{N_1}{N}\sigma_{e1}^2 + \frac{N_2}{N}\sigma_{e2}^2$$

and σ_{et}^2 is the variance of errors in each period t (semester). The variance of the errors for each semester is computed as an independent population as

$$\sigma_{et}^2 = \frac{1}{n_t^p - 1}\sum_{i=1}^{n_t^p}(E_{ti} - \bar{E}_t)^2 \text{ , } t = 1{,}2$$

where E_{ti} represent the individual errors for units in the sample of semester t and $\bar{E}_t$ represent the mean error of the sample in semester t.

Since the value of σ_{et}^2 is unknown, the AA decided to draw a preliminary sample of 20 operations at the end of first semester of the current year. The sample standard deviation of errors in this preliminary sample at first semester is 72,091 €. Based on professional judgement and knowing that usually the expenditure in second semester is larger than in first semester, the AA has made a preliminary prediction of standard deviation of errors for the second semester to be 40% larger than in first semester, that is, 100,927.4 €. Therefore, the weighted average of the variances of the errors is:

$$\begin{aligned}\sigma_w^2 &= \frac{N_1}{N_1 + N_2}\sigma_{e1}^2 + \frac{N_2}{N_1 + N_2}\sigma_{e2}^2 \\ &= \frac{3852}{3852 + 3852} \times 72{,}091^2 + \frac{3852}{3852 + 3852} \times 100{,}927.4^2 \\ &= 7{,}691{,}726{,}176.\end{aligned}$$

Note that the population size in each semester is equal to the number of active operations (with expenditure) in each semester.

At the first semester the global sample size planned for the whole year is:

$$n = \left(\frac{(N_1 + N_2) \times z \times \sigma_w}{TE \ - AE} \right)^2$$

where z is 0.842 (coefficient corresponding to a 60% confidence level), TE, the tolerable error, is 2% (maximum materiality level set by the Regulation) of the book value. The total book value comprises the true book value at the end of the first semester plus the predicted book value for the second semester (1,237,952,015 € + 2,888,554,702 € = 4,126,506,717 €), which means that tolerable error is 2% x 4,126,506,718 € = 82,530,134 €. The preliminary sample on the first semester population yields a sample error rate of 0.6%. The audit authority expects this error rate to remain constant all over the year. Thus AE, the anticipated error, is 0.6% x 4,126,506,718 € = 24,759,040 €. The planned sample size for the whole year is:

$$n = \left(\frac{(3852 + 3852) \times 0.842 \times \sqrt{7{,}691{,}726{,}176}}{82{,}530{,}134 \ - 24{,}759{,}040} \right)^2 \approx 97$$

The allocation of the sample by semester is as follows:

$$n_1 = \frac{N_1}{N_1 + N_2} n \approx 49$$

and

$$n_2 = n - n_1 = 49$$

The first semester sample yielded the following results:

Sample book value - first semester	13,039,581 €
Sample total error - first semester	199,185 €
Sample standard deviation of errors - first semester	69,815 €

At the end of the second semester more information is available, in particular, the number of operations active in the second semester is correctly known, the sample variance of errors s_{e1} calculated from the sample of the first semester is already available and the standard deviation of errors for the second semester σ_{e2} can now be more accurately assessed using a preliminary sample of real data.

The AA realises that the assumption made at the end of the first semester on the total number of operations remains correct. Nevertheless, there are two parameters for which updated figures should be used.

Firstly, the estimate of the standard deviation of errors based on the first semester sample of 49 operations yielded an estimate of 69,815 €. This new value should now be used to reassess the planned sample size. Secondly, based on a new preliminary sample of 20 operations of the second semester population, the audit authority estimates the standard deviation of errors for the second semester to be 108,369 € (close to the predicted value at the end of the first period, but more accurate). We conclude that the standard deviations of errors of both semesters, used to plan the sample size, are close to the values obtained at the end of the first semester. Nevertheless, the audit authority has chosen to recalculate the sample size using the available updated data. As a result, the sample for the second semester is revised.

Further, the predicted total book value of the second semester population should be replaced by the real one, 2,961,930,008 €, instead of the predicted value of 2,888,554,703 €.

Parameter	End of first semester	End of second semester
Standard deviation of errors in the first semester	72,091 €	69,815 €
Standard deviation of errors in the second semester	100,475 €	108,369 €
Total expenditure in the second semester	2,888,554,703 €	2,961,930,008 €

Taking into account these adjustments, the recalculated sample size of the second semester is

$$n_2 = \frac{\left(z \times N_2 \times \sigma_{e2}\right)^2}{(TE - AE)^2 - z^2 \times \frac{N_1^2}{n_1} \times s_{e1}^2}$$

$$= \frac{(0.842 \times 3,852 \times 108,369)^2}{(83,997,640 - 25,199,292)^2 - 0.842^2 \times \frac{3,852^2}{49} \times 69,815^2} = 52$$

Auditing 49 operations is the first semester plus these 52 operations in second semester will provide the auditor with information on the total error for the sampled operations. The previous preliminary sample of 20 operations is used as part of the main sample. Therefore, the auditor has only to select 32 further operations in second semester.

The second semester sample yielded the following results:

Sample book value - second semester	34,323,574 €
Sample total error - second semester	374,790 €
Sample standard deviation of errors - second semester	59,489 €

Based on both samples, the projected error at the level of the population can be computed through the two usual methods: mean-per-unit estimation and ratio estimation. In order to identify whether the mean-per-unit or ration estimation is the best estimation method, the AA calculates the ratio of covariance between the errors and the book values to the variance of the book values of the sampled operations. As this ratio is larger than half of the sample error rate, the audit authority can be sure than ratio estimation is the most reliable estimation method. For pedagogic purposes, both estimation methods are illustrated below.

Mean-per-unit estimation comprises multiplying the average error per operation observed in the sample by the number of operations in the population (N_t); then sum the results obtained for both semesters, yielding the projected error:

$$EE_1 = \frac{N_1}{n_1}\sum_{i=1}^{49} E_{1i} + \frac{N_2}{n_2}\sum_{i=1}^{52} E_{2i} = \frac{3{,}852}{49} \times 199{,}185 + \frac{3{,}852}{52} \times 374{,}790$$
$$= 43{,}421{,}670$$

Ratio estimation comprises multiplying the average error rate observed in the sample by the population book value of the respective semester (BV_t):

$$EE_2 = BV_1 \times \frac{\sum_{i=1}^{n_1} E_{1i}}{\sum_{i=1}^{n_1} BV_{1i}} + BV_2 \times \frac{\sum_{i=1}^{n_2} E_{2i}}{\sum_{i=1}^{n_2} BV_{2i}}$$
$$= 1{,}237{,}952{,}015 \times \frac{199{,}185}{13{,}039{,}581} + 2{,}961{,}930{,}008 \times \frac{374{,}790}{34{,}323{,}574}$$
$$= 51{,}252{,}484$$

Using the mean-per-unit estimation the projected error rate is:

$$r_1 = \frac{43{,}421{,}670}{1{,}237{,}952{,}015 + 2{,}961{,}930{,}008} = 1.03\%$$

and using the ratio estimation is:

$$r_2 = \frac{51{,}252{,}451}{1{,}237{,}952{,}015 + 2{,}961{,}930{,}008} = 1.22\%.$$

The precision is calculated differently according to the method that has been used for projection. For mean-per-unit estimation, the precision is given by the following formula

$$SE_1 = z \times \sqrt{\left(N_1^2 \times \frac{s_{e1}^2}{n_1} + N_2^2 \times \frac{s_{e2}^2}{n_2}\right)}$$

$$= 0.842 \times \sqrt{3{,}852^2 \times \frac{69{,}815^2}{49} + 3{,}852^2 \times \frac{59{,}489^2}{52}} = 41{,}980{,}051$$

For the ratio estimation, the standard deviation of the variable q has to be calculated (Section 6.1.3.4):

$$q_{ti} = E_{ti} - \frac{\sum_{i=1}^{n_t} E_{ti}}{\sum_{i=1}^{n_t} BV_{ti}} \times BV_{ti}.$$

This standard deviation for each semester is, 54,897 € and 57,659 €, respectively. Thus the precision is given by

$$SE_2 = z \times \sqrt{\left(N_1^2 \times \frac{s_{q1}^2}{n_1} + N_2^2 \times \frac{s_{q2}^2}{n_2}\right)}$$

$$= 0.842 \times \sqrt{3{,}852^2 \times \frac{54{,}897^2}{49} + 3{,}852^2 \times \frac{57{,}659^2}{52}} = 36{,}325{,}544$$

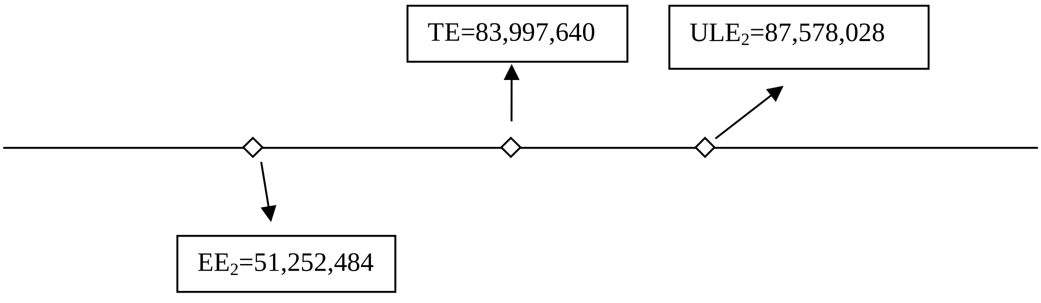
Then the projected error and the upper limit should both be compared to the maximum tolerable error to draw audit conclusions:

$$ULE_1 = EE_1 + SE_1 = 43{,}421{,}670 + 41{,}980{,}051 = 85{,}401{,}721$$

or

$$ULE_2 = EE_2 + SE_2 = 51{,}252{,}484 + 36{,}325{,}544 = 87{,}578{,}028$$

Finally, comparing to the materiality threshold of 2% of the total book value of the population (2% x 4,199,882,023 € = 83,997,640 €) with the projected results from ratio estimation (the selected projection method), we observe that the maximum tolerable error is larger than the projected errors, but smaller than the upper limit. Please refer to section 4.12 for more details on the analysis to be done.



6.2 Difference estimation

6.2.1 Standard approach

6.2.1.1 Introduction

Difference estimation is also a statistical sampling method based on equal probability selection. The method relies on extrapolating the error in the sample and subtracting the projected error to the total declared expenditure in the population in order to assess the correct expenditure in the population (i.e. the expenditure that would be obtained if all the operations in the population were audited).

This method is very close to simple random sampling, having as main difference the use of a more sophisticated extrapolation device.

This method is particularly useful if one wants to project the correct expenditure in the population, if the level of error is relatively constant in the population, and if the book value of different operations tends to be similar (low variability). It tends to be better than MUS when errors have low variability or are weakly or negatively associated with book values. On the other hand, tends to be worse than MUS is when errors have strong variability and are positively associated with book values

As all other methods, this method can be combined with stratification (favourable conditions for stratification are discussed in Section 5.2).

6.2.1.2 Sample size

Computing sample size n within the framework of difference estimation relies on exactly the same information and formulas used in simple random sampling:

- Population size N
- Confidence level determined from systems audit and the related coefficient z from a normal distribution (see Section 5.3)
- Maximum tolerable error TE (usually 2% of the total expenditure)
- Anticipated error AE chosen by the auditor according to professional judgment and previous information
- The standard deviation σ_e of the errors.

The sample size is computed as follows:

$$n = \left(\frac{N \times z \times \sigma_e}{TE - AE}\right)^2$$

where σ_e is the standard-deviation of errors in the population. Please note that, as discussed in the framework of simple random sampling, this standard-deviation is almost never know in advance and audit authorities will have to base it on historical knowledge or on a preliminary/pilot sample of low sample size (sample size is recommended to be not smaller than 20 to 30 units). Also, note that the pilot sample can subsequently be used as a part of the sample chosen for audit. For additional information on how to calculate this standard-deviation see Section 6.1.1.2.

6.2.1.3 Extrapolation

Based on a randomly selected sample of operations, the size of which has been computed according to the above formula, the projected error at the level of the population can be computed by multiplying the average error observed per operation in the sample by the number of operations in the population, yielding the projected error

$$EE = N \times \frac{\sum_{i=1}^{n} E_i}{n}.$$

where E_i represent the individual errors for units in the sample and $\bar{E}$ represent the mean error of the sample.

In a second step the correct book value (the correct expenditure that would be found if all the operations in the population were audited) can be projected subtracting the projected error (EE) from the book value (BV) in the population (declared expenditure). The projection for the correct book value (CBV) is

$$CBV = BV - EE$$

6.2.1.4 Precision

The precision of the projection (measure of the uncertainty associated with the projection) is given by

$$SE = N \times z \times \frac{s_e}{\sqrt{n}}$$

where s_e is the standard-deviation of errors in the sample (now calculated from the same sample used to project the errors to the population)

$$s_e^2 = \frac{1}{n-1} \sum_{i=1}^{n} (E_i - \bar{E})^2$$

6.2.1.5 Evaluation

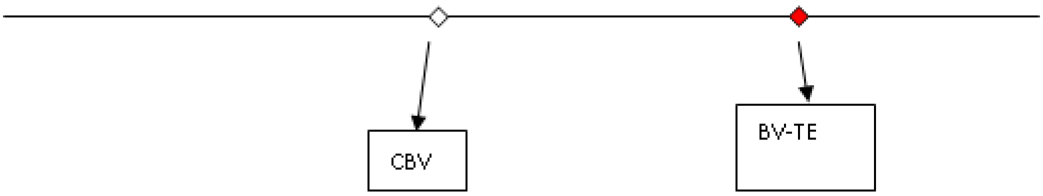
To conclude about the materiality of the errors the lower limit for the corrected book value should firstly be calculated. This lower limit is equal to

$$LL = CBV - SE$$

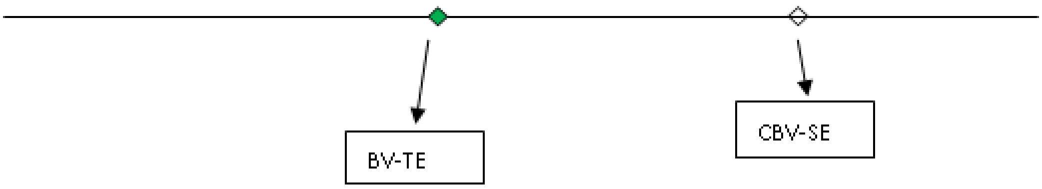
The projection for the correct book value and the lower limit should both be compared to the difference between the book value (declared expenditure) and the maximum tolerable error (TE), which corresponds to the materiality level times the book value:

$$BV - TE = BV - 2\% \times BV = 98\% \times BV$$

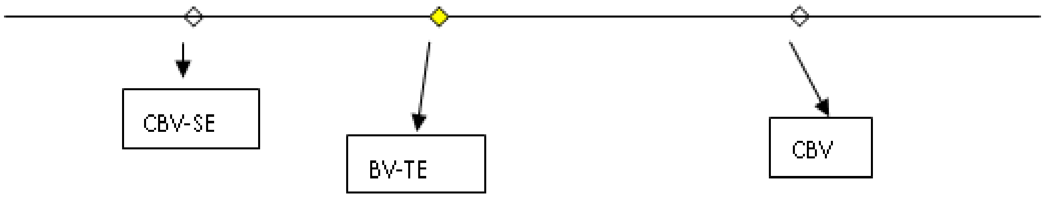
- If $BV - TE$ is larger than CBV the auditor should conclude that there is enough evidence that errors in the programme are larger than materiality threshold:



- If $BV - TE$ is lower than the lower limit $CBV - SE$ than it means there is enough evidence that errors in the programme are lower than materiality threshold.



If $BV - TE$ is between the lower limit $CBV - SE$ and CBV please refer to section 4.12 for more details on the analysis to be done.



6.2.1.6 Example

Let's assume a population of expenditure declared to the Commission in a given year for operations in a programme. The system audits carried out by the audit authority have yielded a high assurance level. Therefore, sampling this programme can be done with a confidence level of 60%.

The following table summarises the population details:

Population size (number of operations)	3,852
Book value (sum of the expenditure in the reference period)	4,199,882,024 €

Based on last year's audit the AA expects an error rate of 0.7% (the last year error rate) and estimates a standard deviation of errors of 168,397 €.

The first step is to compute the required sample size, using the formula:

$$n = \left(\frac{N \times z \times \sigma_e}{TE - AE}\right)^2$$

where z is 0.842 (coefficient corresponding to a 60% confidence level), σ_e is 168,397 €, TE, the tolerable error, is 2% of the book value (maximum materiality level set by the Regulation), i.e. 2% x 4,199,882,024 € = 83,997,640 € and AE, the anticipated error is 0.7%, i.e., 0.7% x 4,199,882,024 € = 29,399,174 €:

$$n = \left(\frac{3{,}852 \times 0.842 \times 168{,}397}{83{,}997{,}640 - 29{,}399{,}174}\right)^2 \approx 101$$

The minimum sample size is therefore 101 operations.

Auditing these 101 operations will provide the auditor with a total error for the sampled operations.

The sample results are summarised in the following table:

Sample book value	124,944,535 €
Sample total error	1,339,765 €
Sample standard deviation of errors	162,976 €

The projected error at the level of the population is:

$$EE = N \times \frac{\sum_{i=1}^{101} E_i}{n} = 3{,}852 \times \frac{1{,}339{,}765}{101} = 51{,}096{,}780,$$

corresponding to a projected error rate of:

$$r = \frac{51{,}096{,}780}{4{,}199{,}882{,}024} = 1.22\%$$

The correct book value (the correct expenditure that would be found if all the operations in the population were audited) can be projected subtracting the projected error (EE) from the book value (BV) in the population (declared expenditure). The projection for the correct book value (CBV) is

$$CBV = 4{,}199{,}882{,}024 - 51{,}096{,}780 = 4{,}148{,}785{,}244$$

The precision of the projection is given by

$$SE = N \times z \times \frac{s_e}{\sqrt{n}} = 3{,}852 \times 0.842 \times \frac{162{,}976}{\sqrt{101}} = 52{,}597{,}044.$$

Combining the projected error and the precision it is possible to compute an upper limit for the error rate. This upper limit is the ratio of the upper limit of error to the book value of the population. Therefore, the upper limit for the error rate is:

$$r_{UL} = \frac{EE + SE}{BV} = \frac{51{,}096{,}780 + 52{,}597{,}044}{4{,}199{,}882{,}024} = 2.47\%$$

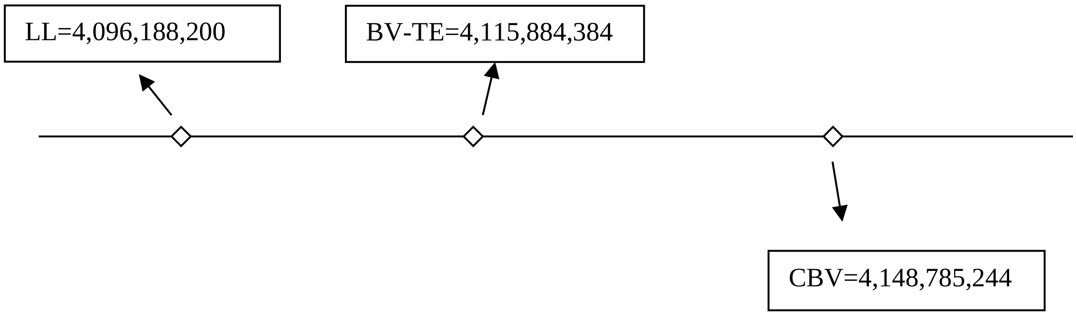
To conclude about the materiality of the errors the lower limit for the correct book value should firstly be calculated. This lower limit is equal to

$$LL = CBV - SE = 4{,}148{,}785{,}244 - 52{,}597{,}044 = 4{,}096{,}188{,}200$$

The projection for the correct book value and the lower limit should both be compared to the difference between the book value (declared expenditure) and the maximum tolerable error (TE):

$$BV - TE = 4{,}199{,}882{,}024 - 83{,}997{,}640 = 4{,}115{,}884{,}384$$

As $BV - TE$ is between the lower limit $LL = CBV - SE$ and CBV, please refer to section 4.12 for more details on the analysis to be done.



6.2.2 Stratified difference estimation

6.2.2.1 Introduction

In stratified difference estimation, the population is divided in sub-populations called strata and independent samples are drawn from each stratum, using the difference estimation method.

The rationale behind stratification and the candidate criteria to implement stratification are the same as presented for simple random sampling (see Section 6.1.2.1). As for simple random sampling, the stratification by level of expenditure per operation is usually a good approach whenever it is expected that the level of error is associated with the level of expenditure.

If stratification by level of expenditure is implemented, and if it is possible to find a few extremely high value operations it is recommended that they are included in a high-value stratum, that will be a 100% audited. In this case, the items belonging to the 100% stratum should be treated separately and the sampling steps will apply only to the population of the low-value items. The reader should be aware that the planned precision for sample size determination should be however based on the total book value of the population. Indeed, as the source of error is the low-value items stratum, but the planned precision is due at population level, the tolerable error and the anticipated error should be calculated at population level, as well.

6.2.2.2 Sample size

The sample size is computed using the same approach as for simple random sampling

$$n = \left(\frac{N \times z \times \sigma_w}{TE - AE}\right)^2$$

where σ_w^2 is the weighted mean of the variances of the errors for the whole set of strata (see Section 6.1.2.2 for further details).

As usual, the variances can be based on historical knowledge or on a preliminary/pilot sample of small sample size. In this later case, the pilot sample can, as usual, subsequently be used as a part of the main sample for audit.

Once the total sample size, n, is computed the allocation of the sample by stratum is as follows:

$$n_h = \frac{N_h}{N} \times n.$$

This is the same general allocation method, also used in simple random sampling, known as proportional allocation. Again, other allocation methods are available and can be applied.

6.2.2.3 Extrapolation

Based on *H* randomly selected samples of operations, the size of each one has been computed according to the above formula, the projected error at the level of the population can be computed in as:

$$EE = \sum_{h=1}^{H} N_h \frac{\sum_{i=1}^{n_h} E_i}{n_h}.$$

In practice, in each group of the population (stratum) multiply the average of observed errors in the sample by the number of operations in the stratum (N_h) and sum all the results obtained for each stratum.

In a second step the correct book value (the correct expenditure that would be found if all the operations in the population were audited) can be projected using the following formula:

$$CBV = BV - \sum_{h=1}^{H} N_h \frac{\sum_{i=1}^{n_h} E_i}{n_h}$$

In the above formula: 1) in each stratum calculate the average of observed errors in the sample; 2) in each stratum multiply the average sample error by the stratum size (N_h); 3) sum these results for all the strata; 4) subtract this value from the total book value of the population (BV). The result of the sum is a projection for the correct book value (CBV) in the population.

6.2.2.4 Precision

Remember that precision (sampling error) is a measure of the uncertainty associated with the projection (extrapolation). In stratified difference estimation is given by the following formula

$$SE = N \times z \times \frac{s_w}{\sqrt{n}}$$

where s_w^2 is the weighted mean of the variance of errors for the whole set of strata calculated from the same sample used to project the errors to the population:

$$s_w^2 = \sum_{i=1}^{H} \frac{N_h}{N} s_{eh}^2, h = 1,2, \ldots, H;$$

and s_{eh}^2 is the estimated variance of errors for the sample of stratum *h*

$$s_{eh}^2 = \frac{1}{n_h - 1} \sum_{i=1}^{n_h} (E_{hi} - \bar{E}_h)^2 , h = 1,2, \ldots, H$$

6.2.2.5 Evaluation

To conclude about the materiality of the errors the lower limit for the corrected book value should firstly be calculated. This lower limit is equal to

$$LL = CBV - SE$$

The projection for the correct book value and the lower limit should both be compared to the difference between the book value (declared expenditure) and the maximum tolerable error (TE)

$$BV - TE = BV - 2\% \times BV = 98\% \times BV$$

Finally, audit conclusions should be drawn using exactly the same approach presented in Section 6.2.1.5 for standard difference estimation.

6.2.2.6 Example

Let us assume a population of expenditure declared to the Commission in a given year for operations in a group of programmes. The management and control system is shared by the group of programmes and the system audits carried out by the audit authority have yielded a high assurance level. Therefore, sampling this programme can be done with a confidence level of 60%.

The AA has reasons to believe that there are substantial risks of error for high value operations, whatever the programme they belong to. Further, there are reasons to expect that there are different error rates across the programmes. Bearing in mind all this information, the AA decides to stratify the population by programme and by expenditure (isolating in a 100% sampling stratum all the operations with book value larger than the materiality).

The following table summarizes the available information:

Population size (number of operations)	4,872
Population size – stratum 1 (number of operations in programme 1)	1,520
Population size – stratum 2 (number of operations in programme 2)	3,347
Population size – stratum 3 (number of operations with BV > materiality level)	5
Book value (sum of the expenditure in the reference period)	6,440,727,190 €
Book value – stratum 1 (total expenditure in programme 1)	3,023,598,442 €
Book value – stratum 2 (total expenditure in programme 2)	2,832,769,525 €
Book value – stratum 3 (total expenditure of operations with BV > Materiality level)	584,359,223 €

The 100% sampling stratum containing the 5 high-value operation should be treated separately as stated in section 6.2.2.1. Therefore, hereafter, the value of N corresponds to the total number of operations in the population, deducted of the number of the operations included in the 100% sampling stratum, i.e. 4,867 (= 4,872 – 5) operations.

The first step is to compute the required sample size, using the formula:

$$n = \left(\frac{N \times z \times \sigma_w}{TE - AE}\right)^2$$

where z is 0.842 (coefficient corresponding to a 60% confidence level) and TE, the tolerable error, is 2% (maximum materiality level set by the Regulation) of the book value, i.e. 2% x 6,440,727,190 € = 128,814,544 €. Based on previous year experience and on the conclusion of the report on managing and control systems the AA expects an error rate not larger than 0.4%, Thus AE, the anticipated error, is 0.4%, i.e., 0.4% x 6,440,727,190 € = 25,762,909 €.

Since the third stratum is a 100% sampling stratum, the sample size for this stratum is fixed and is equal to the size of the population, that is, the 5 high-value operations. The sample size for the remaining two strata is computed using the above formula, where σ_w^2 is the weighted average of the variances of the errors for the two remaining strata:

$$\sigma_w^2 = \sum_{i=1}^{2} \frac{N_h}{N} \sigma_{eh}^2, h = 1,2;$$

and σ_{eh}^2 is the variance of errors in each stratum. The variance of the errors is computed for each stratum as an independent population as

$$\sigma_{eh}^2 = \frac{1}{n_h^p - 1} \sum_{i=1}^{n_h^p} (E_{hi} - \bar{E}_h)^2 , h = 1,2, \ldots, H$$

where E_{hi} represent the individual errors for units in the sample of stratum h and $\bar{E}_h$ represent the mean error of the sample in stratum h. A preliminary sample of 20 operations of stratum 1 yielded an estimate for the standard deviation of errors of 21,312 €.

The same procedure was followed for the population of stratum 2. A preliminary sample of 20 operations of stratum 2 yielded an estimate for the standard deviation of errors of 215,546 €:

Stratum 1 – preliminary estimate of standard deviation of errors	21,312 €
Stratum 2 - preliminary estimate of standard deviation of errors	215,546 €

Therefore, the weighted mean of the variances of the errors of these two strata is

$$\sigma_w^2 = \frac{1,520}{4,867} \times 21,312^2 + \frac{3,347}{4,867} 215,546^2 = 32,092,103,451$$

The minimum sample size is given by:

$$n = \left(\frac{4,867 \times 0.845 \times \sqrt{32,092,103,451}}{128,814,544 \; - 25,762,909} \right)^2 \approx 51$$

These 51 operations are allocated by stratum as follows:

$$n_1 = \frac{1,520}{4,867} \times 51 \approx 16,$$

$$n_2 = n - n_1 = 35$$

and

$$n_3 = N_3 = 5$$

The total sample size is therefore 60 operations:

- 20 operations of stratum 1 preliminary sample, plus
- 35 operations of stratum 2 (the 20 preliminary sample operations plus an additional sample of 15 operations); plus
- 5 high-value operations.

The following table shows the sample results for the whole sample of 60 operations:

Sample results – stratum 1		
A	Sample book value	37,344,981 €
B	Sample total error	77,376 €
C	Sample average error (C=B/16)	3,869 €
D	Sample standard deviation of errors	16,783 €
Sample results – stratum 2		
E	Sample book value	722,269,643 €
F	Sample total error	264,740 €
G	Sample average error (G=F/35)	7,564 €
H	Sample standard deviation of errors	117,335 €
Sample results -100% audit stratum		
I	Sample book value	584,359,223 €
J	Sample total error	7,240,855 €
K	Sample average error (I=J/5)	1,448,171 €

Projecting the error for the two sampling strata is calculated by multiplying the sample average error by the population size. The sum of these two figures, added to the error found in the 100% sampling stratum, is the expected error at population level:

$$EE = \sum_{h=1}^{3} 1520 \times 3{,}869 + 3{,}347 \times 7{,}564 + 7{,}240{,}855 = 38{,}438{,}139$$

The projected error rate is computed as the ratio between the extrapolated error and the book value of the population (total expenditure):

$$r_1 = \frac{39{,}908{,}283}{6{,}440{,}727{,}190} = 0.60\%$$

The correct book value (the correct expenditure that would be found if all the operations in the population were audited) can be projected using the following formula:

$$CBV = BV - EE = 6{,}440{,}727{,}190 - 39{,}908{,}283 = 6{,}402{,}289{,}051$$

Given the standard deviations of errors in the sample of both strata (table with sample results), the weighted mean of the variance of errors for the whole set of sampling strata is:

$$s_w^2 = \sum_{h=1}^{2} \frac{N_h}{N} s_{eh}^2 = \frac{1{,}520}{4{,}867} \times 16{,}783^2 + \frac{3{,}347}{4{,}867} \times 117{,}335^2 = 9{,}555{,}777{,}062$$

The precision of the projection is given by

$$SE = N \times z \times \frac{s_w}{\sqrt{n}} = 4{,}867 \times 0.842 \times \frac{\sqrt{9{,}555{,}777{,}062}}{\sqrt{55}} = 54{,}016{,}333$$

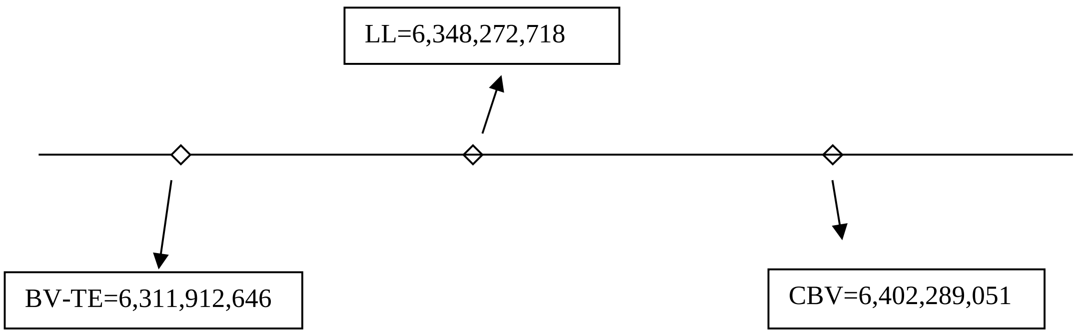
To conclude about the materiality of the errors the lower limit for the corrected book value should firstly be calculated. This lower limit is equal to

$$LL = CBV - SE = 6{,}402{,}289{,}051 - 54{,}016{,}333 = 6{,}348{,}272{,}718$$

The projection for the correct book value and the lower limit should both be compared to the difference between the book value (declared expenditure) and the maximum tolerable error (TE):

$$BV - TE = 6{,}440{,}727{,}190 - 128{,}814{,}544 = 6{,}311{,}912{,}646$$

Since $BV - TE$ is lower than the lower limit $CBV - SE$ than there is enough evidence that errors in the programme are lower than materiality threshold.



6.2.3 Difference estimation – two periods

6.2.3.1 Introduction

The audit authority may decide to carry out the sampling process in several periods during the year (typically two semesters). The major advantage of this approach is not related with sample size reduction, but mainly allowing spreading the audit workload over the year, thus reducing the workload that would be done at the end of the year based on just one observation.

With this approach the year population is divided in two sub-populations, each one corresponding to the operations and expenditure of each semester. Independent samples are drawn for each semester, using the standard simple random sampling approach.

6.2.3.2 Sample size

The sample size is computed using the same approach as for simple random sampling in two semesters. See Section 6.1.3.2 for further details.

6.2.3.3 Extrapolation

Based on the two sub-samples of each semester, the projected error at the level of the population can be computed as:

$$EE = N_1.\frac{\sum_{i=1}^{n_1} E_{1i}}{n_1} + N_2.\frac{\sum_{i=1}^{n_2} E_{2i}}{n_2}$$

In practice, in each semester multiply the average of observed errors in the sample by the number of operations in the population (N_t) and sum the results obtained for both semesters.

In a second step the correct book value (the correct expenditure that would be found if all the operations in the population were audited) can be projected using the following formula:

$$CBV = BV - EE$$

where BV is the yearly book value (including the two semesters) and EE the above projected error.

6.2.3.4 Precision

Remember that precision (sampling error) is a measure of the uncertainty associated with the projection (extrapolation). It is given by the following formula

$$SE \quad = z \times \sqrt{\left(N_1^2 \times \frac{s_{e1}^2}{n_1} + N_2^2 \times \frac{s_{e2}^2}{n_2}\right)}$$

where s_{et} is the standard-deviation of errors in the sample of semester *t*, (now calculated from the same samples used to project the errors to the population)

$$s_{et}^2 = \frac{1}{n_t - 1} \sum_{i=1}^{n_t} (E_{ti} - \bar{E}_t)^2$$

6.2.3.5 Evaluation

To conclude about the materiality of the errors the lower limit for the corrected book value should firstly be calculated. This lower limit is equal to

$$LL = CBV - SE$$

The projection for the correct book value and the lower limit should both be compared to the difference between the book value (declared expenditure) and the maximum tolerable error (*TE*)

$$BV - TE = BV - 2\% \times BV = 98\% \times BV$$

Finally, audit conclusions should be drawn using exactly the same approach presented in Section 6.2.1.5 for standard difference estimation.

6.2.3.6 Example

An AA has decided to split the audit workload between the two semesters of the year. At the end of the first semester the characteristics of the population are the following:

Declared expenditure (DE) at the end of first semester	1,237,952,015 €
Size of population (operations - first semester)	3,852

Based on the past experience, the AA knows that usually all the operations included in the programmes at the end of the reference period are already active in the population of the first semester. Further it is expected that the declared expenditure at the end of the first semester represents about 30% of the total declared expenditure at the end of the reference period. Based on these assumptions a summary of the population is described in the following table:

Declared expenditure (DE) of the first semester	1,237,952,015 €
Declared expenditure (DE) of the second semester (predicted)	2,888,554,702 €
Size of population (operations - period 1)	3,852
Size of population (operations – period 2, predicted)	3,852

The system audits carried out by the audit authority have yielded a low assurance level. Therefore, sampling this programme should be done with a confidence level of 90%.

At the end of the first semester the global sample size (for the set of two semesters) is computed as follows:

$$n = \left(\frac{N \times z \times \sigma_w}{TE - AE}\right)^2$$

where σ_w^2 is the weighted mean of the variances of the errors for in each semester:

$$\sigma_w^2 = \frac{N_1}{N}\sigma_{e1}^2 + \frac{N_2}{N}\sigma_{e2}^2$$

and σ_{et}^2 is the variance of errors in each period t (semester). The variance of the errors for each semester is computed as an independent population as

$$\sigma_{et}^2 = \frac{1}{n_t^p - 1}\sum_{i=1}^{n_t^p}(E_{ti} - \bar{E}_t)^2\,, t = 1,2$$

where E_{ti} represent the individual errors for units in the sample of semester t and $\bar{E}_t$ represent the mean error of the sample in semester t.

Since the value of σ_{et}^2 is unknown, the AA decided to draw a preliminary sample of 20 operations at the end of first semester of the current year. The sample standard deviation of errors in this preliminary sample at first semester is 49,534 €. Based on professional judgement and knowing that usually the expenditure in second semester is larger than in first, the AA has made a preliminary prediction of standard deviation of errors for the second semester to be 20% larger than in first semester, that is, 59,441 €. Therefore, the weighted average of the variances of the errors is:

$$\sigma_w^2 = \frac{N_1}{N_1 + N_2}\sigma_{e1}^2 + \frac{N_2}{N_1 + N_2}\sigma_{e2}^2 = 0.5 \times 69{,}534^2 + 0.5 \times 59{,}441^2 = 2{,}993{,}412{,}930.$$

Note that the population size in each semester is equal to the number of active operations (with expenditure) in each semester.

At the end of first semester the global sample size for the whole year is:

$$n = \left(\frac{N \times z \times \sigma_w}{TE - AE}\right)^2$$

where σ_w^2 is the weighted average of the variances of the errors for the whole set of strata (see Section 7.1.2.2 for further details), z is 1.645 (coefficient corresponding to a 90% confidence level), and TE, the tolerable error, is 2% (maximum materiality level set by the Regulation) of the book value. The total book value comprises the true book value at the end of the first semester plus the predicted book value for the second semester 4,126,506,717, which means that tolerable error is 2% x 4,126,506,717 € = 82,530,134 €. The preliminary sample on the first semester population yields a sample error rate of 0.6%. The audit authority expects these error rate remains constant all over the year. Thus AE, the anticipated error, is 0.6% x 4,126,506,717 € = 24,759,040 €. The sample size for the whole year is:

$$n = \left(\frac{3852 \times 2 \times 1.645 \times \sqrt{5{,}898{,}672{,}130}}{82{,}530{,}134 - 24{,}759{,}040}\right)^2 \approx 145$$

The allocation of the sample by semester is as follows:

$$n_1 = \frac{N_1}{N_1 + N_2}\, n \approx 73$$

and

$$n_2 = n - n_1 = 72$$

The first semester sample yielded the following results:

Sample book value - first semester	41,009,806 €
Sample total error - first semester	577,230 €
Sample standard deviation of errors - first semester	52,815 €

At the end of the second semester more information is available, in particular, the number of operations active in the second semester is correctly known, the sample variance of errors s_{e1} calculated from the sample of the first semester is already available and the standard deviation of errors for the second semester σ_{e2} can now be more accurately assessed using a preliminary sample of real data.

The AA realises that the assumption made at the end of the first semester on the total number of operations remains correct. Nevertheless, there are two parameters for which updated figures should be used.

Firstly, the estimate of the standard deviation of errors based on the first semester sample of 73 operations yielded an estimate of 52,815 €. This new value should now be used to reassess the planned sample size. Secondly, based on a new preliminary sample of 20 operations of the second semester population, the audit authority estimates the standard deviation of errors for the second semester to be 87,369 € (faraway of the predicted value at the end of the first period). We conclude that the standard deviation of errors in the first semester used to plan the sample size is close to the value obtained at the end of the first semester. Nevertheless, the standard deviation of error in the second semester used to plan the sample size is far away from the figure given by the new preliminary sample. As a result, the sample for the second semester should be revised.

Further, the predicted total book value of the second semester population should be replaced by the real one, 5,202,775,175 €, instead of the predicted value of 2,888,554,702 €.

Parameter	End of first semester	End of second semester
Standard deviation of errors in the first semester	49,534 €	52,815 €
Standard deviation of errors in the second semester	59,441 €	87,369 €
Total expenditure in the second semester	2,888,554,702 €	5,202,775,175 €

Taking into consideration these two adjustments, the recalculated sample size of the second semester is

$$n_2 = \frac{\left(z \times N_2 \ \times \sigma_{e2}\right)^2}{(TE - AE)^2 - z^2 \times \frac{N_1^2}{n_1} \times s_{e1}^2}$$

$$= \frac{(1.645 \ \times 3{,}852 \times 107{,}369)^2}{(128{,}814{,}544 \ - 38{,}644{,}363)^2 - 1.645^2 \times \frac{3{,}852^2}{142} \times 65{,}815^2} \approx 47$$

Auditing the 73 operations in the first semester plus these 47 operations in second semester will provide the auditor with information on the total error for the sampled operations. The previous preliminary sample of 20 operations is used as part of the main sample. Therefore, the auditor has only to select 27 further operations in second semester.

The second semester sample yielded the following results:

Sample book value - second semester	59,312,212 €
Sample total error - second semester	588,336 €
Sample standard deviation of errors - first semester	78,489 €

Based on both samples, the projected error at the level of the population can be computed as:

$$EE = N_1 \times \frac{\sum_{i=1}^{n_1} E_{1i}}{n_1} + N_2 \times \frac{\sum_{i=1}^{n_2} E_{2i}}{n_2} = 3{,}852 \times \frac{577{,}230}{142} + 3{,}852 \times \frac{588{,}336}{68}$$

$$= 78{,}677{,}283$$

Corresponding to an projected error rate of 1,22%

In a second step the correct book value (the correct expenditure that would be found if all the operations in the population were audited) can be projected using the following formula:

$$CBV = BV - EE = 6{,}440{,}727{,}190 - 78{,}677{,}283 = 6{,}362{,}049{,}907$$

where BV is the yearly book value (including the two semesters) and EE the above projected error.

The precision (sampling error) is a measure of the uncertainty associated with the projection (extrapolation) and it is given by the following formula:

$$SE = z \times \sqrt{\left(N_1^2 \times \frac{s_{e1}^2}{n_1} + N_2^2 \times \frac{s_{e2}^2}{n_2}\right)}$$

$$= 1.645 \times \sqrt{\left(3852^2 \times \frac{52{,}815^2}{73} + 3852^2 \times \frac{78{,}849^2}{47}\right)} = 82{,}444{,}754$$

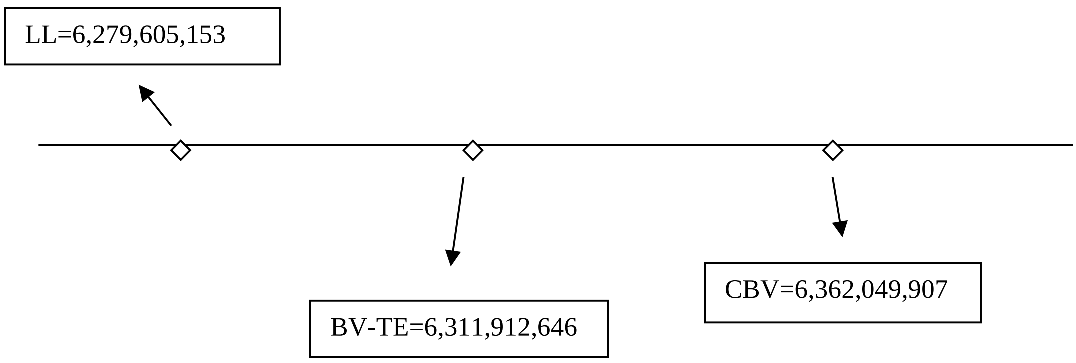
To conclude about the materiality of the errors the lower limit for the corrected book value should firstly be calculated. This lower limit is equal to

$$LL = CBV - SE = 6{,}362{,}049{,}907 - 82{,}444{,}754 = 6{,}279{,}605{,}153$$

The projection for the correct book value and the lower limit should both be compared to the difference between the book value (declared expenditure) and the maximum tolerable error (TE)

$$BV - TE = 6{,}440{,}727{,}190 - 128{,}814{,}544 = 6{,}311{,}912{,}646$$

As $BV - TE$ is between the lower limit $LL = CBV - SE$ and CBV, please refer to section 4.12 for more details on the analysis to be done.



6.3 Monetary unit sampling

6.3.1 Standard approach

6.3.1.1 Introduction

Monetary unit sampling is the statistical sampling method that uses the monetary unit as an auxiliary variable for sampling. This approach is usually based on systematic sampling with probability proportional to size (PPS), i.e. proportional to the monetary value of the sampling unit (higher value items have higher probability of selection).

This is probably the most popular sampling method for auditing and is particularly useful if book values have high variability and there is positive correlation (association) between errors and book values. In other words, whenever it is expected that items with higher values tend to exhibit higher errors, situation that frequently holds in the audit framework.

Whenever the above conditions hold, i.e. book values have high variability and error are positively correlated (associated) with book values, then MUS tends to produce smaller sample sizes than equal probability based methods, for the same level of precision.

It should also be noted that samples produced by this method will typically have an over representation of high value items and an under representation of low value items. This is not a problem by itself as the method accommodates this fact in the extrapolation process, but makes sample results (e.g. sample error rate) as non-interpretable (only extrapolated results can be interpreted).

As equal probability based methods, this method can be combined with stratification (favourable conditions for stratification are discussed in Section 5.2).

6.3.1.2 Sample size

Computing sample size *n* within the framework of monetary unit sampling relies on the following information:

- Population book value (total declared expenditure) *BV*
- Confidence level determined from systems audit and the related coefficient z from a normal distribution (see Section 5.3)
- Maximum tolerable error *TE* (usually 2% of the total expenditure)
- Anticipated error *AE* chosen by the auditor according to professional judgment and previous information
- The standard deviation σ_r of the error rates (produced from a MUS sample).

The sample size is computed as follows:

$$n = \left(\frac{z \times BV \times \sigma_r}{TE - AE}\right)^2$$

where σ_r is the standard-deviation of error rates produced from a MUS sample. To obtain an approximation to this standard-deviation before performing the audit the Member States will have to rely either on historical knowledge (variance of the error rates in a sample of past period) or on a preliminary/pilot sample of low sample size, n^p (sample size for the preliminary sample is recommended to be not less than 20 to 30 operations). In any case, the variance of the error rates (square of the standard-deviation) is obtained through

$$\sigma_r^2 = \frac{1}{n^p - 1}\sum_{i=1}^{n^p}(r_i - \bar{r})^2\,;$$

where $r_i = \frac{E_i}{BV_i}$ is the error rate of an operation[27] and is defined as the ratio between E_i and the book value (the expenditure declared to the Commission, BV_i) of the i-th operation included in sample and $\bar{r}$ represent the mean error rate in the sample, that is:

$$\bar{r} = \frac{1}{n^p}\sum_{i=1}^{n^p}\frac{E_i}{BV_i}$$

As usual, if the standard-deviation is based on a preliminary sample, this sample can be subsequently used as a part of the full sample chosen for audit. Nevertheless, selecting and observing a preliminary sample in MUS framework is a much more complex task than in simple random sampling or difference estimation. This is because high value items are more frequently chosen to the sample. Therefore, observing a 20 to 30 operations sample will frequently constitute a heavy task. Due to this reason, in the framework of MUS it is highly recommended that the estimation of the standard-deviation σ_r is based on historical data, in order to avoid the need to select a preliminary sample.

6.3.1.3 Sample selection

After determining sample size it is necessary to identify the high value population units (if any) that will belong to a high value stratum to be audited a 100%. The cut-off value for determining this top stratum is equal to the ratio between the book value (*BV*) and the planed sample size (*n*). All items whose book value is higher than this cut-off (if $BV_i > BV/n$) will be placed in the 100% audit stratum.

The sampling size to be allocated to the non-exhaustive stratum, n_s , is computed as the difference between n and the number of sampling units (e.g. operations) in the exhaustive stratum (n_e).

Finally the selection of the sample in the non-exhaustive stratum will be made using probability proportional to size, i.e. proportional to the item book values BV_i[28]. A

[27] Whenever the book value of unit i (BV_i) is larger than the cut-off BV/n the ratio $\frac{E_i}{BV_i}$ should be substituted by $\frac{E_i}{BV/n}$, where BV represents the book value of current population if a preliminary sample is used or the book value of the historic population if an historic sample is used. Also, n represents the sample size of the preliminary sample (if used) or the sample size of the historic sample.

[28] This can be performed using specialized software, any statistical package or even a basic software as Excel. Note that in some software the division between the exhaustive high value stratum and the non-

popular way to implement the selection is through systematic selection, using a sampling interval equal to the total expenditure in the non-exhaustive stratum (BV_s) divided by the sample size (n_s), i.e.

$$SI = \frac{BV_s}{n_s}$$

In practice the sample is selected from a randomised list of items (usually operations), selecting each item containing the xth monetary unit, x being equal to the sampling interval and having a random starting point between 1 and SI. For instance, if a population has a book value of 10,000,000€, and we select a sample of 40 operations, every operation containing the 250,000th€ will be selected.

Note that in practice it may happen that after the calculation of the sampling interval based on the expenditure and sample size of the sampling stratum, some population units will still exhibit an expenditure larger than this sampling interval BV_s/n_s (although they have not previously exhibit an expenditure larger than the cut-off (BV/n). In fact, all items whose book value is still higher than this interval ($BV_i > BV_s/n_s$) have also to be added to the high-value stratum. If this happens, and after moving the new items to the high value stratum, the sampling interval has to be recalculated for the sampling stratum taking into consideration the new values for the ratio BV_s/n_s. This iterative process may have to be performed several times until a moment where no further units present expenditure larger than the sampling interval.

6.3.1.4 Projected error

The projection of the errors to the population should be made differently for the units in the exhaustive stratum and for the items in the non-exhaustive stratum.

For the exhaustive stratum, that is, for the stratum containing the sampling units with book value larger than the cut-off, $BV_i > \frac{BV}{n}$, the projected error is just the summation of the errors found in the items belonging to the stratum:

$$EE_e = \sum_{i=1}^{n_e} E_i$$

For the non-exhaustive stratum, i.e. the stratum containing the sampling units with book value smaller or equal to the cut-off value, $BV_i \leq \frac{BV}{n}$ the projected error is

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exhaustive stratum is not necessary as they automatically accommodate the selection of units with a 100% selection probability.

$$EE_s = SI \sum_{i=1}^{n_s} \frac{E_i}{BV_i}$$

To calculate this projected error:
1) for each unit in the sample calculate the error rate, i.e. the ration between the error and the respective expenditure $\frac{E_i}{BV_i}$
2) sum these error rates over all units in the sample
3) multiply the previous result by the sampling interval (SI)

The projected error at the level of population is just the sum of these two components:

$$EE = EE_e + EE_s$$

6.3.1.5 Precision

Precision is a measure of the uncertainty associated with the extrapolation. It represents sampling error and should be calculated in order to subsequently produce a confidence interval.

The precision is given by the formula:

$$SE = z \times \frac{BV_s}{\sqrt{n_s}} \times s_r$$

where s_r is the standard-deviation of error rates in the sample of the non-exhaustive stratum (calculated from the same sample used to extrapolate the errors to the population)

$$s_r^2 = \frac{1}{n_s - 1} \sum_{i=1}^{n_s} (r_i - \bar{r}_s)^2$$

having $\bar{r}_s$ equal to the simple average of error rates in the sample of the stratum

$$\bar{r}_s = \frac{\sum_{i=1}^{n_s} \frac{E_i}{BV_i}}{n_s}$$

Note that the sampling error is only computed for the non-exhaustive stratum, since there is no sampling error to account for in the exhaustive stratum.

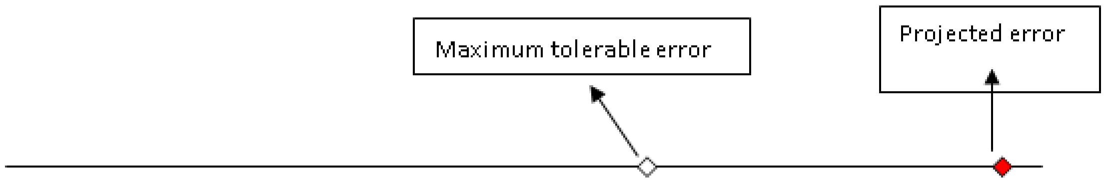
6.3.1.6 Evaluation

To draw a conclusion about the materiality of the errors the upper limit of error (ULE) should be calculated. This upper limit is equal to the summation of the projected error EE itself and the precision of the extrapolation

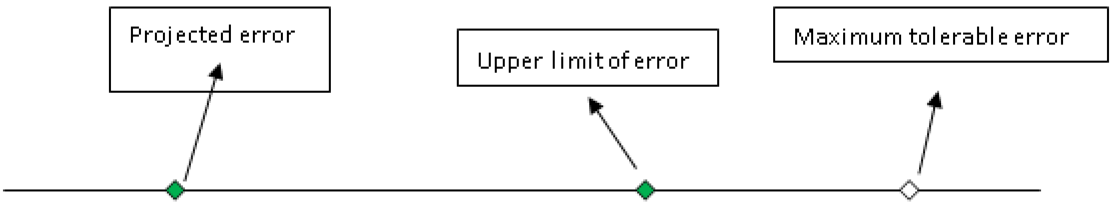
$$ULE = EE + SE$$

Then the projected error and the upper limit should both be compared to the maximum tolerable error to draw audit conclusions:

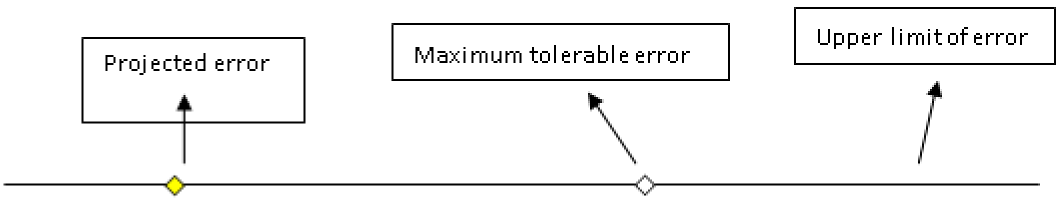
- If projected error is larger than maximum tolerable error, it means that the auditor would conclude that there is enough evidence to support that errors in the population are larger than materiality threshold:



- If the upper limit of error is lower than maximum tolerable error, then the auditor should conclude that errors in the population are lower than materiality threshold.



If the projected error is lower than maximum tolerable error but the upper limit of error is larger please refer to section 4.12 for more details on the analysis to be done.



6.3.1.7 Example

Let's assume a population of expenditure declared to the Commission in a given year for operations in a programme. The system audits performed by the audit authority have yielded a low assurance level. Therefore, sampling this programme should be done with a confidence level of 90%.

The population is summarised in the following table:

Population size (number of operations)	3,852
Book value (sum of the expenditure in the reference period)	4,199,882,024 €

The sample size is computed as follows:

$$n = \left(\frac{z \times BV \times \sigma_r}{TE - AE}\right)^2$$

where σ_r is the standard-deviation of error rates produced from a MUS sample. To obtain an approximation to this standard deviation the AA decided to use the standard deviation of previous year. The sample of the previous year was constituted by 50 operations, 5 of which have a book value larger than the sampling interval.

The following table shows the results of the previous year's audit for these 5 operations.

Operation ID	Book Value (BV)	Correct Book Value (CBV)	Error	Error rate
1850	115,382,867 €	115,382,867 €	- €	-
4327	129,228,811 €	129,228,811 €	- €	-
4390	142,151,692 €	138,029,293 €	4,122,399 €	0.0491
1065	93,647,323 €	93,647,323 €	- €	-
1817	103,948,529 €	100,830,073 €	3,118,456 €	0.0371

Notice that the error rate (last column) is computed as $r_i = \frac{E_i}{BV/n}$ the ratio between the error of the operation and the BV divided by the initial sample size, that is 50, because these operations have a book value larger than the sampling interval (for more details please check Section 6.3.1.2).

The following tables summarises the results of last year's audit for the sample of 45 operations with the book value smaller than the cut-off value.

	A	B	C	D	E
1	**Operation ID**	**Book Value (BV)**	**Audit Value (AV)**	**Error**	**Error rate**
2	239	10,173,875 €	9,962,918 €	210,956 €	0.0207
3	424	23,014,045 €	23,014,045 €	- €	
4	2327	32,886,198 €	32,886,198 €	- €	
5	5009	34,595,201 €	34,595,201 €	- €	
6	1491	78,695,230 €	78,695,230 €	- €	
7	(...)	(...)	(...)	(...)	(...)
39	2596	8,912,999 €	8,909,491 €	3,508 €	0.00039
40	779	26,009,790 €	26,009,790 €	- €	-
41	1250	264,950 €	264,950 €	- €	-
42	3895	30,949,004 €	30,949,004 €	- €	-
43	2011	617,668 €	617,668 €	- €	-
44	4796	335,916 €	335,916 €	- €	-
45	3632	7,971,113 €	7,971,113 €	- €	-
46	2451	17,470,048 €	17,470,048 €	- €	-
47	**Sample standard deviation:=STDEV.S(E2:E46;0;0;0.0491;0;0.0371)------------->**				0.085

Based on this preliminary sample the standard deviation of the error rates, σ_r , is 0.085, (computed in MS Excel as **":=STDEV.S(E2:E46;0;0;0.0491;0;0.0371)**")

Given this estimate for the standard deviation of error rates, the maximum tolerable error and the anticipated error, we are in conditions to compute the sample size. Assuming a tolerable error which is 2% of the total book value, 2% x 4,199,882,024 = 83,997,640, (materiality value set by the regulation) and an anticipated error rate of 0.4%, 0.4% x 4,199,882,024 = 16,799,528 (which corresponds to strong belief of the AA based both on past year's information and the results of the report on assessment of management and control systems),

$$n = \left(\frac{1.645 \times 4{,}199{,}882{,}024 \times 0.085}{83{,}997{,}640 - 16{,}799{,}528}\right)^2 \approx 77$$

In first place, it is necessary to identify the high value population units (if any) that will belong to a high-value stratum to be submitted at a 100% audit work. The cut-off value for determining this top stratum is equal to the ratio between the book value (*BV*) and the planed sample size (*n*). All items whose book value is higher than this cut-off (if $BV_i > BV/n$) will be placed in the 100% audit stratum. In this case the cut-off value is 4,199,882,024/77=54,593,922 €.

The AA put in an isolated stratum all the operations with book value larger than 54,593,922, which corresponds to 8 operations, amounting to 786,837,081 €

The sampling interval for the remaining population is equal to the book value in the non-exhaustive stratum (BV_s) (the difference between the total book value and the book value of the eight operations belonging to the top stratum) divided by the number of operations to be selected (77 minus the 8 operations in the top stratum).

$$Sampling\ interval = \frac{BV_s}{n_s} = \frac{4{,}199{,}882{,}024 - 786{,}837{,}081}{69} = 49{,}464{,}419$$

The AA has checked that there were no operations with book values higher than the interval, thus the top stratum includes only the 8 operations with book-value larger than the cut-off value. The sample is selected from a randomised list of operations, selecting each item containing the 49,464,419th monetary unit.

A file containing the remaining 3,844 operations (3,852 – 8 high value operations) of the population is randomly sorted and a sequential cumulative book value variable is created. A sample value of 69 operations (77 minus 8 high value operations) is drawn using exactly the following procedure.

A random value between 1 and the sampling interval, 49,464,419 has been generated (22,006,651). The first selection corresponds to the first operation in the file with the accumulated book value greater or equal to 22,006,651.

The second selection corresponds to the first operation containing the 71,471,070th monetary unit ($22{,}006{,}651 + 49{,}464{,}419 = 71{,}471{,}070$ starting point plus the sampling interval). The third operation to be selected corresponds to the first operation containing the 120,935,489th monetary unit ($71{,}471{,}070 + 49{,}464{,}419 = 120{,}935{,}489$ previous monetary unit point plus the sampling interval) and so on…

Operation ID	Book Value (BV)	AcumBV	Sample
239	10,173,875 €	10,173,875 €	No
424	23,014,045 €	33,187,920 €	Yes
2327	32,886,198 €	66,074,118 €	No
5009	34,595,201 €	100,669,319 €	Yes
1491	78,695,230 €	179,364,549 €	Yes
(…)	(…)	(…)	…
2596	8,912,999 €	307,654,321 €	No
779	26,009,790 €	333,664,111 €	Yes
1250	264,950 €	333,929,061 €	No
3895	30,949,004 €	364,878,065 €	No
2011	617,668 €	365,495,733 €	No
4796	335,916 €	365,831,649 €	No
3632	7,971,113 €	373,802,762 €	Yes
2451	17,470,048 €	391,272,810 €	No
(…)	(…)	(…)	…

After auditing the 77 operations, the AA is able to project the error.

Out of the 8 high-value operations (total book value of 786,837,081 €), 3 operations contain error corresponding to an amount of error of 7,616,805 €.

For the remaining sample, the error has a different treatment. For these operations, we follow the following procedure:

1) for each unit in the sample calculate the error rate, i.e. the ration between the error and the respective expenditure $\frac{E_i}{BV_i}$

2) sum these error rates over all units in the sample (computed in MS Excel as **":=SUM(E2:E70)**")

3) multiply the previous result by the sampling interval (SI)

$$EE_s = SI \sum_{i=1}^{n_s} \frac{E_i}{BV_i}$$

	A	B	C	D	E
1	**Operation ID**	**Book Value (BV)**	**Audited Value (AV)**	**Error**	**Error rate**
2	5002	48,725,645 €	48,725,645 €	- €	-
3	779	26,009,790 €	333,664,111 €	- €	-
4	2073	859,992 €	859,992 €	- €	-
5	239	10,173,875 €	9,962,918 €	210,956 €	0.02
6	989	394,316 €	394,316 €	- €	-
7	65	25,234,699 €	25,125,915 €	108,784 €	0
8	5010	34,595,201 €	34,595,201 €	- €	-
9	(...)	(...)	(...)	(...)	(...)
64	1841	768,278 €	768,278 €	- €	-
65	3672	624,882 €	624,882 €	- €	-
66	2355	343,462 €	301,886 €	41,576 €	0.12
67	959	204,847 €	204,847 €	- €	-
68	608	15,293,716 €	15,293,716 €	- €	-
69	4124	6,773,014 €	6,773,014 €	- €	-
70	262	662 €	662 €	- €	-
71	**Total:=SUM(E2:E70)**--->				1.096
72	**Sample standard deviation:=STDEV.S(E2:E70)**-------------------------------->				0.09

$$EE_s = 49{,}464{,}419 \times 1.096 = 54{,}213{,}004$$

The projected error at the level of population is just the sum of these two components:

$$EE = 7{,}616{,}805 + 54{,}213{,}004 = 61{,}829{,}809$$

The projected error rate is the ratio between the projected error and the total expenditure:

$$r = \frac{61{,}829{,}809}{4{,}199{,}882{,}024} = 1.47\%$$

The standard deviation of error rates in the sampling stratum is 0.09 (computed in MS Excel as **“:=STDEV.S(E2:E70)**”).

The precision is given by:

$$SE = z \times \frac{BV_s}{\sqrt{n_s}} \times s_r = 1.645 \times \frac{4{,}199{,}882{,}024 - 786{,}837{,}081}{\sqrt{69}} \times 0.09 = 60{,}831{,}129$$

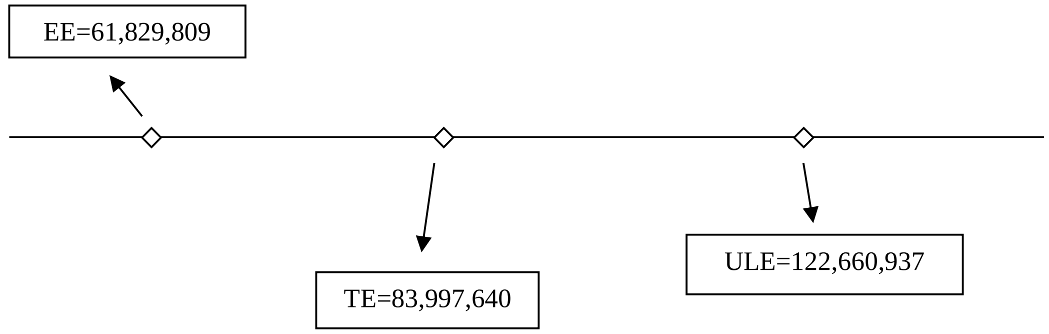
Note that the sampling error is computed for the non-exhaustive stratum only, since there is no sampling error to account for in the exhaustive stratum.

To draw a conclusion about the materiality of the errors the upper limit of error (ULE) should be calculated. This upper limit is equal to the summation of the projected error EE itself and the precision of the extrapolation

$$ULE = 61{,}829{,}809 + 60{,}831{,}129 = 122{,}660{,}937$$

Then the projected error and the upper limit should both be compared to the maximum tolerable error, 83,997,640 €, to draw audit conclusions.

Since the maximum tolerable error is larger than the projected error, but smaller than the upper limit of error, please refer to section 4.12 for more details on the analysis to be done.



6.3.2 *Stratified monetary unit sampling*

6.3.2.1 Introduction

In stratified monetary unit sampling, the population is divided in sub-populations called strata and independent samples are drawn from each stratum, using the standard monetary unit sampling approach.

As usual, candidate criteria to implement stratification should take into account that in stratification we aim to find groups (strata) with less variability than the whole population. Therefore, any variables that we expect to explain the level of error in the operations are also good candidates for stratification. Some possible choices are programmes, regions, responsible bodies, classes based on the risk of the operation, etc.
In stratified MUS, the stratification by level of expenditure is not relevant, as MUS already takes into account the level of expenditure in the selection of sampling units.

6.3.2.2 Sample size

The sample size is computed as follows:

$$n = \left(\frac{z \times BV \times \sigma_{rw}}{TE - AE}\right)^2$$

where σ_{rw}^2 is a weighted mean of the variances of the error rates for the whole set of strata, with the weight for each stratum equal to the ratio between the stratum book value (BV_h) and the book value for the whole population (*BV*).

$$\sigma_{rw}^2 = \sum_{i=1}^{H} \frac{BV_h}{BV} \sigma_{rh}^2, h = 1,2, \dots, H;$$

and σ_{rh}^2 is the variance of error rates in each stratum. The variance of the errors rates is computed for each stratum as an independent population as

$$\sigma_{rh}^2 = \frac{1}{n_h^p - 1} \sum_{i=1}^{n_h^p} (r_{hi} - \bar{r}_h)^2 , h = 1,2, \dots, H$$

where $r_{hi} = \frac{E_i}{BV_i}$ represent the individual error rates for units in the sample of stratum h and $\bar{r}_h$ represent the mean error rate of the sample in stratum h[29].

As previously presented for the standard MUS method these values can be based on historical knowledge or on a preliminary/pilot sample of low sample size. In this later case the pilot sample can as usual subsequently be used as a part of the sample chosen for audit. The recommendation of calculating these parameters using historical data again holds, in order to avoid the need to select a preliminary sample. When starting applying the stratified MUS method for the first time, it may happen that historical stratified data is unavailable. In this case, sample size can be determined using the formulas for the standard MUS method (see Section 6.3.1.2). Obviously the price to by this lack of historical knowledge is that on the first period of audit the sample size will be larger than in fact would be needed if that information were available. Nevertheless, the information collected in the first period of application of the stratified MUS method can be applied in future periods for sample size determination.

Once the total sample size, n, is computed the allocation of the sample by stratum is as follows:

$$n_h = \frac{BV_h}{BV} n.$$

This is a general allocation method, where the sample is allocated to strata proportionally to the expenditure (book value) of the strata. Other allocation methods are available. A more tailored allocation may in some cases bring additional precision gains or reduction of sample size. The adequacy of other allocation methods to each specific population requires some technical knowledge in sampling theory.

6.3.2.3 Sample selection

In each stratum h, there will be two components: the exhaustive group inside stratum h (that is, the group containing the sampling units with book value larger than the cut-off value, $BV_{hi} > \frac{BV_h}{n_h}$); and the sampling group inside stratum h (that is, the group containing the sampling units with book value smaller or equal than the cut-off value, $BV_{hi} \leq \frac{BV_h}{n_h}$)

[29] Whenever the book value of unit i (BV_i) is larger than the cut-off BV_h/n_h the ratio $\frac{E_i}{BV_i}$ should be substituted by the ratios $\frac{E_i}{BV_h/n_h}$.

After determining sample size, it is necessary to identify in each of the original stratum (h) the high value population units (if any) that will belong to a high value group to be audited a 100%. The cut-off value for determining this top group is equal to the ratio between the book value of the stratum (BV_h) and the planed sample size (n_h). All items whose book value is higher than this cut-off (if $BV_{hi} > \frac{BV_h}{n_h}$) will be placed in the 100% audit group.

The sampling size to be allocated to the non-exhaustive group, n_{hs} , is computed as the difference between n_h and the number of sampling units (e.g. operations) in the exhaustive group of the stratum (n_{he}).

Finally the selection of the samples is done in the non-exhaustive group of each stratum using probability proportional to size, i.e. proportional to the item book values BV_i. A common way to implement the selection is through systematic selection, using a selection interval equal to the total expenditure in the non-exhaustive group of the stratum (BV_{hs}) divided by the sample size (n_{hs}) [30], i.e.

$$SI_h = \frac{BV_{hs}}{n_{hs}}$$

Note that several independent samples will be selected, one for each original strata.

6.3.2.4 Projected error

The projection of errors to the population is made differently for units belonging to the exhaustive groups and for items in the non-exhaustive groups.

For the exhaustive groups, that is, for the groups containing the sampling units with book value larger than the cut-off value, $BV_{hi} > \frac{BV_h}{n_h}$, the projected error is the summation of the errors found in the items belonging to those groups:

$$EE_e = \sum_{h=1}^{H} \sum_{i=1}^{n_h} E_{hi}$$

In practice:
1) For each stratum h, identify the units belonging to the exhaustive group and sum their errors

[30] If some population units will still exhibit an expenditure larger than this sampling interval, then the procedure explained in section 6.3.1.3 shall be applied.

2) Sum the previous results over the all set of H strata.

For the non-exhaustive groups, i.e. the groups containing the sampling units with book value lower or equal to the cut-off value, $BV_{hi} \leq \frac{BV_h}{n_h}$, the projected error is

$$EE_s = \sum_{h=1}^{H} \frac{BV_{hs}}{n_{hs}} \sum_{i=1}^{n_{hs}} \frac{E_{hi}}{BV_{hi}}$$

To calculate this projected error:
1) in each stratum h, for each unit in the sample calculate the error rate, i.e. the ratio between the error and the respective expenditure $\frac{E_{hi}}{BV_{hi}}$
2) in each stratum h, sum these error rates over all units in the sample
3) in each stratum h, multiply the previous result by the total expenditure in the population of the non-exhaustive group (BV_{hs}); this expenditure will also be equal to the total expenditure in the stratum minus the expenditure of items belonging to the exhaustive group
4) in each stratum h, divide the previous result by the sample size in the non-exhaustive group (n_{hs})
5) sum the previous results over the whole set of H strata

The projected error at the level of population is just the sum of these two components:

$$EE = EE_e + EE_s$$

6.3.2.5 Precision

As for the standard MUS method, precision is a measure of the uncertainty associated with the extrapolation. It represents sampling error and should be calculated in order to subsequently produce a confidence interval.

The precision is given by the formula:

$$SE = z \times \sqrt{\sum_{h=1}^{H} \frac{BV_{hs}^2}{n_{hs}} . s_{rhs}^2}$$

where s_{rhs} is the standard-deviation of error rates in the sample of the non-exhaustive group of stratum h (calculated from the same sample used to extrapolate the errors to the population)

$$s^2_{rhs} = \frac{1}{n_{hs}-1}\sum_{i=1}^{n_{hs}}(r_{hi}-\bar{r}_{hs})^2\,, h = 1,2,\dots,H$$

having $\bar{r}_{hs}$ equal to the simple average of error rates in the sample of the non-exhaustive group of stratum h.

The sampling error is only computed for the non-exhaustive groups, since there is no sampling error arising from the exhaustive groups.

6.3.2.6 Evaluation

To draw a conclusion about the materiality of the errors the upper limit of error (ULE) should be calculated. This upper limit is equal to the summation of the projected error EE itself and the precision of the extrapolation

$$ULE = EE + SE$$

Then the projected error and the upper limit should both be compared to the maximum tolerable error to draw audit conclusions using exactly the same approach presented in Section 6.3.1.6.

6.3.2.7 Example

Assuming a population as expenditure declared to the Commission in a given year for operations in a group of two programmes. The system audits performed by the AA have yielded a low assurance level. Therefore, sampling this programme should be done with a confidence level of 90%.

The AA has reasons to believe that there are different error rates across the programmes. Bearing in mind all this information, the audit authority decided to stratify the population by programme.

The following table summarizes the available information.

Population size (number of operations)	6,252
Population size – stratum 1	4,520
Population size – stratum 2	1,732
Book value (sum of the expenditure in the reference period)	4,199,882,024 €

Book value – stratum 1	2,506,626,292 €
Book value – stratum 2	1,693,255,732 €

The first step is to compute the required sample size, using the formula:

$$n = \left(\frac{z \times BV \times \sigma_{rw}}{TE - AE}\right)^2$$

where σ_{rw}^2 is a weighted mean of the variances of the error rates for the whole set of strata, with the weight for each stratum equal to the ratio between the stratum book value (BV_h) and the book value for the whole population (BV):

$$\sigma_{rw}^2 = \sum_{i=1}^{H} \frac{BV_h}{BV} \sigma_{rh}^2, h = 1,2,\ldots,H;$$

where σ_{rh} is the standard deviation of error rates produced from a MUS sample. To obtain an approximation to this standard deviation the AA decided to use the standard deviation of previous year. The sample of the previous year was constituted by 110 operations, 70 operations from the first programme (stratum) and 40 from the second programme.

Based on this last year's sample we calculate the variance of the error rates as (see Section 7.3.1.7 for details):

$$\sigma_{r1}^2 = \frac{1}{70-1} \sum_{\mathrm{i}=1}^{70} (r_{1i} - \bar{r}_{1s})^2 = 0.000045$$

and

$$\sigma_{r2}^2 = \frac{1}{40-1} \sum_{\mathrm{i}=1}^{40} (r_{2i} - \bar{r}_{2s})^2 = 0.010909$$

This leads to the following result

$$\sigma_{rw}^2 = \frac{2{,}506{,}626{,}292}{4{,}199{,}882{,}024} \times 0.000045 + \frac{1{,}693{,}255{,}732}{4{,}199{,}882{,}024} \times 0.010909 = 0.004425$$

Given this estimate for the variance of error rates we are in conditions to compute the sample size. As already stated the AA expects significant differences across both strata. Further, based on report on the functioning of the management and control system, the audit authority expects an error rate around 1.1%. Assuming a tolerable error which is 2% of the total book value (materiality level set by the Regulation), that is, *TE*=2% x

4,199,882,024=83,997,640, and the anticipated error, i.e., AE=1.1% x 4,199,882,024=46,198,702, the sample size is

$$n = \left(\frac{1.645 \times 4{,}199{,}882{,}024 \times \sqrt{0.004425}}{83{,}997{,}640 - 46{,}198{,}702} \right)^2 \approx 148$$

The allocation of the sample by stratum is as follows:

$$n_1 = \frac{BV_1}{BV} \times n = \frac{2{,}506{,}626{,}292}{4{,}199{,}882{,}024} \times 148 \approx 89$$

$$n_2 = n - n_1 = 148 - 89 = 59.$$

These two samples sizes lead to the following values of cut-off for high-value strata:

$$Cut - off_1 = \frac{BV_1}{n_1} = \frac{2{,}506{,}626{,}292}{89} = 28{,}164{,}340$$

and

$$Cut - off_2 = \frac{BV_2}{n_2} = \frac{1{,}693{,}255{,}731}{59} = 28{,}699{,}250$$

Using these two cut-off values, 16 and 12 high value operations are found in stratum 1 and stratum 2, respectively.

The sample size for the sampling part of stratum 1 will be given by total sample size (89), deducted from the 16 high-value operations, i.e., 73 operations. Applying the same reasoning for stratum 2, the sample size for the sampling part of stratum 2 is 59-12=47 operations.

The next step will be the calculation of sampling interval for the sampling strata. The sampling intervals are, respectively, given by:

$$SI_1 = \frac{BV_{1s}}{n_{1s}} = \frac{1{,}643{,}963{,}924}{73} = 22{,}520{,}054$$

and

$$SI_2 = \frac{BV_{2s}}{n_{2s}} = \frac{1{,}059{,}467{,}667}{47} = 22{,}541{,}865$$

The following table summarises the previous results:

Population size (number of operations)	6,252
Population size – stratum 1	4,520
Population size – stratum 2	1,732

Book value (sum of the expenditure in the reference period)	4,199,882,024 €
Book value – stratum 1	2,506,626,292 €
Book value – stratum 2	1,693,255,732 €
Sample results – stratum 1	
Cut-off value	28,164,340 €
Number of operations above cut-off value	16
Book value of operations above cut-off value	862,662,369 €
Book value of operations (non-exhaustive population)	1,643,963,923 €
Sampling interval (non-exhaustive population)	22,520,054 €
Number of operations (non-exhaustive population)	4,504
Sample results – stratum 2	
Cut-off value	28,699,250 €
Number of operations above cut-off value	12
Book value of operations above cut-off value	633,788,064 €
Book value of operations (non-exhaustive population)	1,059,467,668 €
Sampling interval (non-exhaustive population)	22,541,865 €
Number of operations (non-exhaustive population)	1,720

For stratum 1, a file containing the remaining 4,504 operations (4,520 minus 16 high value operations) of the population is randomly sorted and a sequential cumulative book value variable is created. A sample of 73 operations (89 minus 16 high value operations) is drawn using exactly the same procedure as described in Section 7.3.1.7.

For stratum 2, a file containing the remaining 1,720 operations (1,732 minus 12 high value operations) of the population is randomly sorted and a sequential cumulative book value variable is created. A sample value of 47 operations (59 minus 12 high value operations) is drawn as described in previous paragraph.

For stratum 1, in the 16 high-value operations no errors were found.

For stratum 2, in 6, out of the 12 high-value operations, errors that amount to 15,460,340 € were found.

For the remaining samples the error has a different treatment. For these operations we follow the following procedure:

1) for each unit in the sample calculate the error rate, i.e. the ration between the error and the respective expenditure $\frac{E_i}{BV_i}$

2) sum these error rates over all units in the sample

3) multiply the previous result by the sampling interval (SI)

$$EE_{hs} = SI_{hs} \sum_{i=1}^{n_{hs}} \frac{E_{hi}}{BV_{hi}}$$

The sum of the error rates for the non-exhaustive population in stratum 1 is 1.0234,

$$EE_{1s} = 22{,}520{,}054 \times 1.0234 = 23{,}047{,}023$$

and for stratum 2 is 1.176,

$$EE_{2s} = 22{,}541{,}865 \times 1.176 = 26{,}509{,}234.$$

The projected error at the level of population is just the sum of all the components, that is, the amount of error found in the exhaustive part of both strata, which is 15,460,340 € and the projected error for both strata:

$$EE = 15{,}460{,}340 + 23{,}047{,}023 + 26{,}509{,}234 = 65{,}016{,}597$$

corresponding to a projected error rate of 1.55%.

To calculate the precision the variances of the error rates for both sampling strata have to be obtained using the same procedure as described in Section 7.3.1.7:

$$s_{r1}^2 = \frac{1}{72-1} \sum_{i=1}^{72} (r_{1i} - \bar{r}_{1s})^2 = 0.000036$$

and

$$s_{r2}^2 = \frac{1}{48-1} \sum_{i=1}^{48} (r_{2i} - \bar{r}_{2s})^2 = 0.0081$$

The precision is given by:

$$SE = z \times \sqrt{\sum_{h=1}^{H} \frac{BV_{hs}^2}{n_{hs}} \times s_{rhs}^2}$$

$$SE = 1.645 \times \sqrt{\frac{1{,}643{,}963{,}923^2}{73} \times 0.000036 + \frac{1{,}059{,}467{,}668^2}{47} \times 0.0081}$$
$$= 22{,}958{,}216$$

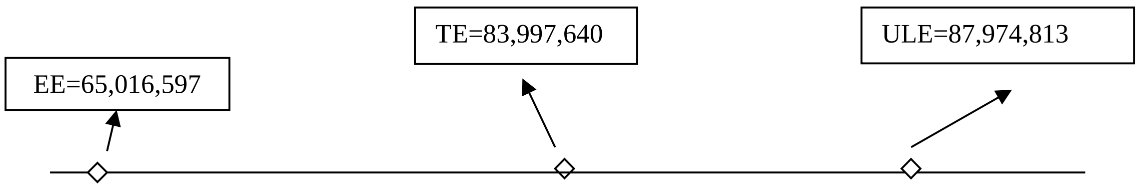
Note that the sampling error is only computed for the non-exhaustive parts of the population, since there is no sampling error to account for in the exhaustive stratum.

To draw a conclusion about the materiality of the errors the upper limit of error (ULE) should be calculated. This upper limit is equal to the summation of the projected error EE itself and the precision of the extrapolation

$$ULE = 65{,}016{,}597 + 22{,}958{,}216 = 87{,}974{,}813$$

Then the projected error and the upper limit should both be compared to the maximum tolerable error to draw audit conclusions:

Comparing to the materiality threshold of 2% of the total book value of the population (2% x 4,199,882,024 € = 83,997,640 €) with the projected results we observe that the maximum tolerable error is larger than the projected error, but smaller than the upper limit. Please refer to section 4.12 for more details on the analysis to be done.



6.3.3 Monetary unit sampling – two periods

6.3.3.1 Introduction

The audit authority may decide to carry out the sampling process in several periods during the year (typically two semesters). As happens with all other sampling methods, the major advantage of this approach is not related with sample size reduction, but mainly allowing spreading the audit workload over the year, thus reducing the workload that would be done at the end of the year based on just one observation.

With this approach, the year population is divided in two sub-populations, each one corresponding to the operations and expenditure of each semester. Independent samples are drawn for each semester, using the standard monetary unit sampling approach.

6.3.3.2 Sample size

First semester

At the first period of auditing (e.g. semester) the global sample size (for the set of two semesters) is computed as follows:

$$n = \left(\frac{z \times BV \times \sigma_{rw}}{TE - AE}\right)^2$$

where σ_{rw}^2 is a weighted mean of the variances of the error rates in each semester, with the weight for each semester equal to the ratio between the semester book value (BV_t) and the book value for the whole population (*BV*).

$$\sigma_{rw}^2 = \frac{BV_1}{BV}\sigma_{r1}^2 + \frac{BV_2}{BV}\sigma_{r2}^2$$

and σ_{rt}^2 is the variance of error rates in each semester. The variance of the errors rates is computed for each semester as

$$\sigma_{rt}^2 = \frac{1}{n_t^p - 1}\sum_{i=1}^{n_t^p}(r_{ti} - \bar{r}_t)^2\,, t = 1,2$$

where $r_{ti} = \frac{E_{ti}}{BV_{ti}}$ represent the individual error rates for units in the sample of semester t and $\bar{r}_t$ represent the mean error rate of the sample in semester t[31].

Values for the expected standard-deviations of error rates in both semesters have to be set using professional judgments and must be based on historical knowledge. The option to implement a preliminary/pilot sample of low sample size as previously presented for the standard monetary unit sampling method is still available, but can only be performed for the first semester. In fact, at the first moment of observation expenditure for the second semester has not yet taken place and no objective data (besides historical) is available. If pilot samples are implemented, they can, as usual, subsequently be used as a part of the sample chosen for audit.

If no historical data or knowledge is available to assess the variability of data in the second semester, a simplified approach can be used, computing the global sample size as

[31] Whenever the book value of unit *i* (BV_i) is larger than BV_t/n_t the ratio $\frac{E_{ti}}{BV_{ti}}$ should be substituted by the ratios $\frac{E_{ti}}{BV_t/n_t}$.

$$n = \left(\frac{z \times BV \times \sigma_{r1}}{TE - AE}\right)^2$$

Note, that in this simplified approach only information about the variability of error rates in the first period of observation is needed. The underlying assumption is that the variability of error rates will be of similar magnitude in both semesters.

Note that problems related to the lack of auxiliary historical information will usually be confined to the first year of the programming period. In fact, the information collected in the first year of auditing can be used in future year for sample size determination.

Also note that the formulas for sample size calculation require values for BV_1 and BV_2, i.e. total book value (declared expenditure) of the first and second semesters. When calculating sample size, the value for BV_1 will be known, but the value of BV_2 will be unknown and has to be imputed according to the expectations of the auditor (also based on historical information).

Once the total sample size, n, is computed the allocation of the sample by semester is as follows:

$$n_1 = \frac{BV_1}{BV}n$$

and

$$n_2 = \frac{BV_2}{BV}n$$

Second semester

At the first observation period, some assumptions were made relatively the following observation periods (typically the next semester). If characteristics of the population in the following periods differ significantly from the assumptions, sample size for the following period may have to be adjusted.

In fact, at the second period of auditing (e.g. semester) more information will be available:

- The total book value in the second semester BV_2 is correctly known;
- The sample standard-deviation of error rates s_{r1} calculated from the sample of the first semester could be already available;
- The standard deviation of error rates for the second semester σ_{r2} can now be more accurately assessed using real data.

If these parameters are not dramatically different from the ones estimated at the first semester using the expectations of the auditor, the originally planned sample size, for the second semester (n_2), won't require any adjustments. Nevertheless, if the auditor considers that the initial expectations significantly differ from the real population characteristics, the sample size may have to be adjusted in order to account for these inaccurate estimates. In this case, the sample size of the second semester should be recalculated using

$$n_2 = \frac{\left(z \times BV_2 \times \sigma_{r2}\right)^2}{(TE - AE)^2 - z^2 \times \frac{BV_1^2}{n_1} \times s_{r1}^2}$$

where s_{r1} is the standard-deviation of error rates calculated from the sample of the first semester and σ_{r2} an estimate of the standard-deviation of error rates in the second semester based on historical knowledge (eventually adjusted by information from the first semester) or a preliminary/pilot sample of the second semester.

6.3.3.3 Sample selection

In each semester, the sample selection will exactly follow the procedure described for the standard monetary unit sampling approach. The procedure will be reproduced here for the sake of the reader.

For each semester, after determining sample size, it is necessary to identify the high value population units (if any) that will belong to a high value group to be audited a 100%. The cut-off value for determining this top group is equal to the ratio between the book value of the semester (BV_t) and the planed sample size (n_t). All items whose book value is higher than this cut-off (if $BV_{ti} > \frac{BV_t}{n_t}$) will be placed in the 100% audit group.

The sampling size to be allocated to the non-exhaustive group, n_{ts} , is computed as the difference between n_t and the number of sampling units (e.g. operations) in the exhaustive group (n_{te}).

Finally, in each semester, the selection of the samples is done in the non-exhaustive group using probability proportional to size, i.e. proportional to the item book values BV_{ti}. A popular way to implement the selection is though systematic selection, using a selection interval equal to the total expenditure in the non-exhaustive group (BV_{ts}) divided by the sample size (n_{ts})[32], i.e.

[32] If some population units will still exhibit an expenditure larger than this sampling interval, then the procedure explained in section 6.3.1.3 shall be applied.

$$SI_t = \frac{BV_{ts}}{n_{ts}}$$

6.3.3.4 Projected error

The projection of errors to the population is calculated differently for units belonging to the exhaustive groups and for items in the non-exhaustive groups.

For the exhaustive groups, that is, for the groups containing the sampling units with book value larger than the cut-off value, $BV_{ti} > \frac{BV_t}{n_t}$, the projected error is the summation of the errors found in the items belonging to those groups:

$$EE_e = \sum_{i=1}^{n_1} E_{1i} + \sum_{i=1}^{n_2} E_{2i}$$

In practice:

1) For each semester *t*, identify the units belonging to the exhaustive group and sum their errors
2) Sum the previous results over the two semesters.

For the non-exhaustive groups, i.e. the groups containing the sampling units with book value lower or equal to the cut-off value, $BV_{ti} \leq \frac{BV_t}{n_t}$, the projected error is

$$EE_s = \frac{BV_{1s}}{n_{1s}} \times \sum_{i=1}^{n_{1s}} \frac{E_{1i}}{BV_{1i}} + \frac{BV_{2s}}{n_{2s}} \times \sum_{i=1}^{n_{2s}} \frac{E_{2i}}{BV_{2i}}$$

To calculate this projected error:

1) in each semester *t*, for each unit in the sample calculate the error rate, i.e. the ratio between the error and the respective expenditure $\frac{E_{ti}}{BV_{ti}}$
2) in each semester *t*, sum these error rates over all units in the sample
3) in semester *t*, multiply the previous result by the total expenditure in the population of the non-exhaustive group (BV_{ts}); this expenditure will also be equal to the total expenditure of the semester minus the expenditure of items belonging to the exhaustive group
4) in each semester *t*, divide the previous result by the sample size in the non-exhaustive group (n_{ts})

5) sum the previous results over the two semesters

The projected error at the level of population is just the sum of these two components:

$$EE = EE_e + EE_s$$

6.3.3.5 *Precision*

As for the standard MUS method, precision is a measure of the uncertainty associated with the extrapolation. It represents sampling error and should be calculated in order to subsequently produce a confidence interval.

The precision is given by the formula:

$$SE = z \times \sqrt{\frac{BV_{1s}^2}{n_{1s}} \times s_{r1s}^2 + \frac{BV_{2s}^2}{n_{2s}} \times s_{r2s}^2}$$

where s_{r2s} is the standard-deviation of error rates in the sample of the non-exhaustive group of semester *t* (calculated from the same sample used to extrapolate the errors to the population)

$$s_{rts}^2 = \frac{1}{n_{ts} - 1} \sum_{i=1}^{n_{ts}} (r_{ti} - \bar{r}_{ts})^2 \, , t = 1,2$$

having $\bar{r}_{ts}$ equal to the simple average of error rates in the sample of the non-exhaustive group of semester *t*.

The sampling error is only computed for the non-exhaustive groups, since there is no sampling error arising from the exhaustive groups.

6.3.3.6 *Evaluation*

To draw a conclusion about the materiality of the errors the upper limit of error (ULE) should be calculated. This upper limit is equal to the summation of the projected error EE itself and the precision of the extrapolation

$$ULE = EE + SE$$

Then the projected error and the upper limit should both be compared to the maximum tolerable error to draw audit conclusions using exactly the same approach presented in Section 6.3.1.6.

6.3.3.7 Example

In order to anticipate the audit workload that usually is concentrated at the end of the audit year the AA decided to spread the audit work in two periods. At the end of the first semester the AA considered the population divided into two groups corresponding to each one of the two semesters. At the end of the first semester the characteristics of the population are the following:

Declared expenditure at the end of first semester	1,827,930,259 €
Size of population (operations - first semester)	2,344

Based on the past experience, the AA knows that usually all the operations included in the programmes at the end of the reference period are already active in the population of the first semester. Moreover, it is expected that the declared expenditure at the end of the first semester represents about 35% of the total declared expenditure at the end of the reference period. Based on these assumptions a summary of the population is described in the following table:

Declared expenditure (DE) at the end of first semester	1,827,930,259 €
Declared expenditure (DE) at the end of the second semester (predicted) 1,827,930,259€ / 35%-1,827,930,259€) = 3,394,727,624€)	3,394,727,624 €
Total expenditure forecasted for the year	5,222,657,883€
Size of population (operations – first semester)	2,344
Size of population (operations – second semester, predicted)	2,344

For the first period, the global sample size (for the set of two semesters) is computed as follows:

$$n = \left(\frac{z \times BV \times \sigma_{rw}}{TE - AE}\right)^2$$

where σ_{rw}^2 is a weighted average of the variances of the error rates in each semester, with the weight for each semester equal to the ratio between the semester book value (BV_t) and the book value for the whole population (BV).

$$\sigma_{rw}^2 = \frac{BV_1}{BV}\sigma_{r1}^2 + \frac{BV_2}{BV}\sigma_{r2}^2$$

and σ_{rt}^2 is the variance of error rates in each semester. The variance of the errors rates is computed for each semester as

$$\sigma_{rt}^2 = \frac{1}{n_t^p - 1}\sum_{i=1}^{n_t^p}(r_{ti} - \bar{r}_t)^2, t = 1,2,\ldots,T$$

Since these variances are unknown, the AA decided to draw a preliminary sample of 20 operations at the end of first semester of the current year. The sample standard deviation of error rates in this preliminary sample at first semester is 0.12. Based on professional judgement and knowing that usually the expenditure in second semester is larger than in first semester, the AA has made a preliminary prediction of standard deviation of error rates for the second semester to be 110% larger than in first semester, that is, 0.25. Therefore, the weighted average of the variances of the error rates is:

$$\sigma_{rw}^2 = \frac{1{,}827{,}930{,}259}{1{,}827{,}930{,}259 + 3{,}394{,}727{,}624} \times 0.12^2$$

$$+\frac{3{,}394{,}727{,}624}{1{,}827{,}930{,}259 + 3{,}394{,}727{,}624} \times 0.25^2 = 0.0457$$

In the first semester, the AA, given the level of functioning of the management and control system, considers adequate a confidence level of 60%. The global sample size for the whole year is:

$$n = \left(\frac{0.842 \times (1{,}827{,}930{,}259 + 3{,}394{,}727{,}624) \times \sqrt{0.0457}}{104{,}453{,}158 - 20{,}890{,}632}\right)^2 \approx 127$$

where z is 0.842 (coefficient corresponding to a 60% confidence level), TE, the tolerable error, is 2% (maximum materiality level set by the Regulation) of the book value. The total book value comprises the true book value at the end of the first semester plus the predicted book value for the second semester 3,394,727,624 €, which means that tolerable error is 2% x 5,222,657,883 € = 104,453,158 €. The last year's audit projected an error rate of 0.4%. Thus AE, the anticipated error, is 0.4% x 5,222,657,883 € = 20,890,632 €.

The allocation of the sample by semester is as follows:

$$n_1 = \frac{BV_1}{BV_1 + BV_2} = \frac{1{,}827{,}930{,}259}{1{,}827{,}930{,}259 + 3{,}394{,}727{,}624} \times 127 \approx 45$$

and

$$n_2 = n - n_1 = 82$$

For the first semester, it is necessary to identify the high value population units (if any) that will belong to a high-value stratum to be submitted at a 100% audit work. The cut-off value for determining this top stratum is equal to the ratio between the book value (BV_1) and the planned sample size (n_1). All items whose book value is higher than this cut-off (if $BV_{i1} > BV_1/n_1$) will be placed in the 100% audit stratum. In this case the cut-off value is 40,620,672 €. There are 11 operations which book value is larger than this cut-off value. The total book value of these operations amounts to 891,767,519 €.

The sampling size to be allocated to the non-exhaustive stratum (n_{1s}) is computed as the difference between n_1 and the number of sampling units in the exhaustive stratum (n_e), that is 34 operations.

The selection of the sample in the non-exhaustive stratum will be made using probability proportional to size, i.e. proportional to the item book values BV_{is1}, through systematic selection, using a sampling interval equal to the total expenditure in the non-exhaustive stratum (BV_{1s}) divided by the sample size (n_{1s}), i.e.

$$SI_{1s} = \frac{BV_{1s}}{n_{1s}} = \frac{1{,}827{,}930{,}259 - 891{,}767{,}519}{34} = 27{,}534{,}198$$

The book value in the non-exhaustive stratum (BV_{1s}) is just the difference between the total book value and the book value of the 11 operations belonging to the top stratum.

The following table summarises these results:

Cut-off value – first semester	40,620,672 €
Number of operations with book value larger than cut-off value - first semester	11
Book value of operations with book value larger than cut-off value - first semester	891,767,519 €
BV_{s1}- first semester	936,162,740 €
n_{s1}- first semester	34
SI_{s1}- first semester	27,534,198 €

Out of the 11 operations with book value larger than the sampling interval, 6 of them have error. The total error found in this stratum is 19,240,855 €.

A file containing the remaining 2,333 operations of the population is randomly sorted and a sequential cumulative book value variable is created. A sample of 34 operations is drawn using the systematic proportional to size procedure.
The value of the 34 operations is audited. The sum of the error rates for the first semester is:

$$\sum_{i=1}^{34} \frac{E_{i1s}}{BV_{i1s}} = 1.4256$$

The standard-deviation of error rates in the sample of the non-exhaustive population of the first semester is (see Section 6.3.1.7 for details):

$$s_{r1s} = \sqrt{\frac{1}{34-1}\sum_{i=1}^{34}(r_{i1s} - \bar{r}_{1s})^2} = 0.085$$

having $\bar{r}_{1s}$ equal to the simple average of error rates in the sample of the non-exhaustive group of first semester.

At the end of the second semester more information is available, in particular, the total expenditure of operations active in the second semester is correctly known, the sample variance of error rates s_{r1} calculated from the sample of the first semester could be already available and the standard deviation of error rates for the second semester σ_{r2} can now be more accurately assessed using a preliminary sample of real data.

The AA realises that the assumption made at the end of the first semester on the total expenditure, 3,394,727,624 €, overestimates the true value of 2,961,930,008. There are also two additional parameters for which updated figures should be used.

Firstly, the estimate of the standard deviation of error rates based on the first semester sample of 34 operations yielded an estimate of 0.085. This new value should now be used to reassess the planned sample size. Secondly, based on the increased expenditure of the second semester compared to the initial estimate, the AA considers more prudent to estimate the standard deviation of error rates for the second semester as 0.30 instead of the initial value of 0.25. The updated figures of standard deviation of error rates for both semesters are far from the initial estimates. As a result, the sample for the second semester should be revised.

Parameter	**Forecast done in the first semester**	**End of second semester**
Standard deviation of error rates in the first semester	0.12	0.085
Standard deviation of error rates in the second semester	0.25	0.30
Total expenditure in the second semester	3,394,727,624 €	2,961,930,008 €

Taking into consideration these three adjustments, the recalculated sample size of the second semester is

$$n_2 = \frac{\left(z \times BV_2 \ \times \sigma_{r2}\right)^2}{(TE - AE)^2 - z^2 \times \frac{BV_1^2}{n_1} \times s_{r1}^2}$$

where s_{r1} is the standard-deviation of error rates calculated from the sample of the first semester (the sample also used to produce the projected error) and σ_{r2} an estimate of the standard-deviation of error rates in the second semester:

$$n_2 = \frac{(0.842 \times 2{,}961{,}930{,}008 \times 0.30)^2}{(95{,}797{,}205 - 19{,}159{,}441)^2 - 0.842^2 \times \frac{1{,}827{,}930{,}259^2}{45} \times 0.085^2} \approx 102$$

where:

- TE = (1,827,930,259€ + 2,961,930,008 €) * 2% = 95,797,205 €
- AE = (1,827,930,259€ + 2,961,930,008 €) * 0,4% = 19,159,441 €

It is necessary to identify the high value population units (if any) that will belong to a high-value stratum to be submitted at a 100% audit work. The cut-off value for determining this top stratum is equal to the ratio between the book value (BV_2) and the planed sample size (n_2). All items whose book value is higher than this cut-off (if $BV_{i2} > BV_2/n_2$) will be placed in the 100% audit stratum. In this case, the cut-off value is 29,038,529 €. There are 6 operations which book value is larger than this cut-off value. The total book value of these operations amounts to 415,238,983 €.

The sampling size to be allocated to the non-exhaustive stratum, n_{2s} , is computed as the difference between n_2 and the number of sampling units (e.g. operations) in the exhaustive stratum (n_{2e}), that is 96 operations (102, the sample size, minus the 6 high-value operations). Therefore, the auditor has to select in the sample using the sampling interval:

$$SI_{2s} = \frac{BV_{2s}}{n_{2s}} = \frac{2{,}961{,}930{,}008 - 415{,}238{,}983}{96} = 26{,}528{,}032$$

The book value in the non-exhaustive stratum (BV_{2s}) is just the difference between the total book value and the book value of the 6 operations belonging to the top stratum.

The following table summarises these results:

Cut-off value - second semester	29,038,529 €
Number of operations with book value larger than cut-off value - second semester	6
Book value of operations with book value larger than cut-off value- second semester	415,238,983 €
BV_{2s}- second semester	2,546,691,025 €
n_{2s}- second semester	96
SI_{2s}- second semester	26,528,032 €

Out of the 6 operations with book value larger than the cut-off value, 4 of them have error. The total error found in this stratum is 9,340,755 €.

A file containing the remaining 2,338 operations of the second semester population is randomly sorted and a sequential cumulative book value variable is created. A sample of 96 operations is drawn using the systematic proportional to size procedure.

The value of these 96 operations is audited. The sum of the error rates for the second semester is:

$$\sum_{i=1}^{96} \frac{E_{2i}}{BV_{2i}} = 1.1875$$

The standard-deviation of error rates in the sample of the non-exhaustive population of the second semester is:

$$s_{r2s} = \sqrt{\frac{1}{96-1}\sum_{i=1}^{96}(r_{i2s} - \bar{r}_{2s})^2} = 0.29$$

having $\bar{r}_{2s}$ equal to the simple average of error rates in the sample of the non-exhaustive group of second semester.

The projection of errors to the population is made differently for units belonging to the exhaustive strata and for items in the non-exhaustive strata.

For the exhaustive strata, that is, for the strata containing the sampling units with book value larger than the cut-off, $BV_{ti} > \frac{BV_t}{n_t}$, the projected error is the summation of the errors found in the items belonging to those strata:

$$EE_e = \sum_{i=1}^{n_1} E_{1i} + \sum_{i=1}^{n_2} E_{2i} = 19{,}240{,}855 + 9{,}340{,}755 = 28{,}581{,}610$$

In practice:
1) For each semester *t*, identify the units belonging to the exhaustive group and sum their errors
2) Sum the previous results over the two semesters.

For the non-exhaustive group, i.e. the strata containing the sampling units with book value smaller or equal to the cut-off value, $BV_{ti} \leq \frac{BV_t}{n_t}$, the projected error is

$$EE_s = \frac{BV_{1s}}{n_{1s}} \times \sum_{i=1}^{n_{1s}} \frac{E_{1i}}{BV_{1i}} + \frac{BV_{2s}}{n_{2s}} \times \sum_{i=1}^{n_{2s}} \frac{E_{2i}}{BV_{2i}}$$
$$= \frac{936{,}162{,}740}{34} \times 1.4256 + \frac{2{,}546{,}691{,}025}{96} \times 1.1875 = 70{,}754{,}790$$

To calculate this projected error:
1) in each semester *t*, for each unit in the sample calculate the error rate, i.e. the ratio between the error and the respective expenditure $\frac{E_{ti}}{BV_{ti}}$
2) in each semester *t*, sum these error rates over all units in the sample
3) in semester *t*, multiply the previous result by the total expenditure in the population of the non-exhaustive group (BV_{ts}); this expenditure will also be equal to the total expenditure of the semester minus the expenditure of items belonging to the exhaustive group
4) in each semester *t*, divide the previous result by the sample size in the non-exhaustive group (n_{ts})
5) sum the previous results over the two semesters

The projected error at the level of population is just the sum of these two components:

$$EE = EE_e + EE_s = 28{,}581{,}610 + 70{,}754{,}790 = 99{,}336{,}400$$

corresponding to a projected error rate of 2.07%.

The precision is a measure of the uncertainty associated with the projection. The precision is given by the formula:

$$\begin{aligned} SE &= z \times \sqrt{\frac{BV_{1s}^2}{n_{1s}} \times s_{r1s}^2 + \frac{BV_{2s}^2}{n_{2s}} \times s_{r2s}^2} \\ &= 0.842 \times \sqrt{\frac{936{,}162{,}740^2}{34} \times 0.085^2 + \frac{2{,}546{,}691{,}025\ ^2}{96} \times 0.29^2} \\ &= 64{,}499{,}188 \end{aligned}$$

where s_{rts} are the standard-deviation of error rates already computed.

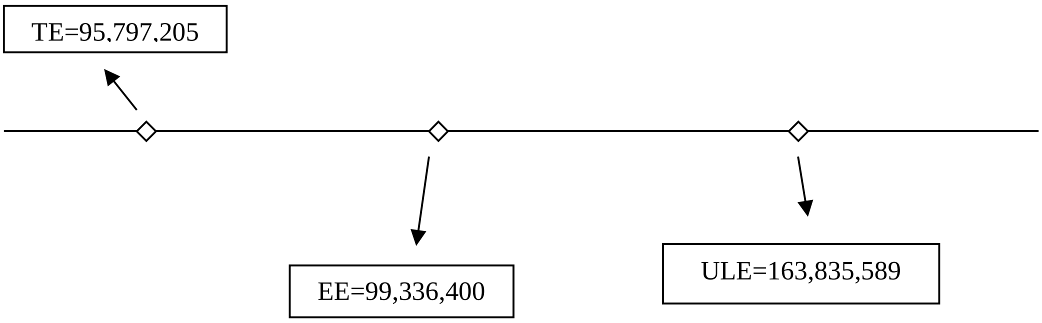
The sampling error is only computed for the non-exhaustive strata, since there is no sampling error arising from the exhaustive groups.

To draw a conclusion about the materiality of the errors the upper limit of error (ULE) should be calculated. This upper limit is equal to the summation of the projected error EE itself and the precision of the projection

$$ULE = EE + SE = 99{,}336{,}400 + 64{,}499{,}188 = 163{,}835{,}589$$

Then the projected error and the upper limit should both be compared to the maximum tolerable error to draw audit conclusions.

In this particular case, the projected error is larger than maximum tolerable error. It means that the auditor would conclude that there is enough evidence to support that errors in the population are larger than materiality threshold:



6.3.4 Two-periods stratified monetary unit sampling

6.3.4.1 Introduction

The audit authority may decide to use a stratified sampling design and simultaneously spread the audit work in several periods during the year (typically two semesters, but the same logic would also apply to more periods). Formally, this will constitute a new sampling design that includes features of stratified MUS and two-period MUS. In this section a method will be proposed to combine this two features into one single sampling design.

Firstly note that by implementing this combined design, the AA will be able to benefit from the advantages offered by stratification and multi-period sampling. By using stratification it will potentially be possible to improve precision in comparison with a non-stratified design (or the use a smaller sample size for the same level of precision). By simultaneously using a multi-period approach, the AA will be able to spread the audit workload over the year, thus reducing the workload that would be done at the end of the year based on just one observation period.

With this approach, the population of the reference period is divided in two sub-populations, each one corresponding to the operations and expenditure of each semester. Independent samples are drawn for each semester, using the stratified monetary unit sampling approach. Please note that it is not necessary to use exactly the same stratification in each audit period. In fact, the type of stratification and even the number of strata may vary from one audit period to the other.

6.3.4.2 Sample size

First semester

At the first period of auditing (e.g. semester) the global sample size (for the set of two semesters) is computed as follows:

$$n = \left(\frac{z \times BV \times \sigma_{rw}}{TE - AE}\right)^2$$

where σ_{rw}^2 is a weighted mean of the variances of the error rates for the whole set of strata and for both periods. The weight for each stratum in each semester is equal to the ratio between the stratum book value (BV_{ht}) and the book value for the whole population, $BV=BV_1+BV_2$ (including both semesters).

$$\sigma_{rw}^2 = \sigma_{rw1}^2 + \sigma_{rw2}^2$$

$$\sigma_{rw1}^2 = \sum_{i=1}^{H_1} \frac{BV_{h1}}{BV} \sigma_{rh1}^2, h = 1,2, \dots, H_1;$$

$$\sigma_{rw2}^2 = \sum_{i=1}^{H_2} \frac{BV_{h2}}{BV} \sigma_{rh2}^2, h = 1,2, \dots, H_2;$$

BV_{ht} represents the expenditure of stratum *h* in period *t*, H_t is the number of strata in period *t*, and σ_{rht}^2 is the variance of error rates in each stratum of each semester. The variance of the errors rates is computed for each stratum in each semester as

$$\sigma_{rht}^2 = \frac{1}{n_{ht}^p - 1} \sum_{i=1}^{n_{ht}^p} (r_{hti} - \bar{r}_{ht})^2, h = 1,2, \dots, H_t, t = 1,2$$

where $r_{hti} = \frac{E_{hti}}{BV_{hti}}$ represents the individual error rates for units in the sample of stratum h in semester t and $\bar{r}_{ht}$ represents the mean error rate of the sample in stratum h and semester *t*[33].

Values for the expected standard-deviations of error rates in both semesters have to be set using professional judgments and be based on historical knowledge. The option to implement a preliminary/pilot sample of low sample size to obtain approximations to the parameters of first semester, as previously presented for the standard two-period monetary unit sampling method, is still available. Again, at the first moment of

[33] Whenever the book value of unit *i* (BV_i) is larger than BV_{ht}/n_{ht} the ratio $\frac{E_{hti}}{BV_{hti}}$ should be substituted by the ratio $\frac{E_{hti}}{BV_{ht}/n_{ht}}$.

observation expenditure for the second semester has not yet taken place and no objective data (besides historical) is available. If pilot samples are implemented, they can, as usual, be used subsequently as a part of the sample chosen for audit.

If no historical data or knowledge is available to assess the variability of data in the second semester, a simplified approach can be used, computing the global sample size as

$$n = \left(\frac{z \times BV \times \sigma_{rw1}}{TE - AE} \right)^2$$

Note, that in this simplified approach only information about the variability of error rates in the first period of observation is needed. The underlying assumption is that the variability of error rates will be of similar magnitude in both semesters.

Note that problems related to the lack of auxiliary historical information will usually be confined to the first year of the programming period. In fact, the information collected in the first year of auditing can be used in future year for sample size determination.

Also note that the formulas for sample size calculation require values for BV_{h1} $(h = 1,2, \ldots, H_1)$ and BV_{h2} $(h = 1,2, \ldots, H_2)$ i.e. total book value (declared expenditure) in each stratum for the first and second semesters. When calculating the sample size, the values for BV_{h1} $(h = 1,2, \ldots, H_1)$ will be known, but the values of BV_{h2} $(h = 1,2, \ldots, H_2)$ will be unknown and have to be imputed according to the expectations of the auditor (also based on historical information and/or forecasts from the programme managing or certifying authorities).

Once the total sample size, n, is computed, the allocation of the sample by stratum and semester is as follows:

$$n_{h1} = \frac{BV_{h1}}{BV} n$$

and

$$n_{h2} = \frac{BV_{h2}}{BV} n$$

where $BV=BV_1+BV_2$ is the total forecasted expenditure for the reference period.

As before, one should note that this is a general allocation method, where the sample is allocated to strata proportionally to the expenditure (book value) of the strata, but that other allocation methods are available. A more tailored allocation may in some cases bring additional precision gains or reduction of sample size. The adequacy of other allocation methods to each specific population requires some technical knowledge in sampling theory and is outside the scope of this guidance note.

Second semester

At the first observation period, some assumptions were made concerning the following observation periods (typically the next semester). If characteristics of the population in the following periods differ significantly from the assumptions, the sample size for the following period may have to be adjusted.

In fact, at the second period of auditing (e.g. semester) more information will be available:

- The total book value in each stratum of the second semester BV_{h2} ($h = 1,2,\dots,H_2$) is correctly known;
- The sample standard-deviations of error rates s_{rh1} ($h = 1,2,\dots,H_1$) calculated from the sample of the first semester could be already available;
- The standard-deviations of error rates of strata in the second semester σ_{rh2} ($h = 1,2,\dots,H_2$) can now be more accurately assessed using real data (e.g. based on pilot samples).

If initial forecasts regarding these population parameters significantly differ from the real population characteristics, the sample size may have to be adjusted for the 2nd semester, in order to take into account these inaccurate estimates. In this case, the sample size of the second semester should be recalculated using

$$n_2 = \frac{z^2 \times BV_2 \times \sum_{h=1}^{H_2}\left(BV_{h2}.\sigma_{rh2}^2\right)}{(TE - AE)^2 - z^2 \times \sum_{h=1}^{H_2}\left(\frac{BV_{h1}^2}{n_{h1}}.s_{rh1}^2\right)}$$

where s_{rh1} is the standard-deviations of error rates calculated from the subsamples of the first semester for each stratum h (if already available), and σ_{rh2} estimates of the standard-deviations of error rates in each stratum of the second semester based on historical knowledge (eventually adjusted by information from the first semester) or a preliminary/pilot sample of the second semester.

After recalculating the global sample size for the 2nd semester, the allocation per stratum is straightforward as:

$$n_{h2} = \frac{BV_{h2}}{BV_2} n_2, (h = 1,2,\dots,H_2)$$

6.3.4.3 Sample selection

In each semester, the sample selection will exactly follow the procedure described for the stratified monetary unit sampling approach. The procedure will be reproduced here for ease of reference.
For each semester and in each stratum h, there will be two components: the exhaustive group inside stratum h (that is, the group containing the sampling units with book value larger than the cut-off value, $BV_{hti} > \frac{BV_{ht}}{n_{ht}}$); and the sampling group inside stratum h (that is, the group containing the sampling units with book value smaller than or equal the cut-off value, $BV_{hti} \leq \frac{BV_{ht}}{n_{ht}}$, or other recalculated cut-off value if there are items with book values above the interval and below cut-off values).

For each semester, after determining the sample size, in each of the original stratum (h) all the high value population units (if any) are to be audited. The cut-off value for determining this top group is equal to the ratio between the book value of the stratum (BV_{ht}) and the planed sample size (n_{ht}). In each stratum, h, all items whose book value is higher than this cut-off (if $BV_{hti} > \frac{BV_{ht}}{n_{ht}}$) will be placed in the 100% audit group.

The sample size to be allocated to the non-exhaustive group, n_{hts} , is computed as the difference between n_{ht} and the number of sampling units (e.g. operations) in the exhaustive group of the stratum (n_{hte}).

Finally, in each semester, the selection of the samples is done in the non-exhaustive group of each stratum, by using probability proportional to size, i.e. proportional to the item book values BV_{hti}. A popular way to implement the selection is through systematic selection, using a selection interval equal to the total expenditure in the non-exhaustive group of the stratum (BV_{hts}) divided by the sample size (n_{hts})[34], i.e.

$$SI_{hts} = \frac{BV_{hts}}{n_{hts}}$$

Note that, in each semester, several independent samples will be selected, one for each original stratum.

6.3.4.4 Projected error

The projection of errors to the population is calculated differently for units belonging to the exhaustive groups and for items in the non-exhaustive groups.

[34] If some population units will still exhibit an expenditure larger than this sampling interval, then the procedure explained in section 6.3.1.3 shall be applied.

For the exhaustive groups, that is, for the groups containing the sampling units with book value larger than the cut-off values, $BV_{hti} > \frac{BV_{ht}}{n_{ht}}$, the projected error is the sum of the errors found in the items belonging to those groups:

$$EE_e = \sum_{h=1}^{H_1} \sum_{i=1}^{n_{h1}} E_{h1i} + \sum_{h=1}^{H_2} \sum_{i=1}^{n_{h2}} E_{h2i}$$

In practice:

1) For each semester *t*, and in each stratum *h*, identify the units belonging to the exhaustive group and sum their errors;
2) Sum the previous results over the set of $H_1 + H_2$ strata.

For the non-exhaustive groups, i.e. the groups containing the sampling units with book value lower or equal to the cut-off values, $BV_{hti} \leq \frac{BV_{ht}}{n_{ht}}$, the projected error is

$$EE_s = \sum_{h=1}^{H_1} \left(\frac{BV_{h1s}}{n_{h1s}} . \sum_{i=1}^{n_{h1s}} \frac{E_{h1i}}{BV_{h1i}} \right) + \sum_{h=1}^{H_2} \left(\frac{BV_{h2s}}{n_{h2s}} . \sum_{i=1}^{n_{h2s}} \frac{E_{h2i}}{BV_{h2i}} \right)$$

To calculate this projected error:

1) in each stratum *h* in each semester *t*, for each unit in the sample calculate the error rate, i.e. the ratio between the error and the respective expenditure $\frac{E_{hti}}{BV_{hti}}$
2) in each stratum *h* in each semester *t*, sum these error rates over all units in the sample
3) in each stratum *h* in semester *t*, multiply the previous result by the total expenditure in the population of the non-exhaustive group (BV_{hts}); this expenditure will also be equal to the total expenditure of the stratum minus the expenditure of items belonging to the exhaustive group of the stratum
4) in each stratum *h* in each semester *t*, divide the previous result by the sample size in the non-exhaustive group (n_{hts})
5) sum the previous results over the whole set of $H_1 + H_2$ strata

The projected error at the level of population is just the sum of these two components:

$$EE = EE_e + EE_s$$

6.3.4.5 *Precision*

As for the standard two-period MUS method, precision is a measure of the uncertainty associated with the extrapolation (projection). It represents sampling error and should be calculated in order to subsequently produce a confidence interval.

The precision is given by the formula:

$$SE = z \times \sqrt{\sum_{h=1}^{H_1} \left(\frac{BV_{h1s}^2}{n_{h1s}} . s_{rh1s}^2 \right) + \sum_{h=1}^{H_2} \left(\frac{BV_{h2s}^2}{n_{h2s}} . s_{rh2s}^2 \right)}$$

where s_{rhts} is the standard-deviation of error rates in the sample of the non-exhaustive group of stratum *h* of semester *t* (calculated from the same sample used to extrapolate the errors to the population)

$$s_{rhts}^2 = \frac{1}{n_{hts} - 1} \sum_{i=1}^{n_{hts}} (r_{hti} - \bar{r}_{hts})^2$$

having $\bar{r}_{hts}$ equal to the simple average of error rates in the sample of the non-exhaustive group of stratum *h* of semester *t*.

The sampling error is only computed for the non-exhaustive groups, since there is no sampling error arising from the exhaustive groups.

6.3.4.6 *Evaluation*

To draw a conclusion about the materiality of the errors the upper limit of error (ULE) should be calculated. This upper limit is equal to the summation of the projected error EE itself and the precision of the extrapolation

$$ULE = EE + SE$$

Then the projected error and the upper limit should both be compared to the maximum tolerable error to draw audit conclusions using exactly the same approach presented in Section 6.3.3.6.

6.3.4.7 *Example*

In order to anticipate the audit workload that usually is concentrated at the end of the audit year the AA decided to spread the audit work in two periods. At the end of the first semester the AA considers the population divided into two groups corresponding to each one of the two semesters. Moreover, the population comprises two different programmes and the AA has reasons to believe that there are different error rates across the programmes. Bearing in mind all this information, besides splitting the workload in two periods, the AA decided to stratify the population by programme.

At the end of the first semester the characteristics of the population are the following:

Declared expenditure at the end of first semester	42,610,732 €
Programme 1	27,623,498 €
Programme 2	14,987,234 €
Size of population (operations - first semester)	5,603
Programme 1	3,257
Programme 2	2,346

Based on the past experience, the AA knows that usually all the operations included in the programmes at the end of the reference period are already active in the population of the first semester. Moreover, based on the past experience AA expects the expenditure declared in the second semester goes up for two programmes, although at different rates. It is expected that the declared expenditure for second semester goes up by 40% and 10%, for programmes 1 and 2, respectively. Based on these assumptions a summary of the population is described in the following table:

Declared expenditure at the end of first semester	42,610,732 €
Programme 1	27,623,498 €
Programme 2	14,987,234 €
Declared expenditure at the end of the second semester (predicted)	55,158,855 €
Programme 1 (27,623,498 € x 1.4)	38,672,897 €
Programme 2 (14,987,234 € x 1.1)	16,485,957 €
Total expenditure forecasted for the year	97,769,587 €
Programme 1	66,296,395 €
Programme 2	31,473,191 €
Size of population (operations – first semester)	5,603
Programme 1	3,257
Programme 2	2,346
Size of population (operations – second semester, predicted)	5,603
Programme 1	3,257
Programme 2	2,346

For the first semester of auditing the global sample size (for the set of two semesters) is computed as follows:

$$n = \left(\frac{z \times BV \times \sigma_{rw}}{TE - AE}\right)^2$$

where σ_{rw}^2 is a weighted mean of the variances of the error rates for the whole set of strata and for both periods. The weight for each stratum in each semester is equal to the ratio between the stratum book value (BV_{ht}) and the book value for the whole population, $BV=BV_1+BV_2$ (including both semesters).

$$\sigma_{rw}^2 = \sigma_{rw1}^2 + \sigma_{rw2}^2$$

$$\sigma_{rw1}^2 = \sum_{i=1}^{2} \frac{BV_{h1}}{BV} \sigma_{rh1}^2, h = 1,2;$$

$$\sigma_{rw2}^2 = \sum_{i=1}^{2} \frac{BV_{h2}}{BV} \sigma_{rh2}^2, h = 1,2;$$

BV_{ht} represents the expenditure of stratum *h, h=1,2,* in period *t* and σ_{rht}^2 is the variance of error rates in each stratum of each semester. The variance of the errors rates is computed for each stratum in each semester as

$$\sigma_{rht}^2 = \frac{1}{n_{ht}^p - 1} \sum_{i=1}^{n_{ht}^p} (r_{hti} - \bar{r}_{ht})^2, h = 1,2, t = 1,2$$

where $r_{hti} = \frac{E_{hti}}{BV_{hti}}$ represents the individual error rates for units in the sample of stratum h in semester t and $\bar{r}_{ht}$ represent the mean error rate of the sample in stratum h and semester *t*[35].

Since these variances are unknown, the AA decided to draw, in each stratum (programme) a preliminary sample of 20 operations at the end of first semester of the current reference period. The sample standard deviation of error rates in this preliminary sample at first semester is 0.0924 and 0.0515 for programmes 1 and 2, respectively. Based on professional judgement, the AA expects the standard deviations of error rates for the second semester to grow by 40% and 10%, that is, to 0.1294 and 0.0567. Therefore, the weighted average of the variances of the error rates is:

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[35] Whenever the book value of unit *i* (BV_i) is larger than BV_{ht}/n_{ht} the ratio $\frac{E_{hti}}{BV_{hti}}$ should be substituted by the ratio $\frac{E_{hti}}{BV_{ht}/n_{ht}}$.

$$\sigma_{rw}^2 = 0.0028188 + 0.0071654 = 0.009984,$$

provided the weighted average for both semesters are:

$$\sigma_{rw1}^2 = \frac{27{,}623{,}498}{97{,}769{,}587} \times 0.0924^2 + \frac{14{,}987{,}234}{97{,}769{,}587} \times 0.0515^2 = 0.0028188$$

$$\sigma_{rw2}^2 = \frac{38{,}672{,}897}{97{,}769{,}587} \times 0.1294^2 + \frac{16{,}485{,}957}{97{,}769{,}587} \times 0.0567^2 = 0.0071654$$

In the first semester, given the level of functioning of the management and control system, the AA considers adequate a confidence level of 90%. The global sample size for the whole year is:

$$n = \left(\frac{z \times BV \times \sigma_{rw}}{TE - AE}\right)^2$$

$$n = \left(\frac{1.645 \times 97{,}769{,}587 \times \sqrt{0.009984}}{1{,}955{,}392 - 391{,}078}\right)^2 \approx 106$$

where z is 1.645 (coefficient corresponding to a 90% confidence level), TE, the tolerable error, is 2% (maximum materiality level set by the Regulation) of the book value. The total book value comprises the true book value at the end of the first semester plus the predicted book value for the second semester, which means that tolerable error is 2% x 97,769,587 € = 1,955,392 €. The last year's audit projected an error rate of 0.4%. Thus AE, the anticipated error, is 0.4% x 97,769,587 € = 391,078 €.

The allocation of the sample by semester and stratum is as follows:

$$n_{h1} = \frac{BV_{h1}}{BV} n, h = 1{,}2;\ n_{11} = \frac{27{,}623{,}498}{97{,}769{,}587} \times 106 \cong 30; n_{21} = \frac{14{,}987{,}234}{97{,}769{,}587} \times 106 \cong 17$$

and

$$n_{h2} = \frac{BV_{h2}}{BV} n, h = 1{,}2;\ n_{12} = \frac{38{,}672{,}897}{97{,}769{,}587} \times 106 \cong 42; n_{22} = \frac{16{,}485{,}957}{97{,}769{,}587} \times 106 \cong 18$$

For the first semester, it is necessary to identify the high value population units of both programmes (if any) that will belong to a high-value stratum to be submitted at a 100%

audit work. The cut-off value for determining this top stratum is equal to the ratio between the book value (BV_{h1}) and the planned sample size (n_{h1}). All items whose book value is higher than this cut-off (if $BV_{ih1} > BV_{h1}/n_{h1}$) will be placed in the 100% audit stratum.

These two samples sizes of first semester (30 and 17) lead to the following values of cut-off for high-value strata, for both programmes:

$$Cut - off_{11} = \frac{BV_{11}}{n_{11}} = \frac{27{,}623{,}498}{30} = 920{,}783$$

and

$$Cut - off_{21} = \frac{BV_{21}}{n_{21}} = \frac{14{,}987{,}234}{17} = 881{,}602$$

Using these two cut-off values, 3 and 4 high value operations are found in programme 1 and 2, totalling a book value of 3,475,552 € and 4,289,673 €, respectively.

The sampling size to be allocated to the non-exhaustive stratum (n_{h1s}) is computed as the difference between n_{h1} and the number of sampling units in the exhaustive stratum. The sample size for the sampling part of programme 1 will be given by total sample size (30), from which the 3 high-value operations are deducted, i.e., 27 operations. Applying the same reasoning for programme 2, the sample size for the sampling part of is 17-4=13 operations.

The next step will be the calculation of sampling interval for the sampling strata. The sampling intervals are, respectively, given by:

$$SI_{11} = \frac{BV_{11s}}{n_{11s}} = \frac{27{,}623{,}498 - 3{,}475{,}552}{27} = 894{,}368$$

and

$$SI_{21} = \frac{BV_{21s}}{n_{21s}} = \frac{14{,}987{,}234 - 4{,}289{,}673}{13} = 822{,}889$$

The following table summarises these results:

Book value (sum of the expenditure at the end of first semester)	42,610,732 €
Book value – programme 1	27,623,498 €
Book value – programme 2	14,987,234 €
Sample results – programme 1	
Cut-off value	920,783 €

Number of operations above cut-off value	3
Book value of operations above cut-off value	3,475,552 €
Book value of operations (non-exhaustive population)	24,147,946 €
Sampling interval (non-exhaustive population)	894,368 €
Number of operations (non-exhaustive population)	3,254
Sample results – programme 2	
Cut-off value	881,602 €
Number of operations above cut-off value	4
Book value of operations above cut-off value	4,289,673 €
Book value of operations (non-exhaustive population)	10,697,561 €
Sampling interval (non-exhaustive population)	822,889 €
Number of operations (non-exhaustive population)	2,342

The selection of the sample in the non-exhaustive strata will be made using probability proportional to size, i.e. proportional to the item book values BV_{ih1s}, through systematic selection.

For programme 1, at the end of the first semester, a file containing the remaining 3,254 operations (3,257 minus 3 high value operations) of the population is randomly sorted and a sequential cumulative book value variable is created. A sample of 27 operations (30 minus 3 high value operations) is drawn using exactly the same procedure as described in Section 6.3.1.7.

For programme 2, at the end of the first semester, a file containing the remaining 2,342 operations (2,346 minus 4 high value operations) of the population is randomly sorted and a sequential cumulative book value variable is created. A sample value of 13 operations (17 minus 4 high value operations) is drawn as described in previous paragraph.

For programme 1, in the 3 high-value operations a total error of 13,768 € was found. For programme 2, no errors were found in the high-value stratum.

The expenditure of the 40 sampled operations (27 + 13) is audited. The sum of the sample error rates for programme 1, at the end of first semester is:

$$\sum_{i=1}^{27} \frac{E_{i11s}}{BV_{i11s}} = 0.0823.$$

The sum of the sample error rates for programme 2, at the end of first semester is:

$$\sum_{i=1}^{13} \frac{E_{i21s}}{BV_{i21s}} = 0.1145$$

The standard-deviation of error rates in the sample of the non-exhaustive population of the first semester, for both programmes is:

$$s_{r11s} = \sqrt{\frac{1}{27-1}\sum_{i=1}^{27}(r_{i11s} - \bar{r}_{11s})^2} = 0.0868$$

$$s_{r21s} = \sqrt{\frac{1}{13-1}\sum_{i=1}^{13}(r_{i21s} - \bar{r}_{21s})^2} = 0.0696$$

having $\bar{r}_{h1s}, h = 1{,}2$, equal to the simple average of error rates in the sample of the non-exhaustive group of first semester.

At the end of the second semester more information is available, in particular, the total expenditure of operations active in the second semester is correctly known, the sample variance of error rates for both programmes, s_{r11} and s_{r21}, based on the first semester stratum samples could be already available and the standard deviation of error rates of the second semester, for both programmes, σ_{r12} and σ_{r22}, can now be more accurately assessed using a preliminary samples of real data.

The AA realises that the assumption made at the end of the first semester on the second semester expenditure, 55,158,855 €, overestimates the true value of 49,211,269. There are also two additional parameters for which updated figures should be used.

First, the estimate of the standard deviation of error rates based on the first semester programme samples of 27 and 13 operations, respectively, yielded estimates of 0.0868 and 0.0696. This new values should now be used to reassess the planned sample size. Second, based on two preliminary samples of the second semester, for both programmes, the AA considers more prudent to estimate the standard deviation of error rates for the second semester as 0.0943 and 0.0497 instead of the initial values of 0.1294 and 0.0567. The updated figures of standard deviation of error rates for the two programmes in both semesters are far from the initial estimates. As a result, the sample for the second semester should be revised.

The following table summarises these results

Parameter	Forecast done at the end first semester	End of second semester
Standard deviation of error rates in the first semester		
Programme 1	0.0924	0.0868
Programme 2	0.0515	0.0696
Standard deviation of error rates in the second semester		
Programme 1	0.1294	0.0943
Programme 2	0.0567	0.0497
Total expenditure in the second semester		
Programme 1	38,672,897 €	32,976,342 €
Programme 2	16,485,957 €	16,234,927 €

Taking into consideration these three types of adjustments, the recalculated sample size of the second semester is

$$n_2 = \frac{z^2 \times BV_2 \times \sum_{h=1}^{2}\left(BV_{h2} . \sigma_{rh2}^2\right)}{(TE - AE)^2 - z^2 \times \sum_{h=1}^{2}\left(\frac{BV_{h1}^2}{n_{h1}} . s_{rh1}^2\right)}$$

where s_{rh1} are the standard-deviations of error rates calculated from the subsamples of the first semester for each stratum *h*, *h=1,2*, and σ_{rh2} estimates of the standard-deviations of error rates in each stratum of the second semester based on preliminary samples:

$$n_2 = \frac{1.645^2 \times 49{,}211{,}269 \times (32{,}976{,}342 \times 0.0943^2 + 16{,}234{,}927 \times 0.0497^2)}{(1{,}836{,}440 - 367{,}288)^2 - 1.645^2 \times \left(\frac{27{,}623{,}498^2}{30} \times 0.0868^2 + \frac{14{,}987{,}234^2}{17} \times 0.0696^2\right)} \cong 31$$

Based on these updated figures the samples size to achieve the desired precision is 31 operations, instead of the 60 planned at the end of the first semester. The allocation by programme is now straightforward:

$$n_{12} = \frac{BV_{12}}{BV_2} n_2 = \frac{32{,}976{,}342}{49{,}211{,}269} \times 31 \cong 21$$

$$n_{22} = 31 - 21 = 10$$

It is necessary to identify the high value population units (if any) that will belong to a high-value strata to be submitted at a 100% audit work. The cut-off values for determining this top strata is equal to the ratio between the book value (BV_{h2}) and the planed sample size (n_{h2}). All items whose book value is larger than these cut-offs (if $BV_{ih2} > BV_{h2}/n_{h2}, h = 1{,}2$) will be placed in the 100% audit stratum. In these cases, the cut-off values are:

The two updated samples sizes of second semester (21 and 10) lead to the following values of cut-off for high-value strata, for both programmes:

$$Cut - off_{12} = \frac{BV_{12}}{n_{12}} = \frac{32{,}976{,}342}{21} = 1{,}570{,}302$$

and

$$Cut - off_{22} = \frac{BV_{22}}{n_{22}} = \frac{16{,}243{,}927}{10} = 1{,}624{,}393$$

There are 3 operations, in programme 1, and 2 operations, in programme 2, which book value is larger than the respective cut-off value. The total book value of these operations amounts to 7,235,619 €, in programme 1, and 4,329,527 €, in programme 2.

The sampling sizes to be allocated to the non-exhaustive strata, n_{12s} and n_{22s}, are computed as the difference between $n_{h2}, h = 1{,}2$ and the number of sampling units (e.g. operations) in the respective exhaustive stratum, that is 14 operations for programme 1 (21, the updated sample size of programme 1 in second semester, minus the 7 high-value operations) and 6 operations for programme 2 (10, the updates sample size of programme 2 in second semester, minus 4 high-value operations). Therefore, the auditor has to select the remaining samples using the sampling intervals:

$$SI_{12s} = \frac{BV_{12s}}{n_{12s}} = \frac{32{,}976{,}342 - 7{,}235{,}619}{18} = 1{,}430{,}040$$

$$SI_{22s} = \frac{BV_{22s}}{n_{22s}} = \frac{16{,}234{,}927 - 4{,}329{,}527}{8} = 1{,}489{,}300$$

The book value in the non-exhaustive strata (BV_{12s} and BV_{22s}) is just the difference between the total book value of the stratum and the book value of the respective high-value operations.

The following table summarises these results:

Book value (declared expenditure in the second semester)	49,211,269 €
Book value – programme 1	32,976,342 €
Book value – programme 2	16,234,927 €

Sample results – programme 1	
Cut-off value	1,570,302 €
Number of operations above cut-off value	3
Book value of operations above cut-off value	7,235,619 €
Book value of operations (non-exhaustive population)	25,740,723 €
Sampling interval (non-exhaustive population)	1,430,040 €
Number of operations (non-exhaustive population)	3,254
Sample results – programme 2	
Cut-off value	1,623,493 €
Number of operations above cut-off value	2
Book value of operations above cut-off value	4,329,527 €
Book value of operations (non-exhaustive population)	11,914,400 €
Sampling interval (non-exhaustive population)	1,489,300 €
Number of operations (non-exhaustive population)	2,344

No errors were found in the expenditure of both programmes' high-value operations.

For programme 1, a file containing the 3,254 operations (3,257 minus 3 high value operations) and the corresponding expenditure declared in the second semester is sorted randomly and a sequential cumulative book value variable is created. A sample of 18 operations (21 minus 3 high value operations) is drawn using exactly the same procedure as before.

For programme 2, a file containing the 2,344 operations (2,346 minus 2 high value operations) and the corresponding expenditure declared in the second semester is randomly sorted and a sequential cumulative book value variable is created. A sample value of 8 operations (10 minus 3 high value operations) is drawn using probability proportional to size.

The expenditure of the 26 (18 + 8) operations is audited. The sum of the sample error rates for programme 1, at the end of second semester is:

$$\sum_{i=1}^{18} \frac{E_{i12s}}{BV_{i12s}} = 0.1345.$$

The sum of the sample error rates for programme 2, at the end of first semester is:

$$\sum_{i=1}^{8} \frac{E_{i22s}}{BV_{i22s}} = 0.0934$$

The standard-deviation of error rates in the sample of the non-exhaustive population of the first semester, for both programmes is:

$$s_{r12s} = \sqrt{\frac{1}{18-1}\sum_{i=1}^{18}(r_{i12s} - \bar{r}_{12s})^2} = 0.0737$$

$$s_{r22s} = \sqrt{\frac{1}{8-1}\sum_{i=1}^{8}(r_{i22s} - \bar{r}_{22s})^2} = 0.0401$$

having $\bar{r}_{h2s}, h = 1{,}2$, equal to the simple average of error rates in the sample of the non-exhaustive group of second semester.

The projection of errors to the population is calculated differently for units belonging to the exhaustive groups and for items in the non-exhaustive groups.

For the high-value strata, that is, for the groups containing the sampling units with book value larger than the cut-off values, $BV_{hti} > \frac{BV_{ht}}{n_{ht}}$, the projected error is the summation of the errors found in the items belonging to those groups:

$$EE_e = \sum_{h=1}^{2}\sum_{i=1}^{n_{h1}} E_{h1i} + \sum_{h=1}^{2}\sum_{i=1}^{n_{h2}} E_{h2i} = 13{,}768$$

In practice:

1) For each semester, and in each stratum h, identify the units belonging to the exhaustive group and sum their errors;
2) Sum the previous results over the set of strata.

For the non-exhaustive groups, i.e. the groups containing the sampling units with book value lower or equal to the cut-off values, $BV_{hti} \leq \frac{BV_{ht}}{n_{ht}}$, the projected error is

$$EE_s = \sum_{h=1}^{2}\left(\frac{BV_{h1s}}{n_{h1s}}.\sum_{i=1}^{n_{h1s}}\frac{E_{h1i}}{BV_{h1i}}\right) + \sum_{h=1}^{2}\left(\frac{BV_{h2s}}{n_{h2s}}.\sum_{i=1}^{n_{h2s}}\frac{E_{h2i}}{BV_{h2i}}\right)$$
$$= 894{,}368 \times 0.0823 + 822{,}889 \times 0.1145 + 1{,}430{,}040 \times 0.1345 + 1{,}489{,}300 \times 0.0934 = 499{,}268$$

To calculate this projected error:

1) in each stratum h in each semester t, for each unit in the sample calculate the error rate, i.e. the ratio between the error and the respective expenditure $\frac{E_{hti}}{BV_{hti}}$
2) in each stratum *h* in each semester *t*, sum these error rates over all units in the sample
3) in each stratum *h* in semester *t*, multiply the previous result by the total expenditure in the population of the non-exhaustive group (BV_{hts}); this expenditure will also be equal to the total expenditure of the stratum minus the expenditure of items belonging to the exhaustive group of the stratum
4) in each stratum *h* in each semester *t*, divide the previous result by the sample size in the non-exhaustive group (n_{hts})
5) sum the previous results over the whole set of strata

The projected error at the level of population is just the sum of these two components:

$$EE = 13{,}768 + 499{,}268 = 513{,}036,$$

corresponding to a projected error rate of 0.56%.

The precision is a measure of the uncertainty associated with the projection. The precision is given by the formula:

$$SE = z \times \sqrt{\sum_{h=1}^{2}\left(\frac{BV_{h1s}^2}{n_{h1s}} . s_{rh1s}^2\right) + \sum_{h=1}^{2}\left(\frac{BV_{h2s}^2}{n_{h2s}} . s_{rh2s}^2\right)}$$

$$= 1.645 \times \sqrt{\frac{24{,}147{,}946^2}{27} 0.0823^2 + \frac{10{,}697{,}561^2}{13} 0.0696^2 + \frac{25{,}740{,}723^2}{18} 0.0737^2 + \frac{11{,}914{,}400^2}{8} 0.0401^2}$$

$$= 1{,}062{,}778$$

where s_{rhts} are the standard-deviation of error rates of the non-exhaustive group of stratum *h* of semester *t* already computed.

The sampling error is only computed for the non-exhaustive groups, since there is no sampling error arising from the exhaustive groups.

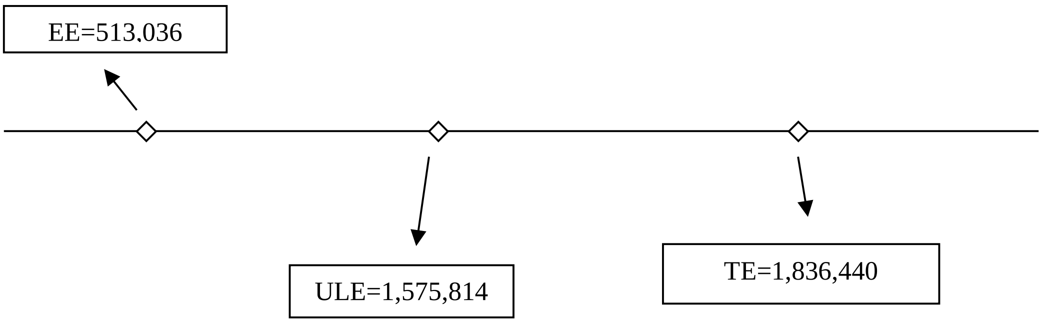
To draw a conclusion about the materiality of the errors the upper limit of error (ULE) should be calculated. This upper limit is equal to the summation of the projected error EE itself and the precision of the projection

$$ULE = EE + SE = 513{,}036 + 1{,}062{,}778 = 1{,}575{,}814$$

Then the projected error and the upper limit should both be compared to the maximum tolerable error to draw audit conclusions.

Then the projected error and the upper limit should both be compared to the maximum tolerable error to draw audit conclusions.

In this particular case, both the projected error and the upper limit are smaller than maximum tolerable error. It means that the auditor would conclude that there is not enough evidence to support that errors in the population are larger than materiality threshold:



6.3.5 *Conservative approach*

6.3.5.1 Introduction

In the context of auditing it is usual to use a conservative approach to monetary unit sampling. This conservative approach has the advantage of requiring less knowledge about the population (for ex. no information about population variability is needed for sample size calculation). Also, several software packages used in the audit world automatically implement this approach turning easier its application. In fact, when adequately supported by these packages the application of the conservative method requires significantly less technical and statistical knowledge than the so-called standard approach. The main disadvantage of this conservative approach is in fact related with this easiness of application: as it uses less detailed information for sample size calculation and for precision determination it usually produces larger samples sizes and larger estimated sampling errors than the more exact formulas used in the standard approach. Nevertheless, whenever sample is already of a manageable size and not a major concern of the auditor, this approach can be a good option due to its simplicity. Also it is important to stress that this method is only applicable to situations where the frequency of errors is small and the error rates clearly bellow materiality[36]. Finally, one should note that as a consequence of the fact that this method usually produces large sample sizes, the users are sometimes tempted to feed it with very small and unrealistic anticipated errors. This practice will unavoidably result in inconclusive results for the audit due to the too large upper error limit and it imperative to remember that as for any other sampling method, the anticipated error should be chosen to be realistic based on the auditor best knowledge and opinion.

This method cannot be combined with stratification or spreading the audit work in two or more periods within the reference period as it would result in unworkable formulas for precision determination. Therefore, the audit authorities are encouraged to use the standard approach for these purposes.

6.3.5.2 Sample size

The calculation of sample size n within the framework of monetary unit sampling conservative approach relies on the following information:

- Population book value (total declared expenditure) *BV*
- A constant called reliability factor (*RF*) determined by the confidence level
- Maximum tolerable error *TE* (usually 2% of the total expenditure)
- Anticipated error *AE* chosen by the auditor according to professional judgment and previous information
- The expansion factor, EF, which is a constant also associated with the confidence level and used when errors are expected

The sample size is computed as follows:

$$n = \frac{BV \times RF}{TE - (AE \times EF)}$$

The reliability factor RF is a constant from the Poisson distribution for an expected zero error. It is dependent on the confidence level and the values to apply in each situation can be found in the following table.

Confidence level	99%	95%	90%	85%	80%	75%	70%	60%	50%
Reliability Factor (RF)	4.61	3.00	2.31	1.90	1.61	1.39	1.21	0.92	0.70

Table 4. Reliability factors by confidence level

[36] In particular, it is not possible to calculate the sample size if the anticipated error is larger or close to materiality.

The expansion factor, $EF,$ is a factor used in the calculation of MUS sampling when errors are expected, which is based upon the risk of incorrect acceptance. It reduces the sampling error. If no errors are expected, the anticipated error (AE) will be zero and the expansion factor is not used. Values for the expansion factor are found in the following table.

Confidence level	99%	95%	90%	85%	80%	75%	70%	60%	50%
Expansion Factor (EF)	1.9	1.6	1.5	1.4	1.3	1.25	1.2	1.1	1.0

Table 5. Expansion factors by confidence level

The formula for sample size determination shows why this approach is called conservative. In fact sample size is neither dependent on the population size nor on the population variability. This means that the formula aims to fit any kind of population despite its specific characteristics, therefore usually producing sample sizes that are larger than the ones needed in practice.

6.3.5.3 Sample selection

After determining sample size, the selection of the sample is made using probability proportional to size, i.e. proportional to the item book values BV_i. A popular way to implement the selection is through systematic selection, using a sampling interval equal to the total expenditure (BV) divided by the sample size (n), i.e.

$$SI = \frac{BV}{n}$$

Typically, the sample is selected from a randomised list of all items, selecting each item containing the x^{th} monetary unit, **x being the step corresponding to the book value divided by the sample size**, that is, the sampling interval.

Some items can be selected multiple times (if its value is above the size of the sampling interval). In this case, the auditor should create an exhaustive stratum where all the items with book value larger than the sampling interval should belong. This stratum will have a different treatment for error projection, as usual.

6.3.5.4 *Projected error*

The projection of the errors to the population follows the procedure presented in the context of the standard MUS approach. Again, the extrapolation is done differently for the units in the exhaustive stratum and for the items in the non-exhaustive stratum.

For the exhaustive stratum, that is, for the stratum containing the sampling units with book value larger than the sampling interval, $BV_i > \frac{BV}{n}$, the projected error is just the summation of the errors found in the items belonging to the stratum:

$$EE_e = \sum_{i=1}^{n_e} E_i$$

For the non-exhaustive stratum, i.e. the stratum containing the sampling units with book value lower or equal to the sampling interval, $BV_i \leq \frac{BV}{n}$ the projected error is

$$EE_s = SI \sum_{i=1}^{n_s} \frac{E_i}{BV_i}$$

To calculate this projected error:
1) for each unit in the sample calculate the error rate, i.e. the ration between the error and the respective expenditure $\frac{E_i}{BV_i}$
2) sum these error rates over all units in the sample
3) multiply the previous result by the sampling interval (SI)

The projected error at the level of population is just the sum of these two components:

$$EE = EE_e + EE_s$$

6.3.5.5 *Precision*

Precision, which is measuring sampling error, has two components: the Basic Precision, $BP,$ and the Incremental allowance, IA.

The basic precision is just the product between sampling interval and the reliability factor (already used for calculating sample size):

$$BP = SI \times RF.$$

The incremental allowance is computed for every sampling unit belonging to the non-exhaustive stratum that contains an error.

Firstly, items with errors should be ordered by decreasing value of the projected error.

Secondly, an incremental allowance is calculated for each one of these items (with errors), using the formula:

$$IA_i = (RF(n) - RF(n-1) - 1) \times SI \times \frac{E_i}{BV_i}.$$

where $RF(n)$ is the reliability factor for the error that appear at n^{th} order at a given confidence level (typically the same used for sample size calculation), and $RF(n-1)$ is the reliability factor for the error at $(n-1)^{th}$ order at a given confidence level. For example, at 90% of confidence the corresponding table of reliability factors is:

Order of the error	**Reliability Factor (RF)**	$\boldsymbol{RF(n) - RF(n-1) - 1}$
Order zero	2.31	
1st	3.89	0.58
2nd	5.33	0.44
3rd	6.69	0.36
4th	8.00	0.31
…		

Table 7. Reliability factors by order of the error

For instance if the larger projected error in the sample is equal to 10,000€ (25% of the expenditure of 40,000€) and we have a sampling interval of 200,000€, the individual incremental allowance for this error is equal to 0.58 x 0.25 x 200,000=29,000€.

A table with reliability factors for several confidence levels and different number of errors found in the sample can be found in appendix.

Finally, the incremental allowance is the sum of all item incremental allowances:

$$IA = \sum_{i=1}^{n_s} IA_i.$$

The global precision (SE) will be equal to the sum of the two components: basic precision (BP) and incremental allowance (IA)

$$SE = BP + IA$$

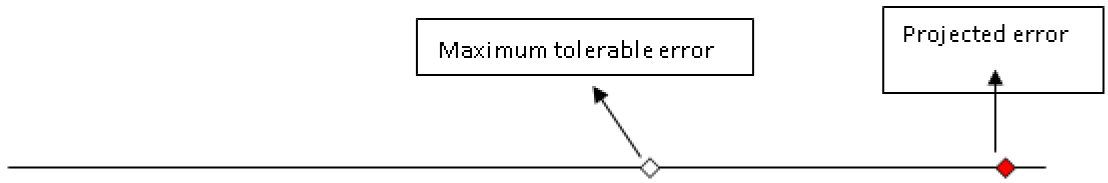
6.3.5.6 Evaluation

To draw a conclusion about the materiality of the errors the upper limit of error (ULE) should be calculated. This upper limit is equal to the summation of the projected error EE itself and the global precision of the extrapolation

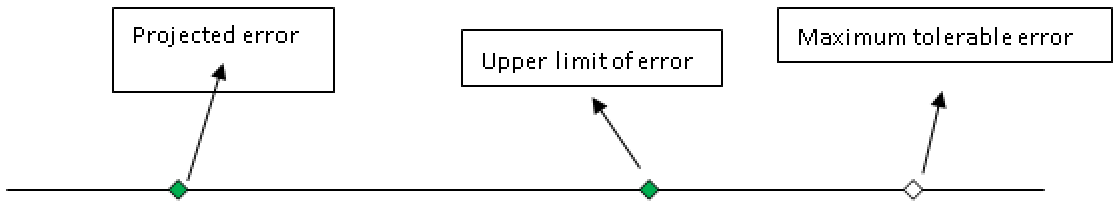
$$ULE = EE + SE$$

Then the projected error and the upper limit should both be compared to the maximum tolerable error to draw audit conclusions:

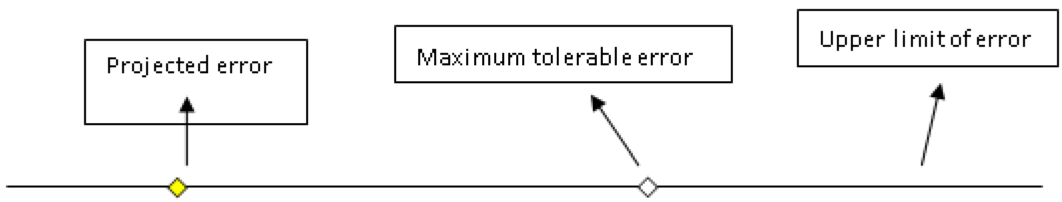
- If projected error is larger than maximum tolerable error, it means that the auditor would conclude that there is enough evidence to support that errors in the population are larger than materiality threshold:



- If the upper limit of error is lower than maximum tolerable error, then the auditor should conclude that errors in the population are lower than materiality threshold.



If the projected error is lower than maximum tolerable error but the upper limit of error is larger, please refer to section 4.12 for more details on the analysis to be done.



6.3.5.7 Example

Let's assume a population as expenditure declared to the Commission in a given year for operations in a programme. The system audits performed by the audit authority have yielded a low assurance level. Therefore, sampling this programme should be done with a confidence level of 90%.

The population is summarised in the table below:

Population size (number of operations)	3,852
Book value (sum of the expenditure in the reference period)	4,199,882,024 €

The sample size is computed as follows:

$$n = \frac{BV \times RF}{TE - (AE \times EF)},$$

where BV is the total book value of the population, that is, the total expenditure declared to the Commission in the reference period, RF is the reliability factor corresponding to the 90% confidence level, 2.31, $EF,$ is the expansion factor corresponding to the confidence level if errors are expected, 1.5. Regarding this particular population the audit authority, based on the past years' experience and on the knowledge of the improvements on the management and control system has decided that an expected error rate of 0.2% is reliable

$$n = \frac{4{,}199{,}882{,}024 \times 2.31}{0.02 \times 4{,}199{,}882{,}024 - (0.002 \times 4{,}199{,}882{,}024 \times 1.5)} \approx 136$$

The selection of the sample is made using probability proportional to size, i.e. proportional to the item book values, BV_i through systematic selection, using a sampling interval equal to the total expenditure (BV) divided by the sample size (n), i.e.

$$SI = \frac{BV}{n} = \frac{4{,}199{,}882{,}024}{136} = 30{,}881{,}485$$

A file containing the 3,852 operations of the population is randomly sorted and a sequential cumulative book value variable is created.

The sample is selected from this randomised list of all operations, selecting each item containing the 30,881,485th monetary unit.

Operation	Book Value (BV)	AcumBV
239	10,173,875 €	10,173,875 €
424	23,014,045 €	33,187,920 €
2327	32,886,198 €	66,074,118 €
5009	34,595,201 €	100,669,319 €
1491	78,695,230 €	179,364,549 €
(…)	**(…)**	**(…)**

A random value between 0 and the sampling interval, 30,881,485 is generated (16,385,476). The first item to be selected is the one that contains the 16,385,476th monetary unit. The second selection corresponds to the first operation in the file with the accumulated book value greater or equal to 16,385,476+30,881,485 and so on…

Operation	Book Value (BV)	AcumBV	Sample
239	10,173,875 €	10,173,875 €	No
424	23,014,045 €	33,187,920 €	Yes
2327	32,886,198 €	66,074,118 €	Yes
5009	34,595,201 €	100,669,319 €	Yes
1491	78,695,230 €	179,364,549 €	Yes
(…)	**(…)**	**(…)**	**(…)**
2596	8,912,999 €	307,654,321 €	Yes
779	26,009,790 €	333,664,111 €	No
1250	264,950 €	333,929,061 €	No
3895	30,949,004 €	364,878,065 €	Yes
2011	617,668 €	365,495,733 €	No
4796	335,916 €	365,831,649 €	No
3632	7,971,113 €	373,802,762 €	No
2451	17,470,048 €	391,272,810 €	Yes
(…)	**(…)**	**(…)**	**(…)**

There are 24 operations whose book value is larger than the sampling interval, meaning that each one is selected at least once (for instance, the operation 1491 is selected 3 times, cf. previous table). The book value of these 24 operations amounts to 1,375,130,377 €. Out of these 24 operations, 4 contain errors corresponding to an error amount of 7,843,574 €.

For the remaining sample the error have a different treatment. For these operations we use the following procedure:

1) for each unit in the sample calculate the error rate, i.e. the ration between the error and the respective expenditure $\frac{E_i}{BV_i}$

2) sum these error rates over all units in the sample
3) multiply the previous result by the sampling interval (SI)

$$EE_s = SI \sum_{i=1}^{n_s} \frac{E_i}{BV_i}$$

Operation	**Book Value (BV)**	**Correct Book Value (CBV)**	**Error**	**Error rate**
2596	8,912,999 €	8,912,999 €	- €	-
459	869,080 €	869,080 €	- €	-
2073	859,992 €	859,992 €	- €	-
239	10,173,875 €	9,962,918 €	210,956 €	0.02
989	394,316 €	394,316 €	- €	-
65	25,234,699 €	25,125,915 €	108,784 €	0.00
5010	34,595,201 €	34,595,201 €	- €	-
…	…	…	…	…
3632	7,971,113 €	7,971,113 €	- €	-
3672	624,882 €	624,882 €	- €	-
2355	343,462 €	301,886 €	41,576 €	0.12
959	204,847 €	204,847 €	- €	-
608	15,293,716 €	15,293,716 €	- €	-
4124	6,773,014 €	6,773,014 €	- €	-
262	662 €	662 €	- €	-
Total				1.077

$$EE_s = 30{,}881{,}485 \times 1.077 = 33{,}259{,}360$$

The projected error at the level of population is just the sum of these two components:

$$EE = 7{,}843{,}574 + 33{,}259{,}360 = 41{,}102{,}934$$

corresponding to a projected error rate of 0.98%.

In order to be able to build the upper limit of error one needs to calculate the two components of the precision, the Basic Precision, BP, and the Incremental allowance, IA.

The basic precision is just the product between sampling interval and the reliability factor (already used for calculating sample size):

$$BP = 30{,}881{,}485 \times 2.31 = 71{,}336{,}231$$

The incremental allowance is computed for every sampling unit belonging to the non-exhaustive stratum that contains an error.

First, items with errors should be ordered by decreasing value of the projected error. Second, an incremental allowance is calculated for each one of these items (with errors), using the formula:

$$IA_i = (RF(n) - RF(n-1) - 1) \times SI \times \frac{E_i}{BV_i}.$$

where $RF(n)$ is the reliability factor for the error that appear at n^{th} order at a given confidence level (typically the same used for sample size calculation), and $RF(n-1)$ is the reliability factor for the error at $(n-1)^{th}$ order at a given confidence level (see table in the appendix).

Finally, the incremental allowance is the sum of all item incremental allowances:

$$IA = \sum_{i=1}^{n_s} IA_i.$$

The following table summarises these results for the 16 operations containing error:

Order	**Error (A)**	**Error rate (B):=(A)/BV**	**Projected error:=(B)*SI**	**RF(n)**	**(RF(n)-RF(n-1))-1**	IA_i
0				2.30		
1	4,705,321 €	0.212	6,546,875 €	3.89	0.59	3,862,656 €
(…)	(…)	(…)	(…)	(…)	(…)	(…)
12	12,332 €	0.024	741,156 €	17.78	0.18	133,408 €
13	6,822 €	0.02	617,630 €	18.96	0.18	111,173 €
14	7,706 €	0.012	370,578 €	20.13	0.17	62,998 €
15	4,787 €	0.008	247,052 €	21.29	0.16	39,528 €
16	26,952 €	0.001	29,488 €	22.45	0.16	4,718 €
Total		1.077	38,264,277 €			14,430,761 €

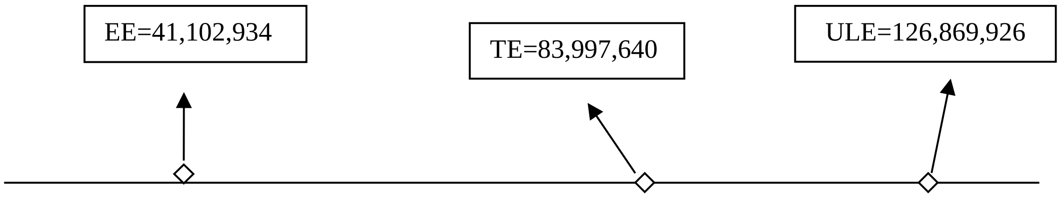
The global precision (SE) will be equal to the sum of the two components: basic precision (BP) and incremental allowance (IA)

$$SE = 71{,}336{,}231 + 14{,}430{,}761 = 85{,}766{,}992$$

To draw a conclusion about the materiality of the errors the upper limit of error (ULE) should be calculated. This upper limit is equal to the summation of the projected error EE itself and the global precision of the projection

$$ULE = 41{,}102{,}933 + 85{,}766{,}992 = 126{,}869{,}926$$

Now the maximum tolerable error, TE=2% x 4,199,882,024=83,997,640 € should be compared with both the projected error and the upper limit of error. The maximum tolerable error is larger than the projected error but smaller than the upper limit of error. Please refer to section 4.12 for more details on the analysis to be done.



6.4 Non statistical sampling

6.4.1 Introduction

A non- statistical sampling method may be used on the professional judgement of the AA, in duly justified cases, in accordance with internationally accepted audit standards and in any case, where the number of operations is insufficient to allow the use of a statistical method.

As explained above in Section 5.2, statistical sampling should be used, as a general rule, to audit the declared expenditure and draw conclusions about the amount of error in a population. Non-statistical sampling does not allow the calculation of precision, and consequently there is no control of the audit risk. Consequently, non-statistical sampling should only be used in cases where statistical sampling is not possible to implement.

In practice, the specific situations that may justify the use of non-statistical sampling are related to the population size. In fact, it may happen to work with a very small population, whose size is insufficient to allow the use of statistical methods (the population is smaller or very close to the recommended sample size).

In summary, non-statistical sampling is considered appropriate for cases where it is not possible to achieve an adequate sample size that would be required to support statistical sampling. It is not possible to state the exact population size below which non-statistical sampling is needed as it depends on several population characteristics, but usually this threshold is somewhere between 50 and 150 sampling units. **The final decision should of course take into consideration the balance between the cost and benefit associated with each of the methods. It is**

recommended that the audit authority seeks the Commission's advice before taking the decision to apply non-statistical sampling in specific circumstances, for cases where the threshold of 150 units is exceeded. The Commission may agree with the use of non-statistical sampling based on a case by case analysis.

For 2014-2020, the regulation also sets criteria to be respected when non-statistical sampling is applied, namely to cover a minimum of 5% operations and 10% of the expenditure declared (Article 127(1) CPR). This may lead in practice to sample sizes equivalent to the ones obtained by statistical sampling methods. In such situations, the AAs are encouraged to use statistical methods instead.

Even in the situations where the AA applied a non-statistical sampling method, the sample shall be selected using a random method[37] [38]. The size of the sample must be determined taking into account the level of assurance provided by the system, and must be sufficient to enable the AA to draw a valid audit opinion on the legality and regularity of the expenditure. **The AA should be able to extrapolate the results to the population from which the sample was drawn.**

When implementing non-statistical sampling, the AA should consider stratifying the population by dividing it into sub-populations, each one being a group of sampling units with similar characteristics, in particular in terms of risk or expected error rate or where the population includes specific types of operations (e.g. financial instruments). Stratification is a very efficient tool to improve the quality of the projections and it is strongly recommendable to use some kind of stratification in the framework of non-statistical sampling.

6.4.2 Stratified and non-stratified non-statistical sampling

Stratified non-statistical sampling should be the first option to consider by the AA when confronted with the impossibility to use statistical sampling. As explained regarding the stratification of statistical sampling designs, the criteria to use for stratification purposes is related with the expectation of the auditor regarding its contribution to explain the level of error in the population. Whenever one expects that the level of error will be different for different groups in the population this classification is a good candidate to implement stratification.

[37] i.e. using a statistical (probabilistic method) cf. Section 4.1 and 4.2 for a distinction between sampling method and selection method. Additionally remember the rule of thumb that settles the minimum sample size for statistical sampling equal to 30.

[38] Non-random (e.g. risk-based) non-statistical sampling selection can only be used for the complementary sample foreseen in Article 17 (§5 and §6) of Regulation (EC) No 1828/2006 (period 2007-2013) and Article 28 of Regulation (EU) No 480/2014 (period 2014-2020).

When using equal probability selection (where each sampling unit has equal chance of being selected regardless of the amount of expenditure declared in the sampling unit), the stratification by level of expenditure is to be recommended as a very efficient tool to improve the quality of the estimates. It should be noted that although this stratification is not obligatory, such a design can also help the AA to ensure the recommended coverage of the expenditure declared required for the programming period 2014-2020.

For this stratification (which could be used both in equal probability selection and probability proportional to size):

- Determine the cut-off value of expenditure for items that will be included in the high value stratum. There is not a general rule to establish the cut-off value. Therefore if the commonly used practice to establish the cut-off value equal to the maximum tolerable error (2% of the total expenditure) of the population, if applied, should only be seen as a starting point that should be adapted to the population characteristics. This cut-off can and should be changed in accordance to population characteristics. In short, this cut-off value should mainly be determined by professional judgments. Whenever the auditor can identify a few number of items whose expenditure is significantly higher than the one observed on the remaining items should consider to create a stratum with these elements. In addition, the auditor is invited to use more than two expenditure-based strata if the division in two strata seems insufficient to generate the desirable level of homogeneity inside each stratum.

- A 100% audit of the high value items is the basic method to consider. Nevertheless, in practice, some situations may arise where the identified cut-off creates a too large high-value stratum, which could hardly be exhaustively observed. In these situations, it is possible to also observe the high-value stratum thought sampling, but as a general rule the sampling rate (i.e. the proportion of units and expenditure of this stratum that is selected to the sampling) has to be larger or equal than the one used for the low-value stratum.

- The sampling size to be allocated to the non-exhaustive stratum is computed as the difference between the total sample size and the number of sampling units (for example operations) in the high-value stratum. In case the AA would like to apply stratification also to the low-value units, allocate this calculated sample size between individual strata in accordance with the methods suggested in section 6.1.2.2. (if the selection is based on equal probabilities) or 6.3.2.2 (if the selection is based on probabilities proportional to size).

If it is not possible to identify any stratification criteria (that in the opinion of the auditor may contribute to create more homogeneous subpopulations in terms of the expected errors or error rates) and in particular if one cannot observe any significant variability in the expenditure of the population items, then the option may be to use a non-stratified non-statistical sampling design. In this case, the sample is selected directly from the whole population without considering any subpopulations.

6.4.3 Sample size

In non-statistical sampling, the sample size is calculated using professional judgment and taking account the level of assurance provided by the system audits. The final goal is to obtain a sample size that is sufficient to enable the AA to achieve valid conclusions about the population and draw up a valid audit opinion (cf. Article 127(1) CPR).

Concerning the programming period 2014-2020 and as established by Article 127(1) CPR, a non-statistical sample should cover a minimum of 5% of operations[39] and 10 % of the expenditure. Since the regulation refers to a minimum coverage, these thresholds correspond therefore to the 'best case scenario' of high assurance from the system. In line with annex 3 of the ISA 530, the higher the auditor's assessment of the risk of material misstatement, the larger the sample size needs to be. The requirement of 10% of expenditure declared (Article 127(1) CPR) refers to the expenditure in the sample, independently on the use of sub-sampling. This means that the sample shall correspond to a minimum of 10% of the expenditure declared, but when sub-sampling is used, the expenditure effectively audited could in fact be less provided the AA can draw a valid audit opinion (cf section 6.4.10).

There is no fixed rule to select the sample size based on the assurance level from the system audits, but as a reference, the AA, when defining the sample size under non-statistical sampling, may consider the following indicative thresholds[40].

Assurance level from the system audits	**Recommended coverage**	
	on operations	**on expenditure declared**

[39] For the programming period 2007-2013, the Commission maintains that the sample size under non-statistical sampling should cover a minimum of 10% of operations (cf. section 7.4.1 of the guidance on sampling COCOF_08-0021-03_EN of 04/04/2013).

[40] These reference values may of course be changed according to the AA's professional judgment and any additional information it may have about the risk of material misstatement.

Assurance level from the	Recommended coverage	
Works well. No, or only minor improvement(s) needed.	5%	10%
Work. Some improvement(s) needed.	Between 5% - 10% (to be defined by the AA on the basis of its professional judgement)	10%
Works partially. Substantial improvement(s) needed.	Between 10% and 15% (to be defined by the AA on the basis of its professional judgement)	Between 10% and 20% (to be defined by the AA on the basis of its professional judgement)
Essentially does not work.	Between 15% and 20% (to be defined by the AA on the basis of its professional judgement)	Between 10% and 20% (to be defined by the AA on the basis of its professional judgement)

Table 6. Recommended coverage for non-statistical sampling

6.4.4 Sample selection

The sample from the positive population shall be selected using a random method. In particular, the selection can be made either using:

- equal probability selection (where each sampling unit has equal chance of being selected regardless of the amount of expenditure declared in the sampling unit), as in simple random sampling (cf. Sections 6.1.1 and 6.1.2 for the reference to simple random sampling and stratified simple random sampling); or
- probability proportional to size (expenditure) (where a random selection is made of the first element for the sample and then subsequent elements are selected using an interval until the desired sample size is reached; it uses the monetary unit as an auxiliary variable for sampling) as done for the MUS case (cf. Sections 6.3.1 and 6.3.2 for the reference to monetary unit sampling and stratified monetary unit sampling).

6.4.5 Projection

Please note that the use of non-statistical sampling does not avoid the need to project the errors observed in the sample to the population. The projection has to take into account the sampling design, i.e. the existence of stratification or not, the type of selection (equal probability or probability proportional to size), and any other relevant characteristics of the design. The use of simple sample statistics (as the sample error rate) is only possible in very specific circumstances where the sampling is compatible with such statistics. For example, the sample error rate can only be used to project the errors to the population under a design without any level of stratification, based on equal probability selection and ratio estimation. Therefore, the only significant difference between statistical and non-statistical sampling is that for the last the level of precision and consequently the upper error limit are not calculated.

6.4.5.1 Equal probability selection

If units were selected with equal probabilities, the projected error should follow one of the projection methods presented in section 6.1.1.3, i.e. mean-per-unit estimation or ratio estimation.

Mean-per-unit estimation (absolute errors)
Multiply the average error per operation observed in the sample by the number of operations in the population, yielding the projected error:

$$EE_1 = N \times \frac{\sum_{i=1}^{n} E_i}{n}.$$

Ratio estimation (error rates)
Multiply the average error rate observed in the sample by the book value at the level of the population:

$$EE_2 = BV \times \frac{\sum_{i=1}^{n} E_i}{\sum_{i=1}^{n} BV_i}$$

The sample error rate in the above formula is just the division of the total amount of error in the sample by the total amount of expenditure of units in the sample (expenditure audited).

It is suggested that the choice between the two projection methods is based on the recommendation included in Section 6.1.1.3 in relation to simple random sampling.

6.4.5.2 Stratified equal probability selection

Based on *H* randomly selected samples of operations (*H* strata) the projected error at the level of the population can be again computed through the two usual methods: mean-per-unit estimation and ratio estimation. The projection follows the procedure described in Section 6.1.2.3 for the stratified simple random sampling.

Mean-per-unit estimation

In each group of the population (stratum) multiply the average error per operation observed in the sample by the number of operations in the stratum (N_h); then sum all the results obtained for each stratum, yielding the projected error:

$$EE_1 = \sum_{h=1}^{H} N_h \times \frac{\sum_{i=1}^{n_h} E_i}{n_h}.$$

Ratio estimation

In each group of the population (stratum) multiply the average error rate observed in the sample by the population book value at the level of the stratum (BV_h):

$$EE_2 = \sum_{h=1}^{H} BV_h \times \frac{\sum_{i=1}^{n_h} E_i}{\sum_{i=1}^{n_h} BV_i}$$

It is suggested that the choice between the two methods should be based upon the considerations presented for the non-stratified method.

If a 100% stratum has been considered and previously taken from the population then the total amount of error observed in that exhaustive stratum should be added to the above estimate (EE_1 or EE_2) in order to produce the final projection of the amount of error in the whole population.

6.4.5.3 Probability proportional to expenditure selection

If units were selected with probabilities proportional to the value of expenditure, the projected error should follow the projection method presented in Section 6.3.1.4 (monetary unit sampling).

For the exhaustive stratum, that is, for the stratum containing the sampling units with book value larger than the cut-off, $BV_i > \frac{BV}{n}$, the projected error is just the summation of the errors found in the items belonging to the stratum:

$$EE_e = \sum_{i=1}^{n_e} E_i$$

For the non-exhaustive stratum, i.e. the stratum containing the sampling units with book value smaller or equal to the cut-off value, $BV_i \leq \frac{BV}{n}$ the projected error is

$$EE_s = \frac{BV_s}{n_s} \sum_{i=1}^{n_s} \frac{E_i}{BV_i}$$

The projected error at the level of population is just the sum of these two components:

$$EE = EE_e + EE_s$$

6.4.5.4 Stratified probability proportional to expenditure selection

If units were selected with probabilities proportional to the value of expenditure and the population is stratified based on any specific criteria, the projected error should follow the projection method presented in Section 6.3.2.4 (stratified monetary unit sampling).

The projection of errors to the population is made differently for units belonging to the exhaustive groups and for items in the non-exhaustive groups.

For the exhaustive groups, that is, for the groups containing the sampling units with book value larger than the cut-off value, $BV_{hi} > \frac{BV_h}{n_h}$, the projected error is the summation of the errors found in the items belonging to those groups:

$$EE_e = \sum_{h=1}^{H} \sum_{i=1}^{n_h} E_{hi}$$

For the non-exhaustive groups, i.e. the groups containing the sampling units with book value lower or equal to the cut-off value, $BV_{hi} \leq \frac{BV_h}{n_h}$, the projected error is

$$EE_s = \sum_{h=1}^{H} \frac{BV_{sh}}{n_{sh}} \sum_{i=1}^{n_{sh}} \frac{E_{hi}}{BV_{hi}}$$

The projected error at the level of population is just the sum of these two components:

$$EE = EE_e + EE_s$$

6.4.6 Evaluation

In any of the previously mentioned strategies the projected error is finally compared to the maximum tolerable error (materiality times the population expenditure):

- If below the tolerable error, then we conclude that the population does not contain material error;
- If above the tolerable error, then we conclude that the population contains material error.

Despite the constraints (i.e. it is not possible to calculate the upper limit of error and consequently there is no control of the audit risk), the projected error rate is the best estimation of the error in the population and can thus be compared with the materiality threshold in order to conclude that the population is (or not) materially misstated.

6.4.7 Example 1 – PPS sampling

Let's assume a positive population of 36 operations for which expenditure 22,031,228 € has been declared.

This population tends to have an insufficient size to be audited through statistical sampling. Further, sampling of payment claims to enlarge population size is not possible. Therefore the AA decides to use a non-statistical approach. Due to the large variability in the expenditure for this population, the AA decides to select the sample using probability proportional to size.

The AA considers that the management and control system "*essentially does not work*", so it decides to select a sample size of 20% of the population of operations. In our case it is 20% x 36=7.2 rounded by excess to 8.

Although the coverage of the population expenditure can only be accessed after the sample selection, the fact that 20% of the population units are selected along with the choice of probability proportional to size selection is expected to results in at least 20% of expenditure coverage.

First, it is necessary to identify the high value population units (if any) that will belong to a high-value stratum to be submitted to a 100% audit work. The cut-off value for determining this top stratum is equal to the ratio between the book value (BV) and the planed sample size (n). All items whose book value is higher than this cut-off (if $BV_i > BV/n$) will be placed in the 100% audit stratum. In this case the cut-off value is 22,031,228/8=2,753,904 €.[41]

The following table summarizes these results:

Declared expenditure (DE) in the reference period	22,031,228 €
Size of population (number of operations)	36
Materiality level (maximum 2%)	2%

[41] Please note that the AA could also decide to apply a lower cut-off value than calculated on the basis of the ratio between the positive population and number of operations to be selected in order to increase the coverage of expenditure declared.

Tolerable misstatement (TE)	440,625 €
Cut off value	2,753,904 €
Number of units above the cut-off value	4
Population book value above the cut-off	12,411,965 €
Remaining population size (number of operations)	32
Remaining population value	9,619,263.00 €

The AA put in an isolated stratum all the operations with book value larger than 2,753,904 € which correspond to 4 operations, amounting to 12,411,965 €. The amount of error found in these four operations amounts to

$$EE_e = 80{,}028.$$

The sampling interval for the remaining population is equal to the book value in the non-exhaustive stratum (BV_s) (the difference between the total book value and the book value of the four operations belonging to the top stratum) divided by the number of operations to be selected (8 minus the 4 operations in the top stratum).

$$Sampling\ interval = \frac{BV_s}{n_s} = \frac{22{,}031{,}228 - 12{,}411{,}965}{4} = 2{,}404{,}816$$[42]

A file containing the remaining 32 operations of the population is randomly sorted and a sequential cumulative book value variable is created. The sample is selected, selecting each item containing the 2,404,816th monetary unit.[43]

The audited expenditure amounts to total book value of the high value projects, 12,411,965 €, plus the audited expenditure in the remaining population sample, 1,056,428 €. Total audited expenditure amounts to 13,468,393 € which represents 61.1% of the total declared expenditure as requested. Bearing in mind the level of assurance of the management and control system, the AA thinks this level of audited expenditure is more than enough to ensure the reliability of the auditing conclusions.

[42] In practice it may happen that after the calculation of the sampling interval based on the expenditure and sample size of the sampling stratum, some population units will still exhibit an expenditure larger than this sampling interval BV_s/n_s (although they have not previously exhibit an expenditure larger than the cut-off (BV/n). In fact, all items whose book value is still higher than this interval ($BV_i > BV_s/n_s$) have also to be added to the high-value stratum. If this happens, and after moving the new items to the high value stratum, the sampling interval has to be recalculated for the sampling stratum taking into consideration the new values for the ratio BV_s/n_s. This iterative process may have to be performed several times until a moment where no further units present expenditure larger than the sampling interval.

[43] In case any of the selected operation had to be replaced due to limitations imposed by Article 148 provisions, the new operation/operations should be selected using probability proportional to size selection. See section 7.10.3.1 for example of such a replacement.

The value of the extrapolated error for the low-value stratum is

$$EE_s = \frac{BV_s}{n_s}\sum_{i=1}^{n_s}\frac{E_{si}}{BV_{si}}$$

where BV_s is the total book value of the remaining population and n_s the corresponding sample size of the remaining population. Notice that this projected error is equal to the sum of the error rates multiplied by the sampling interval. The sum of the error rates is equal to 0.0272:

$$EE_s = \frac{9{,}619{,}623}{4} \times 0.0272 = 65{,}411.$$

The total extrapolated error at the level of population is just the sum of these two components:

$$EE = EE_e + EE_s = 80{,}028 + 65{,}411 = 145{,}439$$

The projected error is finally compared to the maximum tolerable error (2% of 22,031,228 €=440,625 €). The projected error is smaller than the materiality level.

With these results the auditor can reasonably conclude that the population does not contain a material error. Nevertheless, the achieved precision cannot be determined and the confidence of the conclusion is unknown.

Proceeding in the case of insufficient coverage of expenditure

Please note if due to specific characteristics of the population the threshold of the required expenditure coverage was not achieved, the audit authority should select an additional operation/operations using probability proportional to size. In such a situation the new operations/sampling units to be additionally audited should be selected from the population excluding the already selected operations. The interval used for such selection should be calculated using the sampling interval $\frac{BV_{s'}}{n_{s'}}$, where *BVs'* corresponds to the book value of low –value stratum excluding operations already selected in this stratum and $n_{s'}$ corresponds to the number of operations that we want to add for audit of low-value stratum.

6.4.8 Example 2 – Equal probabilities sampling

Let's assume a positive population of 48 operations for which expenditure of 10,420,247 € has been declared.

This population tends to have an insufficient size to be audited through statistical sampling. Further, sampling of payment claims to enlarge population size is not possible. Therefore the AA decides to use a non-statistical approach with stratification of the high-value operations since there are a few operations with extremely large expenditure. The AA decided to identify these operations by setting the cut off level as 5% of 10,420,247 €, that is 521,012 €.

The characteristics of the population are summarized below:

Declared expenditure in the reference period	10,420,247 €
Size of the population (number of operations)	48
Materiality level (maximum 2%)	2%
Tolerable misstatement (TE)	208,405 €
Cut off value (5% of total book value)	521,012 €

The following table summarizes the results:

Number of units above the cut-off value	12
Population book value above cut-off	8,785,634 €
Remaining population size (number of operations)	36
Remaining population value	1,634,613 €

The management and control system was classified in Category 3 "Works partially, substantial improvements needed ", so it decides to select a sample size of 15% of the population of operations. That is, 15% x 48=7.2 rounded by excess to 8. The AA decides that a larger proportion of operations shall be drawn in the high-value stratum. The AA decides to audit 50% of the operations in the high-value stratum, that is 6 operations. The remaining operations (8-6=2) are selected from the remaining population. Nevertheless, the AA decides to increase this sample from 2 to 3 operations in order to achieve a better representation of this stratum.

Due to the small variability in the expenditure for this population in each stratum, the auditor decides to sample the population using equal probabilities in both strata.

Although based on equal probabilities, it is expected that this sample will result in the coverage of at least 20% of the population expenditure due to the high coverage of the high-value stratum. Indeed, by multiplying the sample size by the average book value by operation in each stratum, the AA expects to audit 4,392,817 € in high-value stratum and 136,218 € in the remaining population, which represents around 43.5% of the total expenditure.

A sample of 6 operations is randomly drawn in the high-value stratum. The sample audited expenditure amounts 4,937,894 €. No errors were found in these 6 operations.

A sample of 3 operations of the remaining population of operations is also drawn. The sample audited expenditure in the remaining population amounts to 153,647 €. The identified total sample error in this stratum amounts to 4,374 €.

The total audited expenditure is 153,647 € + 4,937,894 € = 5,091,541 € which represents 48.9% of the total declared expenditure. Bearing in mind the level of assurance of the management and control system, the AA considers this level of audited expenditure is adequate to ensure the reliability of the auditing conclusions.

To decide between the use of mean-per-unit estimation or ratio estimation, the AA has checked the sample data to verify the condition $\frac{\mathrm{COV}_{\mathrm{E,BV}}}{\mathrm{VAR}_{\mathrm{BV}}} > ER/2$, which was confirmed. The decision was then to use ratio estimation.

The value of extrapolated error for both strata is

$$EE = BV_e \times \frac{\sum_{i=1}^{6} E_i}{\sum_{i=1}^{6} BV_i} + BV_s \times \frac{\sum_{i=1}^{3} E_i}{\sum_{i=1}^{3} BV_i} = 0 + 1{,}634{,}613 \times \frac{4{,}374}{153{,}647} = 46{,}534.$$

Where BV_e and BV_s are the total book values of the high and low value strata. Notice that the projected error is equal to the sample error rate multiplied by the stratum book value.

The projected error is finally compared to the maximum tolerable error (2% of 10,420,247€=208,405 €). The projected error is smaller than the materiality level.

The conclusion that can be derived from the exercise is that the auditor can reasonably conclude that the population does not contain a material error. Nevertheless, the achieved precision cannot be determined and the confidence of the conclusion is unknown.

6.4.9 Non-statistical sampling – two periods

Similarly as applied in statistical sampling methods, the audit authority could decide to carry out the sampling process in several periods during the year (typically two semesters) using non-statistical sampling approach. The major advantage of this approach is not related to the sample size reduction, but mainly to allowing spreading the audit workload over the year, thus reducing the workload that would be done at the end of the year based on just one observation.

With this approach the population of the reference period/accounting year is divided into two sub-populations, each one corresponding to the operations/payment claims and expenditure of each semester. Independent samples are drawn for each semester, using either equal probability selection or probability proportional to size (expenditure) selection, referred further as PPS selection.

Two examples below (one on equal probability selection and another on PPS selection) illustrate two-period sampling used with non-statistical sampling methods. It should be noted that the sampling designs and projection methodologies used for two-period sampling in non-statistical sampling are the same as the ones used in statistical sampling, i.e. simple random sampling in the case of equal probability selection and MUS (standard approach) in the case of PPS selection. The only differences are:
- the sample size in not calculated with a specific formula,
- precision is not calculated.

However, the attention is drawn to the specific requirement for non-statistical sampling imposed by the legal provisions for the programming period 2014-2020 concerning expenditure coverage of at least 10 % of the expenditure declared to the Commission during an accounting year[44] and 5% of operations. In the case of using a single period sampling, equal probability selection often results in the expenditure coverage rate close to the sample fraction used to define the number of operations. In the case of two-periods or multi-periods sampling, the coverage rate is usually smaller in view of the fact that some operations (i.e. operations declared in more than one audit period) are checked only on part of the expenditure declared during the year.

Therefore, application of two or multi-period sampling could require covering more operations than in the case of single period sampling in order to comply with the required threshold of expenditure coverage.

It should be noted that since the audit of operations will cover expenditure declared in part of the reference period, the average audit workload per operation in two and multi-period sampling should be less time-consuming. However, in spite of that the overall workload per accounting year could increase in order to reach the desired coverage of expenditure.

In order to address this problem, the AA could decide to apply a high-value stratum which could limit the number of operations to be checked per accounting year to the required minimum (as the operations with larger expenditure will be more represented in the sample).

6.4.9.1 Non-statistical sampling – two periods – equal probability selection

[44] See also section 6.4.3 above.

In order to reduce the audit workload after the end of the reference period, the AA decided to spread the audit work over two periods. At the end of the first semester the AA considered the population divided into two groups corresponding to each of the two semesters. The population at the end of the first semester can be summarized as follows:

Declared expenditure at the end of first semester	19,930,259 €
Size of population (operations - first semester)	41

Based on experience, the AA knows that usually the operations included in the programme at the end of the reference period are not all active in the population of the first semester. Moreover, it is expected that the declared expenditure in the second semester will be twice larger as the declared expenditure in the first semester. This expenditure increases between the two semesters is accompanied by a lower increase in the number of operations. The AA expects that in the second semester there will be 62 active operations (1 operation will be completed in the first semester, the remaining 40 operations of the first semester will continue in the second semester and it is expected to have expenditure declared for 22 new operations in the second semester). Sample selection by payment claim would not increase the population size as in our hypothetical example based on the national programme rules there is one payment claim per semester. The AA decides to use a non-statistical approach by selecting the sample using equal probabilities.

Based on these assumptions a summary of the population is described in the following table:

Declared expenditure at the end of first semester	19,930,259 €
Expenditure to be declared in the second semester (forecast) (19,930,259 €*2 = 39,860,518 €)	39,860,518 €
Total expenditure forecasted for the reference period	59,790,777 €
Size of population (operations – first semester)	41
Size of population (operations – second semester, predicted)	62(40+22)
Materiality level (maximum 2%)	2%
Tolerable error (TE)	1,195,816 €

The AA considers that the management and control system "*works partially, substantial improvements are needed*", so it decides to select a sample size of 15% of the number of operations (see section 6.4.3). In our case in the reference period we have together 63 operations[45] within which expenditure was declared in both sampling periods (41 operations which began in the first semester and 22 new operations in the second semester). Thus, the global sample size for the whole year is:

[45] 62 active operations plus one operation completed in the first semester.

$$n = 0.15 \times 63 \approx 10$$

The allocation of the sample by semester is as follows:

$$n_1 = \frac{N_1}{N_1 + N_2} = \frac{41}{41 + 62} \times 10 \approx 4$$

and

$$n_2 = n - n_1 = 6$$

The AA has decided to apply a high-value stratum which could limit the number of operations to be checked per accounting year to the required minimum (as the operations with larger expenditure will be more represented in the sample).

In the case of the population of the first semester, in our example there is one large operation with the total value of 3,388,144 EUR, the remaining 40 operations being much smaller. Based on professional judgement, the audit authority has decided to apply a high-value stratum with 1 operation (i.e. the largest operation in the population of the first semester). Using this stratification the AA expected to cover at least 20% of the total expenditure in the first semester by auditing 4 operations.

The remaining 3 operations of the sample were selected at random from the first semester population excluding the operation of the high-value stratum (i.e from the population of 16,542,115 EUR). The total value of the 3 operations amounted to 1,150,398 EUR.

Thus, the sample of 4 operations in the first semester covered 22,77% of expenditure declared in the first semester.

The audit authority has detected an error of 127 EUR[46] in the operation of the high-value stratum and a total error 4,801 EUR in the 3 operations selected at random.

At the end of the second semester more information is available, in particular, the total expenditure and the number of operations active in the second semester is correctly known.

The AA realises that the assumption made at the end of the first semester on the total expenditure, 39,860,518 €, slightly underestimates the true value of 40,378,264 €. The

[46] This error could be established on the basis of verification of all invoices (expenditure items) in this operation of the high value stratum declared in the first semester. Alternatively, a sub-sample of at least 30 invoices (expenditure items) could be selected. In the case of a sub-sample of expenditure items, this error would refer to an error extrapolated on the basis of the selected expenditure items to the level of an operation. It should be ensured that the sub-sample of invoices is selected at random, or alternatively stratification at the level of operation could be applied with exhaustive verification of some strata and random selection of expenditure items in the remaining strata.

number of operations active in the second semester is slightly smaller than was initially expected. As a result, the AA does not need to revise the sample size for the second semester as the initial forecasted number of operation in the second semester is close to the real ones. The following table summarises the figures:

Parameter	**Forecast done in the first semester**	**End of second semester**
Number of operations in second semester	62	61
Total expenditure in the second semester	39,860,518 €	40,378,264 €

Taking into consideration the characteristics of the population the AA decides to use again a stratification by expenditure, defining a high-value stratum based on a threshold of 5% of the expenditure of the second semester population. 3 operations exceeds this threshold with the total value of 6,756,739 EUR. The remaining 3 operations (6 operations to be covered in the second semester minus 3 operations of the high value stratum) are selected at random from the population of 58 operations of the low-value stratum of the second semester, i.e. the population of 33,621,525 EUR. The total value of the random sample for the second semester is 1,200,987 EUR. The AA established that the total value of the sample of the second semester (7,957,726 EUR=1,200,987+6,756,739) is slightly below the threshold of 20% for the second semester. However, as the total value of the sample for both semesters exceeds the required minimum of 20%, it was concluded that no additional sample is needed to ensure expenditure coverage.

The AA detected an error 432,076 EUR in the 3 operations of the high value stratum and 5,287 EUR in the low-value stratum.

Taking into consideration the correlation between errors of low strata and expenditure the AA decides to project the error using ratio estimation.

The value of the extrapolated error for both semesters using ratio estimation[47] is

$$EE \quad = EE_{e1} + EE_{e2} + BV_{s1} \times \frac{\sum_{i=1}^{n_{s1}} E_{s1i}}{\sum_{i=1}^{n_{s1}} BV_{s1i}} + BV_{s2} \times \frac{\sum_{i=1}^{n_{s2}} E_{s2i}}{\sum_{i=1}^{n_{s2}} BV_{s2i}}$$

where:

- EE_{e1} and EE_{e2} refer to the errors detected in the high value strata of the first and the second semesters

[47] Using mean-per-unit the formula would be:

$$EE \quad = EE_{e1} + EE_{e2} + \frac{N_{s1}}{n_{s1}} \sum_{i=1}^{n_{s1}} E_{s1i} + \frac{N_{s2}}{n_{s2}} \sum_{i=1}^{n_{s2}} E_{s2i}$$

- BV_{s1} and BV_{s2} refer to the book values of non-exhaustive strata of the first and the second semesters
- $\frac{\sum_{i=1}^{n_{s1}} E_{s1i}}{\sum_{i=1}^{n_1} BV_{s1i}}$ and $\frac{\sum_{i=1}^{n_{s2}} E_{s2i}}{\sum_{i=1}^{n_2} BV_{s2i}}$ reflect respectively an average error rate observed in the non-exhaustive strata of the first semester and the second semester

Notice that the projected error is equal to the sum of the errors detected in the high-value strata of both semester and the error rates of the random samples multiplied by the respective stratum book values of these random samples.

In particular, in our example, the extrapolated error at the level of the population is:

$$EE = 127 + 432{,}076 + 16{,}542{,}115 \times \frac{4{,}801}{1{,}150{,}398} + 33{,}621{,}524 \times \frac{5{,}287}{1{,}200{,}987} = 649{,}247.94$$

(i.e. 1.08% of the population value)

The projected error is finally compared to the maximum tolerable error (2% of 60,308,523 €, i.e. 1,206,170 €). The projected error is smaller than the materiality level. Nevertheless, the achieved precision cannot be determined and the confidence of the conclusion is unknown.

6.4.9.2 Non-statistical sampling – two periods – PPS selection

In order to reduce the audit workload after the end of the reference period, the AA decided to spread the audit work in two periods. At the end of the first semester the AA considered the population divided into two groups corresponding to each of the two semesters. The population at the end of the first semester can be summarized as follows:

Declared expenditure at the end of first semester	16,930,259 €
Size of population (operations - first semester)	34

Based on the past experience, the AA knows that usually the operations included in the programme at the end of the reference period are not all active in the population of the first semester. Moreover, it is expected that the expenditure declared during the second semester will be two and a half times larger than the declared expenditure at the end of the first semester. It is also predicted to have a growth in the number of operations active at the end of the second semester, although smaller than the one predicted for the expenditure. The AA expects that in the second semester there will be 52 active operations (2 operations will be completed in the first semester, the remaining 32 operations of the first semester will continue in the second semester and it is expected to have expenditure declared for 20 new operations in the second semester). Sampling of payment claims to enlarge population size is not possible. Therefore the AA decides to use a non-statistical approach.

Based on these assumptions a summary of the population is described in the following table:

Declared expenditure at the end of first semester	16,930,259 €
Expenditure to be declared in the second semester (forecast) (16,930,259 €*2.5 = 42,325,648 €)	42,325,648 €
Total expenditure forecasted for the year	59,255,907 €
Size of population (operations – first semester)	34
Size of population (operations – second semester, predicted)	52(32+20)
Materiality level (maximum 2%)	2%
Tolerable error (TE)	1,185,118 €

The AA considers that the management and control system "*works partially, substantial improvements are needed*", so it decides to select a sample size of 15% of the number of operations. Moreover, aiming at maximisation of expenditure coverage by random sample, the auditor decides to select the sample using probability proportional to size. In our case in the reference period we have together 54 operations for which expenditure was declared in both sampling periods (34 operations which were included in the first semester and 20 new operations in the second semester). The global sample size for the whole year is:

$$n = 0.15 \times 54 \approx 9$$

The allocation of the sample by semester is as follows:

$$n_1 = \frac{BV_1}{BV_1 + BV_2} = \frac{16{,}930{,}259}{16{,}930{,}259\ + 42{,}325{,}648} \times 9 \approx 3$$

and

$$n_2 = n - n_1 = 6$$

Although the coverage of the population expenditure can only be assessed after the sample selection, the fact that 15% of the operations are selected along with the choice of probability proportional to size selection is expected to results in the case of our population in at least 20% of the expenditure coverage.

Firstly, it is necessary to identify the high value population units (if any) that will belong to a high-value stratum to be submitted to an exhaustive audit work. The cut-off value for determining this top stratum is equal to the ratio between the book value (BV_1) and the planed sample size (n_1). All items whose book value is higher than this cut-off will be placed in the exhaustive audit stratum. In this case the cut-off value is 16,930,259 €/3=5,643,420 €.

There are no operations with book value larger than 5,643,420 , and consequently the sampling interval corresponds to the cut-off value, i.e. 5,643,420 €.

The following table summarises these results:

Cut-off value – first semester	5,643,420 €
Number of operations with book value larger than cut-off value - first semester	0
Book value of operations with book value larger than cut-off value - first semester	0
BV_{s1}- book value of the population of non-exhaustive stratum in the first semester (as we do not have operations above cut-off in first semester, it is all the first semester population)	16,930,259 €
n_{s1}- sample size of non-exhaustive stratum of the first semester	3
SI_{s1}- sampling interval in the first semester	5,643,420 €

A file containing the 34 operations of the population is randomly sorted and a sequential cumulative book value variable is created. The sample is selected, selecting each item containing the 5,643,420th monetary unit. [48] The value of these three operations is audited. The sum of the error rates for the first semester is

$$\sum_{i=1}^{3} \frac{E_{1i}}{BV_{1i}} = 0.066$$

The audited expenditure of the sample amounts to 6,145,892 € which represents 36.3% of the total declared expenditure. Bearing in mind the level of assurance of the management and control system, the AA thinks this level of audited expenditure is more than enough to ensure the reliability of the auditing conclusions.

At the end of the second semester more information is available, in particular, the total expenditure and the number of operations active in the second semester is correctly known.

The AA realises that the assumption made at the end of the first semester on the total expenditure, 42,325,648 €, underestimates the true value of 49,378,264 €. The number of operations active in the second semester is smaller than was initially expected. As a result of the decrease of the number of operations, the sample for the second semester could be reduced. The following table summarises the population of the second semester:

Parameter	**Forecast done**	**End of second**

[48] In case any of the selected operation had to be replaced due to limitations imposed by Article 148 provisions, the new operation/operations should be selected using probability proportional to size selection. See section 7.10.3.1 for example of such a replacement.

	in the first semester	semester
Number of operations in the second semester	52	46
Total expenditure in the second semester	42,325,648 €	49,378,264 €

Thus, the total number of operations declared for both semesters was 48 operations[49] (34 operations included in the first semester and 14 operations which began in the second semester).
Taking into consideration this adjustment, the sample size of the second semester recalculated due to the change in the number of operations is

$$n_2 = 0.15 \times 48 - 3 \approx 5$$

It is necessary to identify the high value population units (if any) that will belong to a high-value stratum to be submitted to a 100% audit work. The cut-off value for determining this top stratum is 9,875,653 € (49,378,264/5).[50] All items whose book value is higher than this cut-off are audited. There are two operations of which book value is larger than this cut-off value. The total book value of these operations amounts to 21,895,357 €. A total error of 56,823 € was found in these two operations.

The sampling size to be allocated to the non-exhaustive stratum, n_{s2} , is computed as the difference between n_2 and the number of sampling units (e.g. operations) in the exhaustive stratum (n_{e2}). In our case it is 3 operations (5, the sample size, minus the 2 high-value operations). Therefore, the auditor has to select the random sample using the sampling interval:

$$SI_{s2} = \frac{BV_{s2}}{n_{s2}} = \frac{49{,}378{,}264 - 21{,}895{,}357}{3} = 9{,}160{,}969^{51}$$

The following table summarises these results:

Cut-off value - second semester	9,875,653 €
Number of operations with book value larger than cut-off value - second semester	2

[49] 46 operations plus 2 operations completed in the 2nd semester.
[50] Please note that the AA could also decide to apply a lower cut-off value than calculated on the basis of the ratio between the semester population and number of operations to be selected in the semester. Application of a lower cut-off value to increase number of operations in the top stratum could be in particular useful for the audit authority if, based on analysis of the specific characteristics of the population it appears that the threshold of expenditure coverage could be difficult to attain even if PPS is applied.
[51] Note that in practice it may happen that after the calculation of the sampling interval based on the expenditure and sample size of the sampling stratum, some population units will still exhibit an expenditure larger than this sampling interval BV_s/n_s (although they have not previously exhibit an expenditure larger than the cut-off (BV/n). In fact, all items whose book value is still higher than this interval ($BV_i > BV_s/n_s$) have also to be added to the high-value stratum. If this happens, and after moving the new items to the high value stratum, the sampling interval has to be recalculated for the sampling stratum taking into consideration the new values for the ratio BV_s/n_s. This iterative process may have to be performed several times until a moment where no further units present expenditure larger than the sampling interval.

Book value of operations with book value larger than cut-off value-second semester	21,895,357 €
BV_{s2}- population of operations with book value below cut-off (non-exaustisve stratum)- second semester	27,482,907 €
n_{s2}- sample size of non-exhaustive stratum of the second semester	3
SI_{s2}- sampling interval in the second semester	9,160,969 €

A file containing the remaining 43 operations of the second semester population is randomly sorted and a sequential cumulative book value variable is created. A sample of 3 operations is drawn using the systematic proportional to size procedure.

The value of these 3 operations is audited. The sum of the error rates for the second semester is:

$$\sum_{i=1}^{3} \frac{E_{2i}}{BV_{2i}} = 0.0475$$

The audited expenditure in the second semester's sample amounts to the total book value of the high value projects, 21,895,357 €, plus the audited expenditure in the remaining population sample, 2,245,892 €. Total audited expenditure in the second semester amounts to 24,141,249 € which represents 48.89% of the total declared expenditure. Bearing in mind the level of assurance of the management and control system, the AA thinks this level of audited expenditure is more than enough to ensure the reliability of the auditing conclusions.[52]

The projection of errors to the population is made differently for (operations) sampling units belonging to the exhaustive strata and for units in the non-exhaustive strata.

For the exhaustive strata, that is, for the strata containing the sampling units with book value larger than the cut-off, $BV_{ti} > \frac{BV_t}{n_t}$, the projected error is the sum of the errors found in the items belonging to those strata:

$$EE_e = \sum_{i=1}^{n_1} E_{1i} + \sum_{i=1}^{n_2} E_{2i} = 0 + 56{,}823 = 56{,}823$$

In practice:

1) For each semester *t*, identify the units belonging to the exhaustive group and sum their errors

2) Sum the previous results over the two semesters.

For the non-exhaustive group, i.e. the strata containing the sampling units with book value smaller or equal to the cut-off value, $BV_{ti} \leq \frac{BV_t}{n_t}$, the projected error is

[52] See example of section 6.4.7 on proceeding in case of insufficient coverage of coverage.

$$EE_s = \frac{BV_{s1}}{n_{s1}} \times \sum_{i=1}^{n_{s1}} \frac{E_{1i}}{BV_{1i}} + \frac{BV_{s2}}{n_{s2}} \times \sum_{i=1}^{n_{s2}} \frac{E_{2i}}{BV_{2i}}$$
$$= 5{,}643{,}420 \times 0.066 + 9{,}160{,}969 \times 0.0475 = 807{,}612$$

To calculate this projected error:
1) in each semester t, for each unit in the sample calculate the error rate, i.e. the ratio between the error and the respective expenditure $\frac{E_{ti}}{BV_{ti}}$
2) in each semester t, sum these error rates over all units in the sample
3) in semester t, multiply the previous result by the sampling interval applied for random selection of operations in the non-exhaustive stratum
4) sum the previous results over the two semesters

The projected error at the level of population is just the sum of these two components:

$$EE = EE_e + EE_s = 56{,}823 + 807{,}612 = 864{,}435$$

(i.e. 1.30% of the population value)

The projected error is finally compared to the maximum tolerable error (2% of 66,308,523 €=1,326,170 €). The projected error is smaller than the materiality level. Nevertheless, the achieved precision cannot be determined and the confidence of the conclusion is unknown.

6.4.10 Two-stage sampling (sub-sampling) in non-statistical sampling methods

Generally, all expenditure declared to the Commission in the sample shall be subject to audit. However, where the selected sampling units include a large number of underlying payment claims or invoices/other expenditure items, the audit authority may audit them through sub-sampling. More detailed information in this regard can be found in section 7.6 *Two-stage sampling* and in section 6.5.3.1 focused on two-stage and three-stage sampling within ETC programmes.

Please note that the items sub-sampled should be selected at random. It is also possible to apply a stratification design at the level of sub-sampling with invoices/expenditure items of some strata verified exhaustively and some strata checked by verification of a random selection of expenditure items. Stratification could be typically carried out based on the type of expenditure or the amount of invoice/expenditure item (for example by verification of all high-value items exhaustively and a stratum of low-value items by randomly selected items).

For the programming period 2014-2020 and in line with Article 28 CDR, where sub-sampling is used with either invoices or payment claims as the sub-sampling units, the AA should cover not less than 30 invoices/other expenditure items or payment claims. Where other sub-sampling units are used under non-statistical sampling (such as for example a project within an operation, a project partner in ETC programmes), the AA may decide, based on professional judgment, the sufficient coverage of a sub-sample. In this case, it is recommended that if less than 30 sub-sampling units are selected, they should cover at least 10% of the expenditure of the sampling unit (for example of an operation).

6.5 Sampling methods for European Territorial Cooperation (ETC) programmes

6.5.1 *Introduction*

ETC programmes have a number of particularities: it is normally not possible to group them because each system and sub-system is different; the number of operations is frequently low. For each operation, there is generally a lead partner (lead beneficiary under Article 13 of Regulation (EU) No 1299/2013) and a number of other project partners (other beneficiaries under Article 13 of Regulation (EU) No 1299/2013). Operations selected under cross-border and transnational cooperation shall involve partners from at least two participating countries, whereas operations under interregional cooperation shall involve partners from at least three countries (Article 12 of Regulation (EU) No 1299/2013).

6.5.2 *Sampling unit*

The sampling unit shall be determined by the audit authority based on professional judgement. It may be an operation, a project within an operation or a payment claim by a beneficiary (Article 28(6) of delegated Regulation No 480/2014). If the AA decides to use a payment claim as a sampling unit, it could opt either for an aggregated payment claim including individual payment claims of lead and other project partners or alternatively it could opt for a payment claim of a project partner (without distinguishing between lead and other project partners). The AA could also decide to use grouped payment claims of a project partner declared within an operation in a given sampling period. In such a case the grouped payment claims by project partner constitute the sampling unit (this sampling unit is later referred in the text as a project partner).

The selection of the sampling unit determines the projection approach. The projection of errors to the level of population is based on the errors in the selected sampling units. Thus, if the AA does not verify all the expenditure in the selected sampling unit (sub-sampling is applied), it needs to extrapolate the errors of the sub-sample to the level of the sampling unit before extrapolation to the level of the population.

In particular, if the AA decides to choose operations as the sampling units, with a sub-sample of project partners, the AA has to project the errors detected in the expenditure of selected partners to the level of the operation before extrapolation to the level of the population.

On the contrary, a simpler projection approach would be ensured by the use of project partners[53] (or of payment claims of project partners) as sampling units. Use of these sampling units allows for projection of the errors detected in the expenditure declared by the selected project partners (or in the selected payment claims of projects partners) directly to the level of the population of all expenditure declared to the EC, without going through the two-stage projection described above. (As the operation does not constitute the sampling unit in such a situation, there is no need to extrapolate detected errors to the level of the operation).

Although there might be other options available, the EC services recommend in particular the use of one of the following sampling units in ETC programmes when designing the sampling methodology:

a) payment claim of an (individual) project partner,
b) project partner (i.e. all the payment claims declared by a project partner within an operation in a given sampling period) or
c) the operation.

All the above sampling units could be used both in statistical sampling and non-statistical sampling methods. However, the use of operations as sampling units under a statistical sampling method could require heavy workload in the context of ETC programmes as compared to the other two sampling units listed above. Therefore, the use of operation as the sampling unit is recommended in non-statistical sampling methods.

Section 6.5.3 below presents in the context of two- and three-stages sampling more detailed information on the possible sampling units and sub-sampling units in the ETC programmes together with additional notes on the relevant methodological constrains and implications.

6.5.3 *Sampling methodology*

In the case of both statistical and non-statistical sampling procedures within ETC programmes, the general sampling methodologies, as described in the relevant sections of this guidance, are applicable. This section provides additional clarifications in view of particularities of the ETC programmes.

The threshold of 50-150 operations may not be reached in ETC programmes characterised by small population sizes, particularly in the beginning of the implementation period. However, even if this threshold is reached, given the specific set-up of the ETC programmes, it may not be cost-effective to use statistical sampling.

[53] without the need to distinguish between lead and other project partners

Therefore the AA, on the basis of its professional judgement, could use non-statistical sampling for ETC, under the conditions of Article 127(1) CPR, while respecting the minimum coverage of 5% of operations and 10% of expenditure. The reasoning and options taken by the AA should be reflected in its audit strategy, which requires an annual update, as established by Article 127(4) CPR.

When statistical sampling methods are used, this allows the calculation of the precision, which gives control over the audit risk. Where an operation constitutes the sampling unit, the application of the statistical sampling methodologies may lead to high costs for auditing ETC programmes, given their specific set-up. Therefore, the AAs are recommended to use other sampling units (a partner or a payment claim of an individual project partner) which could decrease the costs of the audit procedures with statistical sampling. This approach is facilitated once the monitoring system (foreseen in Article 24 of Regulation (EU) No 480/2014) allows for the breakdown of data on expenditure between project partners.

Moreover, it should be noted that in the 2014-2020 programming period the provisions of Article 127 of Regulation (EU) No 1303/2013 require a coverage of a minimum of 5% of operations and 10% of the expenditure declared if a non-statistical sampling method is applied. Since in the case of statistical sampling this requirement is not applicable, the AA should consider that the use of a statistical sampling method could lead in some cases to equivalent or even reduced audit work (as compared to non-statistical sampling), in particular if payment claims of project partners are used as sampling units and simple random sampling is used. If confronted with similar audit costs and efforts, the AA is recommended to opt for statistical sampling.

Finally, due to the specific control system used by ETC programmes (e.g. decentralized vs centralised systems) the AA may consider stratification (for example, using the results from system audits), enabling the AA to draw conclusions per stratum where necessary. The stratification by MS may be considered either *a priori* or *a posteriori* (e.g. when the error rate is above 2%), in order to allow the AA to assess where the error comes from. In this respect, the sampling methodology can take into account the "bottom-up strategy" explained in section 7.8 of this guidance.

6.5.3.1 Two-stage and three-stage sampling (sub-sampling)

When using either statistical or non-statistical sampling methods, the AA needs to establish errors at the level of the selected sampling units before projecting the errors detected in the sample to the population. As a general rule, all expenditure declared to the Commission in the sample should be subject to audit. However, where the selected sampling units include a large number of underlying payment claims or invoices, the audit authority may audit them through sub-sampling. In such cases, to establish the error at the level of the selected sampling units, the AA needs to project errors detected in the sub-sample to the level of the sampling unit. In the next stage, the errors of the selected sampling units (established on the basis of a sub-sample) are projected to the population of operations or payment claims in order to calculate the projected error of the population.

Sub-sampling units

Both in statistical and non-statistical sampling, the AA could use different sub-sampling units within two/three-stage sampling design such as invoices, projects within an operations, aggregated payment claims including individual payment claims of lead and other project partners, payment claims of individual project partners, project partners.

Due to the set-up of operations in the context of ETC programmes, the AA frequently applies a sampling design with either two-stage or three-stage sampling, where a project partner or a payment claim of project partner could constitute a sampling unit at one of the sampling stages.

If the sampling unit is an operation, the AA could decide to have a sampling design with selection of a sub-sample of payment claims of individual project partners (two-stage sampling). Another option of two-stage sampling design, the most frequently used in ETC context, is to group all payment claims of individual project partners per project partner and to select a sub-sample of project partners within the selected operation. In such cases, errors detected at the level of payment claims/project partners need to be projected first to the level of the operation before the final projection of errors to the level of the population of operations.

Invoices as sub-sampling unit

If some sampling units of the selected sub-sample (payment claims/partners) have a large number of invoices/other expenditure items, the AA could decide to audit them on a sample basis leading to a three-stage sampling design. In such a case, the error detected in the sub-sample of invoices should be first projected to the level of a payment claim/a partner. Subsequently, the errors established at the level of payment claims/partners should be projected to the level of the operation as in two-stage sampling design.

The AA could also use invoices as the sampling unit in two-stage sampling, which is in particular applied when either a payment claim of individual project partner or a partner constitute the main sampling unit. In the case of operation as the main sampling unit in two-stage sampling design, the sub-sample of invoices would be selected directly from the population of all invoices of the operation, without the intermediary stage of a sub-sample at the level of partner/payment claim.

Selection of sub-sampling units under statistical and non-statistical methods

All sampling units in sub-samples should be selected at random[54], also in the case of non-statistical sampling methods. Nevertheless, in case stratification is applied at the level of sub-samples, obviously the AA could decide to audit all sampling units of a particular stratum.

Example: if the AA decides to use an operation as the sampling unit of the main sample and project partners as the sub-sampling units, the AA could either:
- make a random selection of project partners (without distinguishing between lead and other project partners) or
- apply stratification at the level of an operation:
- one stratum for the expenditure of the lead partner and
- a second stratum for the expenditure of other project partners.

Since in the latter case, the lead partner is not selected at random but his expenditure constitutes an exhaustive stratum, the projection model should take this into account. To calculate the error at the level of the operation, the errors of the other project partners selected at random in the operation should be projected to the stratum of other project partners, whereas the error of the lead partner should be added to the projected error to establish the total projected error rate of the operation. Section 6.5.3.3 below includes an example based on such a sampling design.

It is also reminded that in case statistical sampling is applied for the main sample, the AA needs to ensure application of the statistical sampling method for the selection of sampling units of the sub-samples at all stages. In particular, in case operations are chosen as the sampling units with a sub-sample of projects partners in the second stage and a sub-sample of invoices in the third stage, the AA needs to ensure observation of at least 30 units in the second stage and also in the third stage. Consequently, if the sub-sample unit selected within an operation is the project partner, this means that 30 project partners should be selected (few cases would be applicable, if any). Otherwise, the method can still be applied but it will lead to the selection of all the partners pertaining to the operation, leading in practice to application of two-stage sampling (operation in first stage and invoice in second stage) instead of three-stage sampling. Similarly, for each selected partner a verification of a sub-sample of at least 30 invoices should be ensured in case the exhaustive audits are too costly.

[54] Using equal probability selection (where each sampling unit has equal chance of being selected regardless of the amount of expenditure declared in the sampling unit) or probability proportional to size (expenditure) (where a random selection is made of the first element for the sample and then subsequent elements are selected using an interval until the desired sample size is reached) with the use of the monetary unit as an auxiliary variable for sampling as done for the MUS case.

For the programming period 2014-2020 and in line with Article 28 CDR, where sub-sampling is used with either invoices or payment claims as the sub-sampling units, the AA should cover not less than 30 invoices/other expenditure items or payment claims also under non-statistical sampling. Where other sub-sampling units are used under non-statistical sampling (such as for example a project within an operation, a project partner), the AA may decide, based on professional judgment, the sufficient coverage of a sub-sample. In this case, it is recommended that if less than 30 sub-sampling units are selected, they should cover at least 10% of the expenditure of the sampling unit (for example of an operation).

6.5.3.2 Main potential configurations of sampling units in two-stage and three-stage sampling

The tables below summarise the main potential configurations of sampling units in two-stage or three-stage sampling within the ETC context. Based on statistical considerations, these configurations could be applied both in statistical and non-statistical sampling methods. However, as clarified in the table, some of the listed configurations could be not feasible due to high-cost of audit and in some cases methodological constraints would hinder using them in statistical sampling methods due to insufficient number of sub-sampling units in practice. **In particular, whereas options 1 and 2 presented in the table below are considered as the most cost-effective in the case of statistical sampling methods and options 2 and 3 in non-statistical sampling methods, the remaining options could require much more audit resources and consequently are often not feasible in practice.**

6.5.3.2.1 Two-stage designs

Option	**Sampling unit of the main sample**	**Sub-sampling unit (if relevant)**	**Recommendation to apply in non-statistical and statistical sampling methods**	**Other remarks/constraints**
1.	Payment claim of a project partner	Invoice/other expenditure item	*Statistical sampling:* yes	Among the presented statistical sampling designs, it is the configuration requiring the least audit resources allowing at the same time calculation of precision and upper error limit, which gives control over the audit risk.
			Non-statistical sampling: It is a significantly less cost-effective approach as compared to the use of project partner as the main sampling unit due to the requirement of covering a minimum of 10% expenditure declared to the EC and 5% of operations in regard to an accounting year. (The AA would need to cover more sampling units to comply with the requirement of covering the minimum expenditure level).	In non-statistical sampling methods options 2 and 3 are more cost-effective.
2.	Project partner	Invoice/other	*Statistical sampling:* yes	It is a recommended approach in

Option	Sampling unit of the main sample	Sub-sampling unit (if relevant)	Recommendation to apply in non-statistical and statistical sampling methods	Other remarks/constraints
		expenditure item		statistical sampling method. It could be more costly than option 1.
			Non-statistical sampling: yes (Art.127 of the CPR requires a coverage of a minimum of 5% of operations and 10% of the expenditure declared.)	It is a recommended approach in non-statistical sampling method. It should be noted that as compared to another cost-effective approach in non-statistical sampling (i.e. option 3 below), option 2 does not require projection from project partners to the level of the operation since the projection to the population is carried out directly from project partners. In the case of project partners whose invoices/expenditure items are not verified exhaustively, the error of a partner would be calculated on the basis of projection of errors detected in the sub-sample of invoices/other expenditure items.
3.	Operation	Project partner[55]	*Statistical sampling:* a) In the case of up to 30 project partners in an operation, this design is not applied. (For statistical methods verification of all or at least 30 partners at the level of sub-sample would be required. Whenever the number of partners is equal or smaller than 30, the method would lead to the selection of all existing partners, leading to one-stage sampling design.) b) In the case of more than 30 project partners: high audit cost of covering at least 30 partners.	In statistical sampling methods, options 1 and 2 are more cost-effective.
			Non-statistical sampling: yes (Art.127 of the CPR requires a coverage of a minimum of 5% of operations and 10% of the expenditure declared.)	Two options could be applied for the selection of project partners: a) random selection of partners without distinction between lead and other project partners, b) for each selected operation verification of expenditure declared by the lead partner and expenditure declared by randomly selected other project partners. The approach requires the projection of errors of the selected project partners to the level of the operation (see option 2 for another cost-effective approach in non-statistical sampling which does not require projection from the level of partners to the level of operation).

[55] This sub-sampling unit groups per partner all the payment claims declared by a project partner within an operation in a given sampling period.

Option	Sampling unit of the main sample	Sub-sampling unit (if relevant)	Recommendation to apply in non-statistical and statistical sampling methods	Other remarks/constraints
				In non-statistical sampling, it is recommended that the sub-sample of project partners covers at least 10% of the expenditure of the operation.
4.	Operation /Aggregated payment claim	Invoice/other expenditure item	*Statistical sampling:* As it could require verification of expenditure incurred by different partners within a selected operation (aggregated payment claim), this configuration is not cost-effective. It requires more audit resources than under options 1 and 2.	In statistical sampling methods, options 1 and 2 are more cost-effective.
			Non-statistical sampling: usually not feasible due to high cost of audit	In non-statistical sampling methods, options 2 and 3 are more cost-effective.
5.	Operation	Aggregated payment claim	*Statistical sampling:* a) In the case of up to 30 aggregated payment claims, this design requires verification of all aggregated payment claims, leading to one-stage design. b) In the case of more than 30 payment claims: high audit cost of covering at least 30 aggregated payment claims.	In statistical sampling methods, options 1 and 2 are more cost-effective.
			Non-statistical sampling: usually not feasible due to high cost of audit	In non-statistical sampling methods, options 2 and 3 are more cost-effective.
6.	Operation or aggregated payment claim	Payment claim of a project partner	*Statistical sampling:* a) In the case of up to 30 payment claims of individual project partners, this design requires verification of all payment claims of individual projects partners, leading to one-stage sampling design. b) In the case of more than 30 payment claims: high audit cost of covering at least 30 payment claims of individual project partners.	In statistical sampling methods, options 1 and 2 are more cost-effective.
			Non-statistical sampling: usually not feasible due to high cost of audit	In non-statistical sampling methods, options 2 and 3 are more cost-effective.

In practice, within the ETC context the most commonly used two-stage sampling designs are:

- the use of an operation as the sampling unit and a project partner as the sub-sampling unit in the case of non-statistical sampling (cf. option 3 above),
- the use of a payment claim of individual project partner as the sampling unit and an invoice/other expenditure items as the sub-sampling unit in the case of statistical sampling (cf. option 1 above).

The configuration of a project partner as the sampling unit and an invoice/other expenditure item as the sub-sampling unit (cf. option 2 above) is also a recommended approach, which could be cost-effective both under statistical and non-statistical sampling methods. In such a case, the error of each partner could be calculated on the basis of projection of errors detected in the sub-sample of invoices. The errors of partners would be extrapolated directly to the level of population (without the need to calculate the error of the relevant operations as the operation does not constitute the sampling unit in such a configuration).

Specific attention should be paid to the case where the AA decides to choose an operation as the sampling unit under a statistical sampling method. Different sub-sample units could be applied in such a case, such as an aggregated payment claim (cf. option 5 above), a project partner (cf. option 3 above) or a payment claim of individual project partner (cf. option 6 above). However, under a statistical sampling method it is required to ensure at least 30 observations at each sampling stage, this may require the verification of all sub-sample units (as normally there are less than 30 sub-sampling units available).

The exception concerns the selection of operation as the sampling unit and an invoice/other expenditure item as the sub-sampling unit (cf. option 4 above). In this case, the statistical sub-sample of invoices would be selected from the population of all invoices declared for the operation within the sampling period (i.e. covering all project partners who declared expenditure in the sampling period). The audit workload would largely decrease as compared to the application of other sub-sample units mentioned above. However, this configuration would generally require much more audit resources as compared to the use of project partners or payment claims of project partners as the sampling units with a sub-sample of invoices (cf. options 1 and 2 above).

6.5.3.2.2 Three-stage designs

Sampling unit of the main sample	**Sub-sampling unit**	**Sampling unit of sub-sample at the lowest stage**	**Remarks**
Operation	Project partner[56]	Invoice/other expenditure item	See option 3 of the table above.
Operation	Aggregated payment claim	Invoice/other expenditure item	See option 5 of the table above.
Operation	Payment claim of individual project partner	Invoice/other expenditure item	See option 6 of the table above.
Aggregated payment claim	Payment claim of individual project partner	Invoice/other expenditure item	See option 6 of the table above.

Within ETC context, the three-stage design is mainly applied in non-statistical sampling methods where operations are selected as sampling units and project partners as sub-sampling unit, for which a random selection of invoices is verified.

[56] This sub-sampling unit groups per partner all the payment claims declared by a project partner within an operation in a given sampling period.

6.5.3.3 A possible approach in two-stage sampling (operation as the sampling unit and sub-sample of project partners whereby the lead partner and a sample of project partners are selected)

6.5.3.3.1 Sampling design

Let's take a case where the AA has decided that, for the operations selected, the audit of the lead partner will always be carried out covering both its own expenditure and the process for aggregating the project partners' payment claims. Where the number of other project partners is such that it is not possible to audit all of them, a random sample shall be selected. Thus the AA has opted for stratification at the level of the sampling unit of the main sample with separated stratum of expenditure declared by the lead partner and stratum of expenditure declared by other project partners. The size of the combined sample of lead partner and project partners must be sufficient to enable the AA to draw valid conclusions.

In such cases, the projection of the errors to the population (or to the corresponding operation) should take into consideration that the lead partner has been audited, while the project partners were audited through sampling.
The following methodology applied by the AA in the present example assumes:

- the use of non-statistical sampling design;
- two-stages design, where the first level is the selection of the operations, the second level the selection of a sample of partners within each operation[57];
- selection of all units (operations, partners) with equal probabilities (other sampling methods are acceptable);
- in each operation the lead partner is always selected;
- a sample of project partners is selected among the list of partners.

Firstly, one should acknowledge that in the first stage of selection (operations) the design should follow one of the previously proposed methods. Inside each operation, the strategy formally corresponds to a stratified design with two strata:

- the first stratum corresponds to the lead partner and is constituted by just one population unit that is always to be selected in the sample. In practice this stratum has to be treated as an exhaustive stratum similar to the high-values strata;
- the second stratum corresponds to the set of project partners and is observed through sampling.

[57] It is also possible to subsample the payments claims or other units of the selected partners if they are too large to be observed exhaustively.

For one specific operation, i, in the sample, the projected error for the exhaustive stratum (corresponding to the lead partner) is:

$$EE_e = E_{LP}$$

where E_{LP} is the amount of error found in the lead partner's expenditure. In other words the projected error of the exhaustive stratum is simply the amount of error found in the lead partner.

Please be aware that it is not mandatory to fully audit the lead partner; subsampling of the lead partner's expenditure is an option if it includes a large number of payment claims (or other subunits). If this is the case, the subsample of payment claims (or other subunits) has to be used in order to project the amount of error of the lead partner.

If a subsample is used and assuming again a selection based on equal probabilities and ratio estimation[58], the projected error of the lead partner will be:

$$EE_{LP} = BV_{LP} \frac{\sum_{j=1}^{n_{LP}} E_j}{\sum_{j=1}^{n_{LP}} BV_j}.$$

where BV_{LP} is the is the expenditure of the lead partner and n_{LP} the sample size of the subunits audited for this partner.

For the stratum containing the other project partners, the error has to be projected taking into consideration that only a sample of these partners has been observed.

Again, if partners were selected with equal probabilities and assuming ratio estimation, the projected error is

$$EE_{PP} = BV_{PP} \frac{\sum_{i=1}^{n_{s,PP}} E_i}{\sum_{i=1}^{n_{s,PP}} BV_i}.$$

where BV_{PP} is the expenditure of the set of project partners and $n_{s,PP}$ the sample size in the project partners stratum.

This projected error is equal to the error rate in the sample of project partners multiplied by the population expenditure of the stratum.

[58] Be aware that this formula has to be adapted to the specific selection and extrapolation process that have been selected in each. We won't burden the reader with the consideration that should be taken into consideration for these choices fully debated in previous sections.

Please be aware that in cases where the project partners selected to the sample are not fully audited, but only audited through a subsample of payments claims (or other units) then the errors E_i have to be projected, as explained for the lead partner.

The total projected error for the operation I is just the sum of these two components:

$$EE_i = EE_{LP} + EE_{PP}$$

This projection procedure should be followed for each operation in the sample in order to obtain the projected errors for each operation ($EE_i, i = 1, \dots n$). Once the projected errors of all operations in the sample have been calculated, the projection to the population is straightforward, using the appropriate methodologies presented in the previous sections.

The projected error (and the upper error limit when using a statistical design) are finally compared to the maximum tolerable error (materiality level rate multiplied by the population expenditure) in order to conclude about the existence of material error in the population.

6.5.3.3.2 Example

Let us assume a population of expenditure declared to the Commission in a given reference period for operations in European Territorial Cooperation (ETC) programmes. As the management and control systems are not common to all the Member-States involved it is not possible to group them. Moreover, as the number of operations is significantly low (only 47) and for each operation there are more than one project partner (the lead partner and at least one other project partner), and there are a few operations with extremely large book values, the AA decided to use a non-statistical sampling approach with stratification of the high-value operations. The AA decided to identify these operations by setting the cut off level as 3% of total book value.

The following table summarizes the available population information.

Declared expenditure (DE) in the reference period	113,300,285 €
Size of population (operations)	47
Materiality level (maximum 2%)	2%
Tolerable misstatement (TE)	2,266,006 €
Cut off value (3% of total book value)	3,399,009 €

This high-value project will be excluded from sampling and will be treated separately. The total value of this project is 4,411,965 €. The amount of error found in this operation amounts to

$$EE_e = 80{,}328.$$

The following table summarizes these results:

Number of units above cut-off value	1
Population book value above cut-off	4,411,965 €
Amount of error found in operations with book larger than cut-off	80,328 €
Remaining population size (no. of operations)	46
Remaining population value	108,888,320 €

The AA considers that the management and control system "*essentially does not work*", so it decides to select a sample size of 20% of the population of operations. That is, 20% x 47=9.4 rounded by excess to 10. Due to the small variability in the expenditure for this population, the auditor decides to sample the remaining population using equal probabilities. Although based on equal probabilities, it is expected that this sample will result in the coverage of at least 20% of the population expenditure stratum (cf. 6.4.3).

A sample of 9 operations (10 minus the high-value operation) is randomly drawn. 100% of the expenditure regarding the leading partner was audited. Two errors were found.

Operation ID	Lead Partner expenditure		
	Book value	Audited expenditure	Amount of error
864	890,563 €	890,563 €	0 €
12895	1,278,327 €	1,278,327 €	0 €
6724	658,748 €	658,748 €	5,274 €
763	234,739 €	234,739 €	20,327 €
65	987,329 €	987,329 €	0 €
3	1,045,698 €	1,045,698 €	0 €
65	895,398 €	895,398 €	0 €
567	444,584 €	444,584 €	0 €
24	678,927 €	678,927 €	0 €
Total	**7,114,313 €**		

Regarding the expenditure submitted by the remaining project partners, the AA decides, for each operation, to randomly select one project partner to be exhaustively audited.

Operation ID	Project Partners expenditure				
	No. partners audited	Book value (for all project partners in low-value stratum)	Audited expenditure	Amount of error	Projected error
864	1	234,567 €	37,147 €	0 €	0 €

Operation ID	Project Partners expenditure				
	No. partners audited	Book value (for all project partners in low-value stratum)	Audited expenditure	Amount of error	Projected error
12895	1	834,459 €	164,152 €	0 €	0 €
6724	1	766,567 €	152,024 €	23 €	116 €
763	1	666,578 €	83,384 €	0 €	0 €
65	1	245,538 €	56,318 €	127 €	554 €
3	1	344,765 €	101,258 €	0 €	0 €
65	1	678,927 €	97,656 €	0 €	0 €
567	1	1,023,346 €	213,216 €	1,264 €	6,067 €
24	1	789,491 €	137,311 €	0 €	0 €
Total		**5,584,238 €**			

The AA projects the error for each operation using ratio estimation. For example, the projected error of operation ID 65 is given by the sample error rate (127/56,318 x 100% = 0.23%) multiplied by the book value of the project partners of the operation (0.23% x 245,538 € = 554 €).

For each operation in the sample the projected error is equal to the error projected for the project partners plus the error observed in the lead partner.

Operation ID	Total book value	Projected error (lead partner)	Projected error (other project partners)	Total projected error by operation
864	1,125,130 €	0 €	0 €	0 €
12895	2,112,786 €	0 €	0 €	0 €
6724	1,425,315 €	5,274 €	116 €	5,390 €
763	901,317 €	20,327 €	0 €	20,327 €
65	1,232,867 €	0 €	554 €	554 €
3	1,390,463 €	0 €	0 €	0 €
65	1,574,325 €	0 €	0 €	0 €
567	1,467,930 €	0 €	6,067 €	6,067 €
24	1,468,418 €	0 €	0 €	0 €
Total	12,698,551 €			32,338 €

The projected error for the whole low-value stratum is given by the sum of the projected errors by operation (32,338€) divided by the total book value of the sampled operations, 7,114,313 € + 5,584,238 € = 12,698,551 €, which leads to a sample error rate at low-value stratum level of 0.25%. Once again, using ratio estimation procedure, this sample error rate applied to the book value of the low-value stratum, 108,888,320 € gives the projected error at low-value stratum level, 277,294 €.

Summing the projected error for both high-value and low-value strata, the AA gets the total projected error.

$$EE \quad = EE_e + EE_s = 80{,}328 + 277{,}294 = 357{,}622€$$

Finally, the projected error will be compared with the materiality threshold (2,266,006€) as usual leading to conclude that the projected error is below the materiality threshold.

7 Selected topics

7.1 How to determine the anticipated error

The anticipated error can be defined as the amount of error the auditor expects to find in the population. Factors relevant to the auditor's consideration of the expected error include the results of the test of controls, the results of audit procedures applied in the prior period and the results of other substantive procedures. One should consider that the more the anticipated error differs from the true error, the higher the risk of reaching inconclusive results after performing the audit (EE <2% and ULE > 2%).

To set the value of the anticipated error the auditor should take into consideration:

1. If the auditor has information on the error rates of previous years, the anticipated error should, in principle, be based on the projected error obtained in the previous year; nevertheless if the auditor has received information about changes in the quality of the control systems, this information can be used either to reduce or increase the anticipated error. For example, if last year projected error rate was 0.7% and no further information exists, this value can be imputed to the anticipated error rate. If, however the auditor has received evidence about an improvement of the systems that reasonably has convinced him/her that the error rate in the current year will be lower, this information can be used to reduce the anticipated error to a smaller value of, for example, 0.4%.
2. If there is no historical information about error rates, the auditor can use a preliminary/pilot sample in order to obtain an initial estimate of the population error rate. The anticipated error rate is considered to be equal to the projected error from this preliminary sample. If a preliminary sample is already being selected, in order to compute the standard-deviations necessary to calculate the formulas for sample size, then this same preliminary sample can also be used to compute an initial projection of the error rate and thus of the anticipated error.
3. If there is no historical information to produce an anticipated error and a preliminary sample cannot be used due to uncontrollable restrictions, then the auditor should set a value to the anticipated error based on professional experience and judgment. The value should mostly reflect the auditor expectation regarding the true level of error in the population.

In summary, the auditor should use historical data, auxiliary data, professional judgement or a mix of the above to choose a value as realistic as possible for the anticipated error.

An anticipated error based on objective quantitative data is usually more accurate and avoids carrying out additional work in the case audit results are inconclusive. For example if the auditor sets an anticipated error of 10% of materiality, i.e. 0.2% of expenditure, and at the end of the audit he obtains a projected error of 1.5%, results will most probably be inconclusive as the upper limit of error will be higher than the materiality level,. To avoid these situations the auditor should use as anticipated error, in future sampling exercises, the most realistic possible measure of the true error in the population.

A special situation may arise when the anticipated error rate is in the neighbourhood of 2% (cf. Figure 6). For example, if the anticipated error is 1.9% and the confidence level is high (e.g. 90%) it may happen that the resulting sample size is extremely large and hardly achievable. This phenomenon is common to all sampling methods and happens when the planned precision is very small (0.1% in the example)[59]. An advisable possibility, under this situation, is to divide the population in two different subpopulations where the auditor expects to find different levels of error. If it is possible to identify one subpopulation with expected error below 2% and other subpopulation for which the expected error is above 2%, the auditor can safely plan two different samples for these subpopulations, without the risk of obtaining too large samples sizes.

Finally, the Audit Authority should plan its audit work in a way to achieve sufficient precision of the MLE even when the anticipated error is well above materiality (i.e. equal or above 4.0%). In this case it is advisable to compute the sample size formulas with an anticipated error resulting in a maximum planned precision of 2.0%, i.e. by imputing the anticipated error to be equal to 4.0% (cf. Figure 6).

Where historical data on audits of operations and possibly system audit results lead to a very low anticipated error rate, the auditor may decide to use this historical data or any higher error as anticipated error, in order to be prudent in regard to the effective precision (e.g. in case that the effective error rate is higher than predicted).

[59] Remember that the planned precision is a function of the anticipated error, i.e. equal to the difference between the maximum tolerable error and the anticipated error.

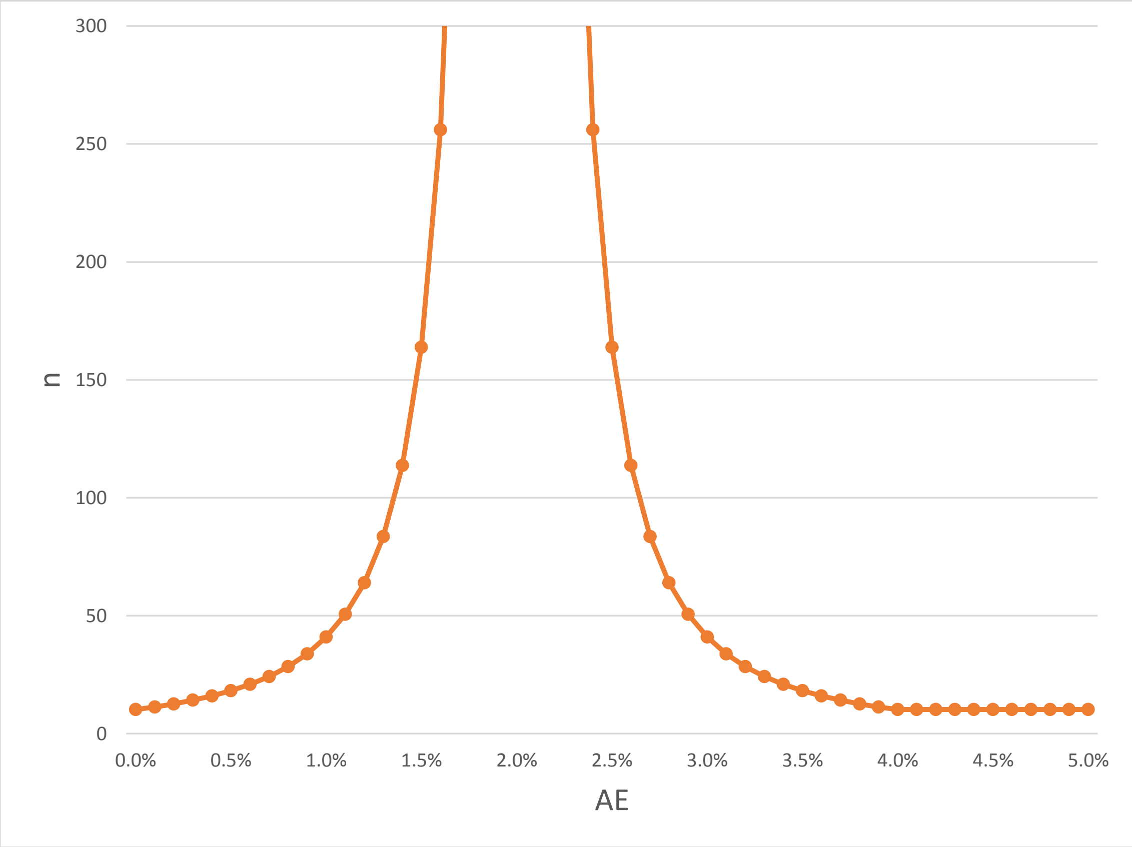


Fig. 6 Sample size as a function of anticipated error

7.2 Additional sampling

7.2.1 Complementary sampling (due to insufficient coverage of high risk areas)

Concerning the programming period 2007-2013, in Article 17(5) of the Commission Regulation (EC) No 1828/2006 (for ERDF, CF and ESF) and Article 43 § 5 of the Commission Regulation (EC) No 498/2007 (for EFF), reference is made to complementary sampling.

A similar provision exists for the programming period 2014-2020, set out in Article 28(12) of Regulation (EU) No 480/2014: "*Where irregularities or a risk of irregularities have been detected, the audit authority shall decide on the basis of professional judgement whether it is necessary to audit a complementary sample of additional operations or parts of operations that were not audited in the random sample in order to take account of specific risk factors identified.*"

The audit assurance should be built from the AA's work on system audits as well on the audits of operations and any complementary audits judged necessary by the AA based on their risk assessment, taking into account the audit work carried out during the programming period.

The results of the random statistical sampling have to be assessed in relation to the results of the risk analysis of each programme. Where it is concluded from this comparison that the random statistical sample does not address some high-risk areas, it should be completed by a further selection of operations, i.e. a complementary sample.

The audit authority should make this assessment on a regular basis during the implementation period.

In this framework, the results of the audits covering the complementary sample are analysed separately from the results of the audits covering the random statistical sample. In particular, the errors detected in the complementary sample are not taken into account for the calculation of the error rate resulting from the audit of the random statistical sample. However, a detailed analysis must also be done of the errors identified in the complementary sample, in order to identify the nature of the errors and to provide recommendations to correct them.

The results of the complementary sample should be reported to the Commission in the Annual Control report immediately following the audit of a complementary sample.

7.2.2 Additional sampling (due to inconclusive results of the audit)

Whenever the results of the audit are inconclusive and, after considering the possibilities offered in Section 7.7, additional work is needed (typically, when the projected error is below the materiality but the upper limit is above), an option is to select an additional sample. For this, the projected error produced from the original sample should be substituted in formulas for sample size determination in the place of the anticipated error (in fact the projected error is at that moment the best estimate of the error in the population). Doing this, a new sample size can be calculated based on the new information arising from the original sample. The size of the additional sample needed can be obtained by subtracting the original sample size from the new sample size. Finally, a new sample can be selected (using the same method as for the original sample), the two samples are grouped together and results (projected error and precision) should be recalculated using data from the final grouped sample.

Imagine that the original sample with sample size equal to 60 operations produced a projected error rate of 1.5%, with a precision of 0.9%. Consequently, the upper limit for the error rate is 1.5+0.9=2.4%. In this situation, we have a projected error rate that is below the 2% materiality level, but an upper limit that it is above. Consequently, the auditor faces a situation where further work is needed to achieve a conclusion (cf. Section 4.12). Among the alternatives one can choose to carry out further testing through additional sampling. If this is the choice, the projected error rate of 1.5% should be imputed in the formula for sample size determination in the place of the anticipated error, leading to a recalculation of the sample size, which would produce in our example a new sample size of n=78. As the original sample had a size of 60 operations, this value should be subtracted from the new sample size resulting in 78-60=18 new observations. Therefore an additional sample of 18 operations should be now selected from the population using the same method as for the original sample (ex. MUS). After this selection, the two samples are grouped together forming a new whole sample of 60+18=78 operations. This global sample will finally be used to recalculate the projected error and the precision of the projection using the usual formulas.

7.3 Sampling carried out during the year

7.3.1 Introduction

The audit authority may decide to carry out the sampling process in several periods during the year (typically two semesters). This approach should not be used with the goal of reducing the global sample size. In general the sum of sample sizes for the several periods of observation will be larger than the sample size that would be obtained by carrying out sampling in one single period at the end of the year. Nevertheless, if calculations are based on realistic assumptions, usually the sum of the partial sample sizes would not be dramatically larger than the one produced in a single observation. The major advantage of this approach is not related with sample size reduction, but mainly allowing spreading the audit workload over the year, thus reducing the workload that would be done at the end of the year based on just one observation.

This approach requires that at the first observation period some assumptions are made in regard to the subsequent observation periods (typically the next semester). For example, the auditor may need to produce an estimate of the total expenditure expected to be found in the population in the next semester. This means that this method is not implemented without risk, due to possible inaccuracies in the assumptions related to following periods. If characteristics of the population in the following periods differ significantly from the assumptions, sample size for the following period may have to be increased and the global sample size (including all periods) may be larger than the one expected and planned.

Chapter 6 of this guidance presents the specific formulas and detailed guidance for implementing sampling in two observation periods within one year. Note that this approach can be followed with any sampling method that has been chosen by the auditor, including possible stratification. It is also acceptable to treat the several periods of the year as different populations from which different samples are planned and extracted[60]. This is not dealt within the methods proposed in Chapter 6 as its application is straightforward using the standard formulas for the several sampling methods. Under this approach the only additional work is to add together the partial projected errors at the end of the year.

The audit authority should aim at using the same sampling method for a given reference period. The use of different sampling methods in the same reference period is not encouraged, as this would result in more complex formulas to extrapolate the error for that year. Namely, global precision measures can be produced, provided that statistical sampling was implemented in the same reference period. However, these more complex formulas are not included in the present document. Hence, if the audit authority uses different sampling methods in the same year, it should seek the adequate expertise in order to obtain the correct calculation of the projected error rate.

In case the AA would decide to use three or four-period sampling designs, please refer to Appendix 2 where the relevant formulas are presented.

7.3.2 Additional notes about multi-period sampling

7.3.2.1 Presentation

The previously proposed methodologies for two-period or multi-period sampling always start with the calculation of the global sample size (for the whole year) that is subsequently allocated to the several periods.

For example in MUS with two-periods one starts by calculating the sample size

$$n = \left(\frac{z \times BV \times \sigma_{rw}}{TE - AE}\right)^2$$

and allocate to the two-periods through

$$n_1 = \frac{BV_1}{BV} n$$

and

$$n_2 = \frac{BV_2}{BV} n$$

[60] This will of course result in sample sizes larger than the ones offered by the approach presented in Chapter 6.

The sample size calculation and allocation relies on certain assumptions about the population parameters (expenditures, standard deviations, etc.) that will be only known at the end of the next auditing period.

Because of this, at the end of the next semester the sample size may have to be recalculated if the assumptions significantly depart from the known population parameters. Therefore, it has been suggested to recalculate the sample size for the second semester by using

$$n_2 = \frac{\left(z \times BV_2 \ \times \sigma_{r2}\right)^2}{(TE - AE)^2 - z^2 \times \frac{BV_1^2}{n_1} \times s_{r1}^2}$$

This recommended approach doesn't exclude the use of other approaches for sample size recalculation that may be still adequate to ensure the required precision at the end of the programming year. In fact, the suggested approach, was developed to avoid the need to recalculate the sample size for the first period (already audited) and consequently avoiding the need to select an additional sample for this period. Nevertheless, should this be a desirable option for the AA[61], it is possible to recalculate the global sample size (after auditing the first period sample) and the proportional allocation by period spreading the correction between the first and second period samples.

A possible approach to achieve this would be to proceed as follows. After the audit of the first period sample, the global sample size is recalculated using

$$n' = \left(\frac{z \times BV \times \sigma_{rw}}{TE - AE}\right)^2$$

where σ_{rw}^2 is a weighted mean of the variances of the error rates in each semester, with the weight for each semester equal to the ratio between the semester book value (BV_t) and the book value for the whole population (*BV*).

$$\sigma_w^2 = \frac{BV_1}{BV} s_{r1}^2 + \frac{BV_2}{BV} \sigma_{r2}^2$$

Note that in this calculation the variance s_{r1}^2 could already be obtained from the first semester sample (already audited), while σ_{r2}^2 is a mere approximation of the variance of the error rates of the second semester based as usual on historical data, a preliminary sample or simply the auditor professional judgment.

[61] This alternative strategy may be used as a mean to avoid that corrections of sample size due to an originally incorrect prediction of population parameters are totally concentrated over the last period of audit.

Also the book value of the population (BV) used in this formula may differ from the one used in the first period. In fact, if this recalculation is done at the end of the second period, the expenditure of both semesters will be correctly known. In the first semester only the book value of the first period was known and the book value of the second semester was based on a prediction done by the auditor.

After recalculating the sample size for the whole year it has to be reallocated to both semesters using the usual approach

$$n'_1 = \frac{BV_1}{BV} n'$$

and

$$n'_2 = \frac{BV_2}{BV} n'$$

Also the balance of this allocation may differ from the original one due to the fact that BV_2 is now known and not a mere prediction.

Finally a sample of size n'_2 from the second period expenditure is selected and audited. Also, if the new recalculated sample size n'_1 is larger than the one originally planned n_1 an additional sample from the first semester expenditure, of size $n'_1 - n_1$, has to be selected and audited. This additional sample will be joined to the originally selected sample of the first period and will be used for projection purposes using the general methodology proposed in Section 7.2.2.

7.3.2.2 Example

In order to anticipate the audit workload that usually is concentrated at the end of the audit year the AA decided to spread the audit work in two periods. At the end of the first semester the AA considered the population divided into two groups corresponding to each one of the two semesters. At the end of the first semester the characteristics of the population are the following:

Declared expenditure at the end of first semester	1,827,930,259 €
Size of population (operations - first semester)	2,344

Based on the past experience, the AA knows that usually all the operations included in the programmes at the end of the reference period are already active in the population of the first semester. Moreover, it is expected that the declared expenditure at the end of the first semester represents about 35% of the total declared expenditure at the end of the reference period. Based on these assumptions a summary of the population is described in the following table:

Declared expenditure (DE) at the end of first semester	1,827,930,259 €
Declared expenditure (DE) at the end of the second semester (predicted) 1,827,930,259€ / 0.35-1,827,930,259€) = 3,394,727,624€)	3,394,727,624 €
Total expenditure forecasted for the year	5.222.657.883€
Size of population (operations – first semester)	2,344
Size of population (operations – second semester, predicted)	2,344

The AA decided to follow a standard MUS sampling design splitting the declared expenditure accordingly to the semester were it was submitted. For the first period, the global sample size (for the set of two semesters) is computed as follows:

$$n = \left(\frac{z \times BV \times \sigma_{rw}}{TE - AE}\right)^2$$

where σ_{rw}^2 is a weighted average of the variances of the error rates in each semester, with the weight for each semester equal to the ratio between the semester book value (BV_t) and the book value for the whole population (BV).

$$\sigma_{rw}^2 = \frac{BV_1}{BV}\sigma_{r1}^2 + \frac{BV_2}{BV}\sigma_{r2}^2$$

and σ_{rt}^2 is the variance of error rates in each semester. The variance of the errors rates is computed for each semester as

$$\sigma_{rt}^2 = \frac{1}{n_t^p - 1}\sum_{i=1}^{n_t^p}(r_{ti} - \bar{r}_t)^2\,, t = 1,2, \dots, T$$

Since these variances are unknown, the AA decided to draw a preliminary sample of 20 operations at the end of first semester of the current year. The sample standard deviation of error rates in this preliminary sample at first semester is 0.12. Based on professional judgement and knowing that usually the expenditure in second semester is larger than in first semester, the AA has made a preliminary prediction of standard deviation of error rates for the second semester to be 110% larger than in first semester, that is, 0.25. Therefore, the weighted average of the variances of the error rates is:

$$\sigma_{rw}^2 = \frac{1{,}827{,}930{,}259}{1{,}827{,}930{,}259 + 3{,}394{,}727{,}624} \times 0.12^2$$

$$+ \frac{3{,}394{,}727{,}624}{1{,}827{,}930{,}259 + 3{,}394{,}727{,}624} \times 0.25^2 = 0.0457$$

In the first semester, the AA, given the level of functioning of the management and control system, considers adequate a confidence level of 60%. The global sample size for the whole year is:

$$n = \left(\frac{0.842 \times (1{,}827{,}930{,}259 \ + 3{,}394{,}727{,}624) \times \sqrt{0.0457}}{104{,}453{,}158 - 20{,}890{,}632}\right)^2 \approx 127$$

where z is 0.842 (coefficient corresponding to a 60% confidence level), TE, the tolerable error, is 2% (maximum materiality level set by the Regulation) of the book value. The total book value comprises the true book value at the end of the first semester plus the predicted book value for the second semester 3,394,727,624 €, which means that tolerable error is 2% x 5,222,657,883 € = 104,453,158 €. The last year's audit projected an error rate of 0.4%. Thus AE, the anticipated error, is 0.4% x 5,222,657,883 € = 20,890,632 €.

The allocation of the sample by semester is as follows:

$$n_1 = \frac{BV_1}{BV_1 + BV_2} = \frac{1{,}827{,}930{,}259}{1{,}827{,}930{,}259 \ + 3{,}394{,}727{,}624} \times 127 \approx 45$$

and

$$n_2 = n - n_1 = 82$$

At the end of the second semester more information is available, in particular, the total expenditure of operations active in the second semester is correctly known, the sample variance of error rates s_{r1} calculated from the sample of the first semester could be already available and the standard deviation of error rates for the second semester σ_{r2} can now be more accurately assessed using a preliminary sample of real data.

The AA realises that the assumption made at the end of the first semester on the total expenditure, 3,394,727,624 €, overestimates the true value of 2,961,930,008. There are also two additional parameters for which updated figures should be used.

The estimate of the standard deviation of error rates based on the first semester sample of 45 operations yielded an estimate of 0.085. This new value should now be used to reassess the planned sample size. Moreover, a preliminary sample of 20 operations the second semester populations has yield a preliminary estimate of the standard deviation of the error rates of 0.32, far from initial value of 0.25. The updated figures of standard deviation of error rates for both semesters are far from the initial estimates. As a result, the sample for the second semester should be revised.

Parameter	Forecast done in the first semester	End of second semester
Standard deviation of error rates in the first semester	0.12	0.085
Standard deviation of error rates in the second semester	0.25	0.32
Total expenditure in the second semester	3,394,727,624 €	2,961,930,008 €

The standard approach to recalculate the sample size (cf. Section 6.3.3.7) would be to recalculate the sample size for the second semester based on the updated population parameters. Nevertheless the AA decides to follow the alternative approach, based on the recalculation of global sample size and reallocation between the two semesters. The recalculates global sample size is:

$$n' = \left(\frac{z \times BV \times \sigma_{rw}}{TE - AE}\right)^2,$$

where σ_{rw}^2 has been defined before but is based on completely know values BV_1, BV_2 and BV and variance s_{r1}^2 was obtained from the first semester sample (already audited), while σ_{r2}^2 is a mere approximation of the variance of the error rates of the second semester based on a preliminary sample of the second semester population:

$$\sigma_{rw}^2 = \frac{BV_1}{BV} s_{r1}^2 + \frac{BV_2}{BV} \sigma_{r2}^2.$$

Therefore,

$$\sigma_{rw}^2 = \frac{1{,}827{,}930{,}259}{4{,}789{,}860{,}267} \times 0.085^2 + \frac{2{,}961{,}930{,}008}{4{,}789{,}860{,}267} 0.32^2 = 0.066,$$

and

$$n' = \left(\frac{0.842 \times 4{,}789{,}860{,}267 \times 0.2571}{95{,}797{,}205 - 19{,}159{,}441}\right)^2 \approx 183.$$

After recalculating the sample size for the whole year it has to be reallocated to both semesters using the usual approach

$$n'_1 = \frac{1{,}827{,}930{,}259}{4{,}789{,}860{,}267} \times 183 \approx 70$$

and

$$n'_2 = 183 - 70 = 113$$

The recalculation of the sampling size implies the first semester sample to be enlarged by 25 operations. To draw an additional sample the AA removes out of the first semester population the previous sampled operations amounting to 1,209,191,248 €. The remaining population has a total book value of 618,739,011 €. Once again, when the AA computes the new cut-off value (the ratio of remaining population book value, 618,739,011 € to the sample size, 25) 2 operations arise with book value larger than it. The book value of these 2 operations amounts to 83,678,923 €. After removing these two operations the AA get the final population to be submitted to sampling using the MUS approach with a sampling interval of:

$$SI'_{s1} = \frac{BV'_{s1}}{n'_{s1}} = \frac{618{,}739{,}011 - 83{,}678{,}923}{23} = 27{,}263{,}482.$$

No errors were found in the 2 operations with book value larger than the cut-off value. Nevertheless, these sampling units have to be grouped with the ones already included in the high-value stratum of the initial sample for the first semester. Out of the 45 operations selected at first semester, 11 belong to the high-value stratum. These operations' total error amounts to 19,240,855 €.

A file containing the remaining (2344 minus 45 operations already selected in first semester minus the 2 operations with book value larger than cut-of value) operations of the population is randomly sorted and a sequential cumulative book value variable is created. A sample of 23 operations is drawn using the systematic proportional to size procedure.

The value of the 23 operations is audited. The sum of the error rates in the whole 57 non-exhaustive stratum sample (34 in the first semester + 23 in the second) first semester sample is:

$$\sum_{i=1}^{57} \frac{E_{is1}}{BV_{is1}} = 0.8391.$$

The standard deviation of error rate of this sample amounts to 0.059.

Regarding the work related to second semester, it is first necessary to identify the high value population units (if any) that will belong to a high-value stratum to be submitted at a 100% audit work. The cut-off value for determining this top stratum is equal to the ratio between the book value (BV_2) and the planed sample size (n_2). All items whose book value is higher than this cut-off (if $BV_{i2} > BV_2/n_2$) will be placed in the 100% audit stratum. In this case, the cut-off value is 26,211,770 €. There are 6 operations which book value is larger than this cut-off value. The total book value of these operations amounts to 415,238,983 €.

The sampling size to be allocated to the non-exhaustive stratum, n_{s2} , is computed as the difference between n_2 and the number of sampling units (e.g. operations) in the exhaustive stratum (n_{e2}), that is 107 operations (113, the sample size, minus the 6 high-value operations). Therefore, the auditor has to select in the sample using the sampling interval:

$$SI_{s2} = \frac{BV_{s2}}{n_{s2}} = \frac{2{,}961{,}930{,}008 - 415{,}238{,}983}{107} = 23{,}800{,}851$$

The book value in the non-exhaustive stratum (BV_{s2}) is just the difference between the total book value and the book value of the 6 operations belonging to the high-value stratum.

Out of the 6 operations with book value larger than the cut-off value, 4 of them have error. The total error found in this stratum is 9,340,755 €.

A file containing the remaining 2,338 operations of the second semester population is randomly sorted and a sequential cumulative book value variable is created. A sample of 107 operations is drawn using the systematic proportional to size procedure.

The value of these 107 operations is audited. The sum of the error rates for the second semester is:

$$\sum_{i=1}^{107} \frac{E_{2i}}{BV_{2i}} = 0.2875.$$

The standard-deviation of error rates in the sample of the non-exhaustive population of the second semester is:

$$s_{rs2} = \sqrt{\frac{1}{107-1} \sum_{i=1}^{107} (r_{is2} - \bar{r}_{s2})^2} = 0.129$$

having $\bar{r}_{s2}$ equal to the simple average of error rates in the sample of the non-exhaustive group of second semester.

The projection of errors to the population is made differently for units belonging to the exhaustive strata and for items in the non-exhaustive strata.

For the exhaustive strata, that is, for the strata containing the sampling units with book value larger than the cut-off, $BV_{ti} > \frac{BV_t}{n_t}$, the projected error is the summation of the errors found in the items belonging to those strata:

$$EE_e = \sum_{i=1}^{n_1} E_{1i} + \sum_{i=1}^{n_2} E_{2i} = 19{,}240{,}855 + 9{,}340{,}755 = 28{,}581{,}610$$

In practice:
1) For each semester t, identify the units belonging to the exhaustive group and sum their errors
2) Sum the previous results over the two semesters.

For the non-exhaustive group, i.e. the strata containing the sampling units with book value smaller or equal to the cut-off value, $BV_{ti} \leq \frac{BV_t}{n_t}$, the projected error is

$$\begin{aligned} EE_s = \frac{BV_{s1}}{n_{s1}} \times \sum_{i=1}^{n_{s1}} \frac{E_{1i}}{BV_{1i}} + \frac{BV_{s2}}{n_{s2}} \times \sum_{i=1}^{n_{s2}} \frac{E_{2i}}{BV_{2i}} &= \frac{1{,}827{,}930{,}259 - 891{,}767{,}519 - 83{,}678{,}923}{57} \times 0.8391 \\ &+ \frac{2{,}546{,}691{,}025}{107} \times 0.2875 = 19{,}392{,}204 \end{aligned}$$

To calculate this projected error:
1) in each semester t, for each unit in the sample calculate the error rate, i.e. the ratio between the error and the respective expenditure $\frac{E_{ti}}{BV_{ti}}$
2) in each semester t, sum these error rates over all units in the sample
3) in semester t, multiply the previous result by the total expenditure in the population of the non-exhaustive group (BV_{st}); this expenditure will also be equal to the total expenditure of the semester minus the expenditure of items belonging to the exhaustive group
4) in each semester t, divide the previous result by the sample size in the non-exhaustive group (n_{st})
5) sum the previous results over the two semesters

The projected error at the level of population is just the sum of these two components:

$$EE = EE_e + EE_s = 28{,}581{,}610 + 19{,}392{,}204 = 47{,}973{,}814$$

corresponding to a projected error rate of 1.0%.

The precision is a measure of the uncertainty associated with the projection. The precision is given by the formula:

$$SE = z \times \sqrt{\frac{BV_{s1}^2}{n_{s1}} \times s_{rs1}^2 + \frac{BV_{s2}^2}{n_{s2}} \times s_{rs2}^2}$$

$$= 0.842 \times \sqrt{\frac{(1{,}827{,}930{,}259 - 891{,}767{,}519 - 83{,}678{,}923)^2}{57} \times 0.059^2 + \frac{2{,}546{,}691{,}025^{\ 2}}{107} \times 0.129^2}$$

$$= 27{,}323{,}507$$

where s_{rst} are the standard-deviation of error rates already computed.

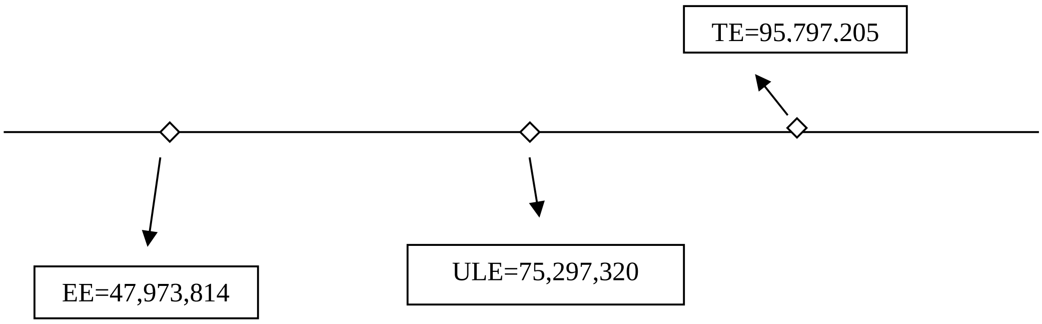
The sampling error is only computed for the non-exhaustive strata, since there is no sampling error arising from the exhaustive groups.

To draw a conclusion about the materiality of the errors the upper limit of error (ULE) should be calculated. This upper limit is equal to the summation of the projected error EE itself and the precision of the projection

$$ULE = EE + SE = 47{,}973{,}814 + 27{,}323{,}507 = 75{,}297{,}320$$

Then the projected error and the upper limit should both be compared to the maximum tolerable error to draw audit conclusions.

In this particular case, the projected error and the upper error limit are smaller than maximum tolerable error. It means that the auditor would conclude that there is evidence to support that the errors in the population are smaller than the materiality threshold.



7.4 Change of sampling method during the programming period

If the audit authority is of the opinion that the sampling method initially selected is not the most appropriate one, it could decide to change the method. However, this should be notified to the Commission in the framework of the Annual Control Report or in a revised audit strategy.

7.5 Error rates

Formulas and methodology presented in Chapter 6 to produce projected error and the respective precision are thought for errors in terms of monetary units, i.e. the difference between the book value in the population (declared expenditure) and the correct/audited book value. Nevertheless, it is common practice to produce results in the form of error rates as they are appealing due to their intuitive interpretation. The conversion of errors into error rates is straightforward and common to all sampling methods.

The projected error rate is simply equal to the projected error divided by the book value in the population

$$EER = \frac{EE}{BV}$$

Similarly, the precision for the estimation of the error rate is equal to the precision of the projected error divided by the book value

$$SER = \frac{SE}{BV}$$

7.6 Two-stage sampling (sub-sampling)

7.6.1 Introduction

In general, all the expenditure declared to the Commission for all the selected operations in the sample should be subject to audit. Nevertheless, whenever the selected operations include a large number of payment claims or invoices, the AA can apply two-stage sampling, selecting the claims/invoices by using the same principles used to select the operations[62]. This offers the possibility to significantly reduce the audit workload, allowing to still control the reliability of the conclusions. Whenever this approach is followed, the sampling methodology should be recorded in the audit report or working papers. It is important to stress that only the expenditure of the secondary units selected to the subsample is audited; this means that in the ACR the audited expenditure is only the one selected to the sample and not the whole expenditure of the selected operation.

[62] In theory, the operation can be subject to subsampling regardless the number of claims/invoices. Of course, whenever the determination of the subsample size produces a number close to the population (operation) size, the subsampling strategy won’t produce any significant reduction in the audit effort. Therefore, the threshold that suggests the use of subsampling at operation level is just the result of the AA subjective evaluation of the gain (reduction of audit effort) that can be brought by this strategy.

The following picture illustrates the process of selection based on a two-stage design. The first stage represents the selection of the operations and the second the selection of the expenditure items inside each sampled operation.

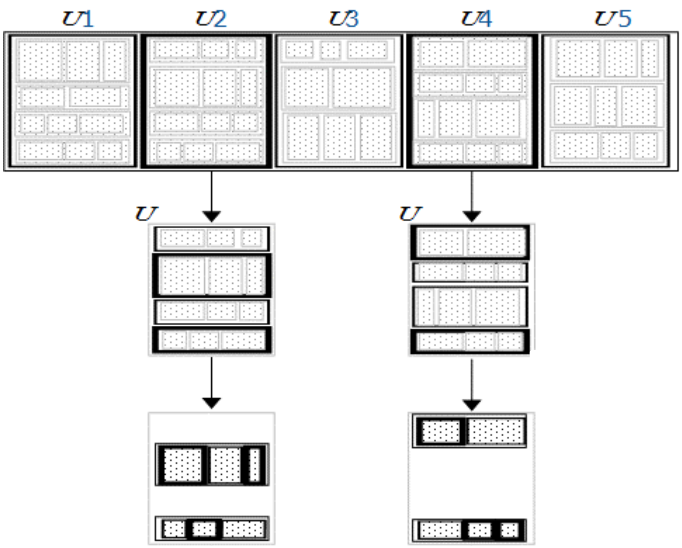


Fig. 7 Illustration of two-stages sampling

In this case, appropriate sample sizes have to be calculated within each operation. A very simple approach to the determination of sub-sample sizes is to use the same sample size determination formulas that are proposed to the main sample under the several sampling designs and based on parameters compatible with expected operation characteristics. Here, one should acknowledge that the reference population is now the operation inside which the subsample is selected and that the population parameters used for the determination the sub-sample size should, whenever possible, reflect the characteristics of the corresponding operation. Despite the sampling methodology used to determine sample sizes, a basic rule of thumb is to never use sample sizes smaller than 30 observations (i.e. invoices or payment claims from beneficiaries).

The AA may choose to use any statistical sampling methods for selecting the claims/invoices within the operations. In fact the sampling method used at the sub-sample level does not need to be equal to the one used for the main sample. For example, it is possible to have a sample selection of operations based on MUS and a subsample of invoices within one operation based on simple random sampling. Therefore, the whole range of sampling methods (including stratification of claims/invoices by level of expenditure, selection based on probabilities proportional to size as in MUS or selection based on equal probabilities) may be applied at this subsample level. Nevertheless, the subsampling strategy (sampling within the primary unit) should always be statistical (unless the sampling of primary units is not itself statistical). The choice between the possible methods is made under the same conditions of applicability that have been proposed in Section 5.2. For example, if within an operation it is expected to have a large variability of the expenditure of the sub-sampled expenditure items and it is expected to have positive correlation between errors and expenditure, then a selection of expenditure items based on MUS may be advisable. Also, when using simple random sampling (SRS) it may happen there are a few units within the operation that stand out due to high level of expenditure. In this case, it is highly advisable to use stratified SRS creating a stratum for the high value items (typically exhaustively observed).

Despite the considerations about the choice of the most suitable sampling design, one should acknowledge that in many situations (mainly due to operational constraints), the easiest way to select the sample of the second stage (claims or invoices) is using simple random sampling. This happens because in many cases the AA wants to perform the selection of the expenditure items on the spot (at the moment of the audit) being more difficult to implement more sophisticated designs (mainly if based on unequal probability selection).

Once the sub-sample is selected and audited, the observed errors have to be projected to the respective operation using a projection method compatible with the selected sampling design. For example, if the expenditure items have been chosen with equal probabilities than the error may be projected to the operation using the usual mean-per-unit estimation or ratio estimation. Please note that errors found in subsamples should NOT have any other kind of treatment (like being treated as systemic unless they have a real systemic nature, i.e. the error detected is systemic within all audit population and can be fully delimited by the audit authority).

Finally, once the errors have been projected for every operation in the sample that has been sub-sampled, the projection for the population follows the usual procedure (as if one had observed the whole expenditure of the operation). For example imagine that an operation in the sample has an expenditure of 2,500,000€ and 400 invoices. One decides to select a sample of 40 invoices based on equal probabilities and without any stratification, and decides to use ratio estimation. Imagine that the total audited expenditure is 290,000€ and the total observed error is 9,280€. The estimated error rate for the operation is 3.2%=(9,280€/290,000€) and the projected error of the operation is 80,000€=3.2%*2,500,000€.

Please note that section 6.5.3 includes additional notes on two- and three-stage sampling in the context of ETC programmes.

7.6.2 Sample size

There are formal ways to calculate sample size at each stage simultaneously using multi-stage sampling formulas. The AA being able to develop such methods are welcomed to do so.

Nevertheless, as already explained, the proposed simple approach can be performed by calculating sample size in two-stages independently:

- First stage: calculate sample size at operations level using the usual appropriate formulas and parameters (should always be larger or equal 30).
- Second stage: for each operation subject to subsampling calculate sample size using again the usual formulas (appropriate to the type of selection used at the second stage). Parameters should be compatible with the ones used at the first stage, although some may be adapted to reflect the reality of the reference operation (for example if there is historical data about the level of variance of the errors within the operation, one should use this variance instead of the variance of the errors used for the sample size calculation at the first stage). At this stage sample size should also be larger or equal to 30.

If the selection in this 2nd stage is based on equal probabilities the sample size is given by

$$n_i = \left(\frac{N_i \times z \times \sigma_{ei}}{TE_i - AE_i}\right)^2$$

where the index *i* represents the operation, N_i is the operation size, σ_{ei} the standard-deviation of errors at the operation level TE_i and AE_i the tolerable and anticipated error at operation level. Please note that the population size should be adapted to the operation level and that the standard-deviation of errors and anticipated error may also be adapted based on historical data and professional judgment if there is any information or expectation that would suggest to adapt this parameters to the operation reality.

If the selection in this 2nd stage is based on MUS the sample size is given by

$$n_i = \left(\frac{z \times BV_i \times \sigma_{ri}}{TE_i - AE_i}\right)^2$$

where the index *i* represents the operation, BV_i is the expenditure of the operation, σ_{ri} the standard-deviation of errors rates at the operation level TE_i and AE_i the tolerable and antecipaed error at operation level. Once again, the book value should be adapted to the operation level and the standard-deviation of errors rates and anticipated error may also be adapted based on historical data and professional judgment.

7.6.3 *Projection*

As for the sample size calculation, also the projection is made under two-stages. Firstly, the subsamples within the operations are used to project the error for those operations. Once the error of the operations are projected (estimated) they are treated as if they were the "true" errors of the operations and will became part of the usual extrapolation process based on the main sample.

In summary:

- For each operation subject to subsampling, estimate its error (or error rate) using the sample of secondary units;
- Once the errors for all operations have been estimated, use the sample of operations to project the total error of the population;
- In both cases the projection should be based on the formulas that correspond to the sample designs that have been used to select the units.

For example, one typical strategy will be to select the operations based on MUS and the subsamples of expenditure items based on equal probabilities. In that case the projection of the errors is:

Subsample level

Mean-per-unit estimation

$$EE_{1i} = N_i \times \frac{\sum_{j=1}^{n_i} E_{ij}}{n_i}.$$

or

Ratio estimation

$$EE_{2i} = BV_i \times \frac{\sum_{j=1}^{n_i} E_{ij}}{\sum_{j=1}^{n_i} BV_{ij}}$$

where all parameters have the usual meaning, *i* represents the operation and *j* the document within the operation.

Main sample level

The projection is made using the usual MUS formulas. The only difference regarding the standard MUS is that some of the errors E_i will be based on a full observation of the operations, while others have been projected based on a subsample of expenditure items. At this stage this fact is ignored, as all the errors will be treated as if they were the “true” errors of the operations, despite they have been fully observed or obtained through a subsample.

$$EE_e = \sum_{i=1}^{n_e} E_i$$

$$EE_s = \frac{BV_s}{n_s}\sum_{i=1}^{n_s}\frac{E_i}{BV_i}$$

7.6.4 Precision

The precision is calculated as usual, i.e. using the formulas in accordance with the sampling design used for the first stage of sampling and ignoring the existence of subsampling. Errors of operations are filled in precision formulas despite its nature (either the true ones when subject to full audit or estimated ones when subject to subsampling).

7.6.5 Example

Let’s assume a population of expenditure declared to the Commission in a given year. The system audits performed by the audit authority have yielded a low assurance level. Therefore, sampling this programme should be done with a confidence level of 90%. This particular programme is characterised by operations that include a large number supporting expenditure items. The AA authority considers the possibility of auditing this population through subsampling, that is, audit only a limited number of payment claims of each operation belonging to the sample. Moreover, due to the expected variability of the errors in the population the AA decides to select the operations in first stage using a probability proportional to size approach (MUS).

The main characteristics of the population are summarised in the following table:

Population size (number of operations)	3,852
Book value (sum of the expenditure in the reference period)	4,199,882,024 €

The sample size is computed as follows:

$$n = \left(\frac{z \times BV \times \sigma_r}{TE - AE}\right)^2$$

where σ_r is the standard-deviation of error rates produced from a MUS sample. To obtain an approximation to this standard deviation the AA decided to use the standard deviation of previous year. The sample of the previous year was constituted by 50 operations, 5 of which have a book value larger than the sampling interval.

Based on this preliminary sample the standard deviation of the error rates, σ_r , is 0.087.

Given this estimate for the standard deviation of error rates, the maximum tolerable error and the anticipated error, we are in conditions to compute the sample size. Assuming a tolerable error which is 2% of the total book value, 2% x 4,199,882,024 = 83,997,640, (materiality value set by the regulation) and an anticipated error rate of 0.4%, 0.4% x 4,199,882,024 = 16,799,528 (which corresponds to strong belief of the AA based both on past year's information and the results of the report on assessment of management and control systems),

$$n = \left(\frac{1.645 \times 4,199,882,024 \times 0.085}{83,997,640 - 16,799,528}\right)^2 \approx 77$$

In first place, it is necessary to identify the high value population units (if any) that will belong to a high-value stratum to be submitted at a 100% audit work. The cut-off value for determining this top stratum is equal to the ratio between the book value (*BV*) and the planed sample size (*n*). All items whose book value is higher than this cut-off (if $BV_i > BV/n$) will be placed in the 100% audit stratum. In this case the cut-off value is 4,199,882,024 €/77=54,593,922 €.

The AA puts in an isolated stratum all the operations with book value larger than 54,593,922, which corresponds to 8 operations, amounting to 786,837,081 €. As referred before, this programme comprises a large number of low book value payment claims by operation. For example, these 8 operations correspond to more than 14,000 payment claims. Therefore the AA decides to draw a sample of payment claims in each of these 8 operations. This procedure involves determination of sample size at operation level. Using equal probabilities, the sample size at operation level is determined by:

$$n_i = \left(\frac{N_i \times z \times \sigma_{ei}}{TE_i - AE_i}\right)^2$$

where the index *i* represents the operation, N_i is the operation size, σ_{ei} the standard-deviation of errors at the operation level TE_i and AE_i the tolerable and anticipated error at operation level. Please note that the population size should be adapted to the operation level and that the standard-deviation of errors and anticipated error may also be adapted based on historical data and professional judgment if there is any information or expectation that would suggest adapting these parameters to the operation reality.

Prior information and experience based on previous years audits has suggested a standard deviation of errors around 8,800 €. Using the same confidence level and the expected error rate as the ones used at population level, 90% and 0.4%, respectively, the AA is able to compute, for example, de sample size for Operation ID 243:

$$n_i = \left(\frac{629 \times 1.645 \times 8{,}800}{1{,}802{,}856 - 360{,}571}\right)^2 \approx 40,$$

which are going to be drawn an equal probabilities design (simple random sampling). As the conditions referred in section 6.1.1.3 are fulfilled, ratio estimation is elected as the projection approach. The following table summarises the results:

Operation ID	Book value	No. payment claims	Audited expenditure	Amount of error in sampled payment claims	Projected error (ratio estimation)
243	90,142,818 €	629	7,829 €	845 €	9,729,299 €
6324	89,027,451 €	1239	1,409 €	76 €	4,802,048 €
734	79,908,909 €	729	56,729 €	1,991 €	2,804,538 €
451	79,271,094 €	769	48,392 €	3,080 €	5,045,358 €
95	89,771,154 €	2839	3,078 €	81 €	2,362,399 €
9458	100,525,834 €	4818	67,128 €	419 €	627,463 €
849	165,336,715 €	1972	12,345 €	1,220 €	16,339,473 €
872	92,853,106 €	1256	29,735 €	1,544 €	4,821,429 €
Total	**786,837,081 €**	**14251**	**226,645 €**	**9,256 €**	**46,532,007 €**

The projected error for this 100% audit stratum amounts to 46,532,007 €

The sampling interval for the remaining population is equal to the book value in the non-exhaustive stratum (BV_s) (the difference between the total book value and the book value of the eight operations belonging to the top stratum) divided by the number of operations to be selected (77 minus the 8 operations in the top stratum).

$$Sampling\ interval = \frac{BV_s}{n_s} = \frac{4{,}199{,}882{,}024 - 786{,}837{,}081}{69} = 49{,}464{,}419$$

The sample is selected from a randomised list of operations, selecting each item containing the 49,464,419th monetary unit.

A file containing the remaining 3,844 operations (3,852 – 8 high value operations) of the population is randomly sorted and a sequential cumulative book value variable is created. A sample value of 69 operations (77 minus 8 high value operations) is drawn using exactly a systematic selection algorithm as described in section 6.3.1.3. The AA determines the sample size of payment claims to be audited in each selected operation exactly has have been done before.

The following table summarises the results of the audit of the 69 operations selected in the first stage:

Book value	**No. payment claims**	**Audited expenditure**	**Amount of error in sampled payment claims**	**Projected error**	**Error rate**
901,818 €	689	616,908 €	58,889 €	86,086 €	0.0955
89,251 €	1989	59,377 €	4,784 €	7,191 €	0.0806
799,909 €	799	308,287 €	17,505 €	45,421 €	0.0568
792,794 €	369	504 €		0 €	0.0000
8,971,154 €	1839	8,613,633 €	406,545 €	423,419 €	0.0472
…	…	…	…	…	…
1,525,348 €	5618	1,483,693 €	74,604 €	76,699 €	0.0503
1,653,365 €	1272	82,240 €	1,565 €	31,461 €	0.0190
853,106 €	1396	69,375 €		0 €	0.0000
…	…	…	…	…	…
Total					**1.034**

For the remaining sample, the error has a different treatment. For these operations, we follow the following procedure:

1) for each unit in the sample calculate the error rate, i.e. the ration between the error and the respective expenditure $\frac{E_i}{BV_i}$; in this case the error rates have been calculated using subsamples of payment claims, but for the purpose of this projection they are treated as if they are the true ones

2) sum these error rates over all units in the sample

3) multiply the previous result by the sampling interval (SI)

$$EE_s = SI \sum_{i=1}^{n_s} \frac{E_i}{BV_i}$$

$$EE_s = 49{,}464{,}419 \times 1.034 = 51{,}146{,}209$$

The projected error at the level of population is just the sum of these two components:

$$EE = 46{,}532{,}007 + 51{,}146{,}209 = 97{,}678{,}216$$

The projected error rate is the ratio between the projected error and the total expenditure:

$$r = \frac{97{,}678{,}216}{4{,}199{,}882{,}024} = 2.33\%$$

As the projected error is larger than the maximum tolerable error, the AA is able to conclude that the population contains material error.

7.7 Recalculation of the confidence level

When after performing the audit, the AA finds that the projected error is lower than the materiality level but the upper limit is larger than that threshold, it may want to recalculate the confidence level that would generate conclusive results (i.e. to have both the projected error and the upper limit below materiality).

When this recalculated confidence level is still compatible with an assessment of the quality of the management and control systems (see table in Section 3.2), it will be perfectly safe to conclude that the population is not materially misstated even without carrying out additional audit work. Therefore, only in situations where the recalculated confidence is not acceptable (not in accordance with the assessment of the systems) is necessary to proceed with the additional work suggested in Section 4.12.

The recalculation of the confidence interval is performed as follows:

- Calculate the materiality level in value, i.e. the materiality level (2%) times the total book value of the population.

- Subtract the projected error (EE) from the materiality value.

- Divide this result by the precision of the projection (SE). This precision is dependent on the sampling method and presented in the sections devoted to the presentation of the methods.

- Multiply the above result by the z parameter used both for sample size and precision calculation and obtain a new value z^*

$$z^* = z \times \frac{(0.02 \times BV) - EE}{SE}$$

- Look for the confidence level associated to this new parameter (z^*) in a table of the normal distribution (in appendix). Alternatively you can use the following excel formula "=1-(1-NORMSDIST(z^*))*2".

Example: after auditing a population with a book value of 1,858,233,036€ and a confidence level of 90% (corresponding to $z = 1.645$, cf. Section 5.3), we have obtained the following results

Characteristic	**Value**
BV	1,858,233,036€
Materiality (2% of BV)	37,164,661€
Projected error (EE)	14,568,765€ (0.8%)
Precision (SE)	26,195,819€ (1.4%)
Upper error limit (ULE)	40,764,584€ (2.2%)

The new z^* parameter is obtained as

$$z^* = 1.645 \times \frac{37{,}164{,}661€ - 14{,}568{,}765€}{26{,}195{,}819€} = 1.419$$

Using the MS Excel function "=1-(1-NORMSDIST(1.419))*2", we obtain the new confidence level 84,4%.

Being this recalculated confidence level compatible with the assessment about the quality of the management and control systems, one can conclude that the population is not materially misstated.

7.8 Strategies for auditing groups of programmes and multi-fund programmes

7.8.1 Introduction

Frequently the AA decide to group two or more operational programmes that share a common system in order to be able to select one single sample representative of the grouped population.

Also, in some cases the operational programme is co-financed by more than one fund. In these cases, also one single sample may be selected and results may be projected for the group of operations.

In both cases a single opinion should be disclosed for the group of OPs or the different funds, but different sampling strategies are possible to achieve this goal and the sample strategy may take into consideration this heterogeneity in the population. This may be performed using stratification (by OP or fund) and also taking into consideration the levels of representativeness that are desired when calculating the samples sizes.

The two typical alternative strategies are:

- Select one single sample;
- Use different samples (associated to different strata) for each OP or each Fund.

If one selects one single sample, the sample size is calculated for the whole group (with no distinction between OPs or Funds). This option, also called top-down approach, will allow a smaller sample size, but the sample is only guaranteed to be representative of the "grouped" population. This means that the sample results may be projected to the group of OPs or the different Funds, but usually won’t allow any projection for the individual Funds or the individual programmes. Although only planned to be representative of the grouped population it is advisable to have the sample stratified by fund (or OP). If this is the case, the global sample size is firstly calculated and subsequently allocated between strata only after global sample size is calculated. The sample size calculation and allocation uses the usual strategies that have been previously proposed for the several stratified sampling designs.

The following figure summarizes this strategy:

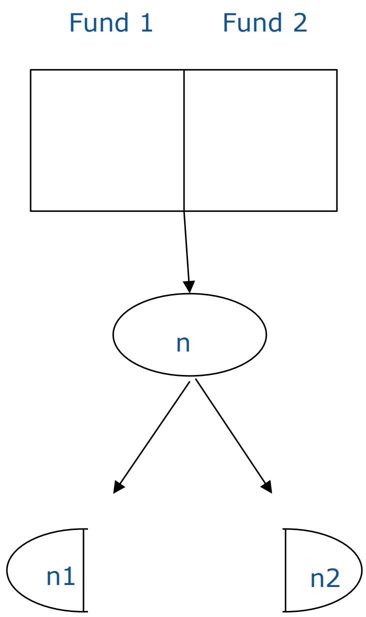


Fig. 8 Top-down strategy

If one uses different samples (one for each OP or fund) then the sample sizes are calculated separately for each stratum (OP or fund). This option, also called bottom-up approach, will generate a larger sample size (as several samples have to be selected), but the sample is guaranteed to be representative not only of the "grouped" population, but also of each stratum (OP or fund). This means that the sample results may be projected to the group of OPs or the group of Funds, and they may also be projected for the individual funds or the individual programmes allowing to obtain conclusive results at the stratum level. These samples should of course be stratified by fund (or OP). In this strategy, the global sample size will simply be the sum of the sample sizes obtained for the calculation at each stratum.

The following figure summarizes this strategy:

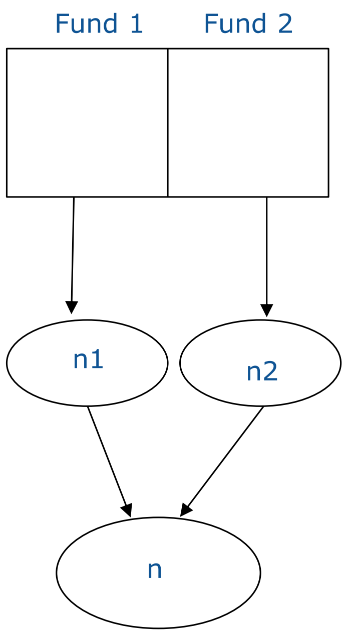


Fig. 9 Bottom-up strategy

It results from what has been presented that the approach based on one single sample (top-down approach) has the main advantage to allow a smaller sample size but as main disadvantage the fact that it does not ensure *a priori* representativeness by stratum (i.e. separate conclusions by stratum may not be possible). If the AA doesn't expect to need to extrapolate the results at stratum level, this will certainly be the suggested option.

The strategy based on different samples allows the projection at stratum level but it will have a significantly increased sample size. Therefore, it is advisable when significantly different results are expected by OP or Fund, in order to ensure representativeness of results by stratum and therefore differentiated conclusions.

It is also important to note that when the sample is only designed to ensure representativeness of the "grouped" population, it may be still possible to project results by stratum or at least for some strata, under the following conditions:

- Each stratum has at least 30 observations (advisable to foresee this sample size from the start);
- The precision for each stratum is adequate to achieve conclusive results (relation between upper error limit and the 2% threshold).

When using this strategy and when calculated *a posteriori*, the results will often be representative for some strata (typically the larger ones) but not for others (typically the smallest ones), i.e. they will allow to produce conclusive projections only for some strata. For example if the population is co-financed by two Fund and to one of the Funds corresponds the major proportion of expenditure, the sample will typically be representative of this larger Fund, but not of the other. If this happens, i.e. if results are conclusive (representative) for some strata but not for others, additional work can still be done in order to obtain representative results for all the strata. This can be achieved through the selection of an additional sample for the stratum without representative results that combined with the original one will provide conclusive results. The strategy is not different from the one already presented in the Section 7.2. Also, the recalculation of confidence level (Section 7.7) may be an option in order to obtain representative results at the stratum level.

As a summary, one could recommend the following strategy:

- when the AA plans to project results at stratum level, it should use the bottom up approach;
- when AA plans to project results at population level (for the group of OPs or Funds), and believes that no projections will be necessary at stratum level, it may use the top-down approach;
- when AA does not have a clear decision about the strategy, it may use the top-down approach but introduce some "over-sampling" of the smaller strata allowing at least 30 observations for those strata. By doing this it will increase the chance of having representative results. Additionally, if the results are not representative, by over-sampling the smallest strata the AA will reduce the amount of additional work that will be necessary in order to be able to conclude about these strata.

7.8.2 Example

Let us assume a population of expenditure declared to the Commission in a given reference period for operations in a group of programmes. The management and control system is common to the group of programmes and the system audits carried out by the Audit Authority have yielded a moderate assurance level. Therefore, the audit authority decided to carry out audits of operation using a confidence level of 80%. The audit authority only foresees to issue one single opinion about the grouped population, reason why it decides to use a top-down approach, i.e. to use a stratified sample by programme, but only ensuring the representativeness at the aggregated level.

The AA has reasons to believe that there are substantial risks of error for high value operations, whatever the programme they belong to. Moreover, there are reasons to expect that there are different error rates across the programmes. Bearing in mind all this information, the AA decides to stratify the population by programme and by expenditure (isolating in a 100% sampling stratum all the operations with book value larger than a cut-off level of 3% of the whole expenditure).

The following table summarizes the available information.

Population size (number of operations)	6,723
Population size – stratum 1 (number of operations in programme 1)	4,987
Population size – stratum 2 (number of operations in programme 2)	1,728
Population size – stratum 3 (number of operations with BV > materiality level)	8
Book value (sum of the expenditure in the reference period)	123,987,653 €
Book value – stratum 1 (total expenditure in programme 1)	85,672,981 €
Book value – stratum 2 (total expenditure in programme 2)	19,885,000 €
Book value – stratum 3 (total expenditure of operations with BV > Materiality level)	18,429,672 €

The high-value projects will be excluded from sampling and will be treated separately. The amount of error found in these 8 operations amounts to 2,975 €.

Population size (number of operations)	6,723
Book value (total declared expenditure in the reference period)	123,987,653 €
Cut-off value	3,719,630
Number of units above cut-off value	8
Population book value above cut-off	18,429,672 €
Remaining population size (no. of operations)	6,715
Remaining population value	105,557,981 €

The first step is to compute the required sample size, using the formula:

$$n = \left(\frac{N \times z \times \sigma_w}{TE - AE}\right)^2$$

where z is 1.282 (coefficient corresponding to a 80% confidence level) and TE, the tolerable error, is 2% (maximum materiality level set by the Regulation) of the book value, i.e. 2% x 123,987,653 € = 2,479,753 €. Based either on previous year experience and on the conclusions of the report on managing and control systems the audit authority expects an error rate not larger than 1.4%, Thus, AE, the anticipated error, is 1.4% of the total expenditure, i.e., 1.4% x 123,987,653 € = 1,735,827 €.

A preliminary sample of 20 operations of programme 1 yielded a preliminary estimate for the standard deviation of errors of 1,008 €. The same procedure was followed for the population of programme 2. The estimate of the standard deviation of errors of 876 €:

Therefore, the weighted average of the variances of the errors of these two strata is

$$\sigma_w^2 = \frac{4{,}987}{6{,}715} 1{,}008^2 + \frac{1{,}728}{6{,}715} 876^2 = 950{,}935$$

The sample size is given by

$$n = \left(\frac{6{,}715 \times 1.282 \times \sqrt{950{,}935}}{2{,}479{,}753 - 1{,}735{,}827} \right)^2 \approx 128$$

The total sample size is given by these 128 operations plus the 8 operations of the exhaustive stratum, that is, 136 operations.

The allocation of the sample by stratum is as follows:

$$n_1 = \frac{N_1}{N_1 + N_2} \times n = \frac{4{,}987}{6{,}715} \times 128 \approx 95,$$

$$n_2 = n - n_1 = 33$$

and

$$n_3 = N_3 = 5$$

Auditing 95 operations in programme 1, 33 operations in programme 2 and 8 operations in stratum 3 will provide the auditor with a total error for the sampled operations. The previous preliminary samples of 20 units in programmes 1 and 2 are used as part of the main sample. Therefore, the auditor has only to randomly select 75 further operations in programme 1 and 13 in programme 2. In order to identify whether the mean-per-unit or ration estimation is the best estimation method, the AA calculates the ratio of covariance between the errors and the book values to the variance of the book values of the sampled operations, which is equal to 0.0109, for programme 1. As the ratio is smaller than the half of the sample error rate, the audit authority can be sure than mean-per-unit estimation is a reliable estimation method. This was also confirmed for programme 2 stratum.

The following table shows the sample results the operations audited:

Sample results – Programme 1		
A	Sample book value	1,667,239 €
B	Sample total error	47,728 €
C	Sample average error (C=B/95)	502.4 €
D	Sample standard deviation of errors	674 €
Sample results – Programme 2		
E	Sample book value	404,310 €
F	Sample total error	3,298 €
G	Sample average error (G=F/33)	100 €
H	Sample standard deviation of errors	1,183 €
Sample results – exhaustive stratum		
I	Sample book value	18,429,672
J	Sample total error	2,975 €

Extrapolating the error for the two sampling strata is done by multiplying the sample average error by the population size. The sum of these two figures has to be added to the error found in the 100% sampling strata, in order to project error to the population:

$$EE \quad = \sum_{h=1}^{3} N_h \times \frac{\sum_{i=1}^{n_h} E_i}{n_h} = 4{,}987 \times 502 + 1{,}728 \times 100 + 2{,}975 = 2{,}681{,}139$$

The projected error rate is computed as the ratio between the projected error and the book value of the population (total expenditure). Using the mean-per-unit estimation the projected error rate is

$$r_1 = \frac{2{,}681{,}139}{123{,}987{,}653} = 2.16\%.$$

The projected error is larger than the materiality level. Therefore, the AA can be reasonable sure that the population contains material error. However, the audit work has raised suspicions that the errors may be particularly concentrated in one of the programmes. Indeed, the AA suspects that programme 1 is the responsible for this result. The AA decides to assess the results at programme level. The following table summarises the characteristics of populations at programme level:

		Programme 1	Programme 2
(A)	Total book value (declared expenditure in the reference period in low-value stratum)	85,672,981 €	19,885,000 €
(B)	Total book value (declared expenditure in the reference period in high-value stratum)	12,286,448 €	6,143,224 €
(C)	Population size (number of operations in low-value stratum)	4987	1728
(D)	Population size (number of operations in high-value stratum)	6	2

The following table summarises the results of the whole sample by programme:

		Programme 1 (low-value stratum)	Programme 2 (low-value stratum)
(E)	Audited expenditure	1,667,239 €	404,310 €
(F)	Sample size (number of operations)	95	33
(G)	Sample total error	47,728 €	3,298 €
(H)	Sample average error	502.4 €	100 €
(I)	Sample standard deviation of errors	674 €	1,183 €

Besides the information belonging to the low-value strata the AA must consider the information on the exhaustive stratum. The following table summarises the results:

		Programme 1 (exhaustive stratum)	Programme 2 (exhaustive stratum)
(J)	Audited expenditure	12,286,448 €	6,143,224 €
(K)	Sample total error	1,983 €	992 €

Using these data the AA is able to project error rates and compute precision at programme level. The following table summarises the results for mean-per-unit estimation:

		Programme 1	Programme 2
(L)	Precision:= $(C) \times 1.282 \times \frac{(I)}{\sqrt{(F)}}$	442,105 €	456,204 €
(M)	Projected error (mean-per-unit estimation):= $(C) \times (H) + (K)$	2,507,452 €	173,687 €
(N)	Upper limit of error:= $(M) + (L)$	2,949,557 €	629,892 €
(O)	Projected error rate (%):=$\frac{(M)}{(A)+(B)}$	2.56%	0.67%
(P)	Upper limit of the projected error rate:= $\frac{(N)}{(A)+(B)}$	2.90%	2.42%

The results for programme 1 seem to be conclusive as the projected error is larger than the maximum tolerable error (computed at programme level, that is 2% of 97,959,429 €). This conclusion is obvious just by looking to the projected error rate (above 2% of materiality level). Nevertheless, the results for programme 2 are not fully conclusive. Indeed, although the projected error is below materiality level (2% of 26,028,224 €), the upper limit of error is larger than it, giving a clear indication that additional analysis would be needed to reach a definite conclusion. Using data of the programme 2, 33 sampled operations (excluding 2 operations of the exhaustive stratum), the AA decides to plan the adequate sample. The following table summarises the information needed for planning the sample size:

	Programme 2
Total book value (declared expenditure in the reference period excluding exhaustive stratum operations)	19,885,000 € (excluding expenditure of 2 operations in exhaustive stratum)
Population size (number of operations, including exhaustive stratum)	1728 (excluding 2 operations of exhaustive stratum)
Materiality level	2%
Maximum tolerable error	397,700 €
Expected error rate	0,6%
Expected error	119,310 €
Sample standard deviation of errors	1,183 €

The planned sample size to obtain reliable results is therefore:

$$n = \left(\frac{1{,}728 \times 1.282 \times 1{,}183}{397{,}700 - 149{,}138}\right)^2 \approx 89$$

The AA is able to have definitive results on Programme 2, using the previous 33 operations and drawing an additional sample of 56 operations. The following table summarises the results of all 89 operations (including the 33 operations of the first sample):

		Programme 2 (low-value stratum)
(E1)	Audited expenditure	1,236,789 €
(F1)	Sample size (number of operations)	89
(G1)	Sample total error	8,278 €
(H1)	Sample average error	93 €
(I1)	Sample standard deviation of errors	1,122 €

The calculations made by the AA are reproduced in the following table:

		Programme 2
(L1)	Precision (mean-per-unit estimation):= $(C) \times 1.282 \times \frac{(I1)}{\sqrt{(F1)}}$	263,469 €
(M1)	Projected error (mean-per-unit estimation):= $(H1) \times (C) + (K)$	161,715 €
(N1)	Upper limit of error:= $(M1) + (L1)$	425,184 €
(O1)	Projected error rate (%):=$\frac{(M1)}{(A)+(B)}$	0.62%
(P1)	Upper limit of the projected error rate:= $\frac{(N1)}{(A)+(B)}$	1.63%

With the results of this extended sample (89 operations) the AA is able to conclude that population of declared expenditure of Programme 2 is not material misstated.

7.9 Sampling technique applicable to system audits

7.9.1 Introduction

Article 62 of Council Regulations (EC) N_o 1083/2006 states: "The audit authority of an operational programme should be responsible in particular for: (a) ensuring that audits are carried out to verify the effective functioning of the management and control system of an operational programme…". These audits are called system audits. System audits aim at testing the effectiveness of controls in the management and control system and concluding on the assurance level that can be obtained from the system. Whether or not to use a statistical sampling approach for the test of controls is a matter of professional judgement regarding the most efficient manner to obtain sufficient appropriate audit evidence in the particular circumstances.

Since for system audits the auditor's analysis of the nature and cause of errors is important, as well as, the mere absence or presence of errors, a non-statistical approach could be appropriate. The auditor can in this case choose a fixed sample size of the items to be tested for each key control. Nonetheless, professional judgment will have to be used in applying the relevant factors[63] to consider. If a non-statistical approach is used then the results cannot be extrapolated.

Attribute sampling is a statistical approach which can help the auditor to determine the level of assurance of the system and to assess the rate at which errors appear in a sample. Its most common use in auditing is to test the rate of deviation from a prescribed control to support the auditor's assessed level of control risk. The results can then be projected to the population.

As a generic method encompassing several variants, attribute sampling is the basic statistical method to apply in the case of system audits; any other method that can be applied to system audits will be based on the concepts developed below.

Attribute sampling tackles binary problems such as yes or no, high or low, true or false answers. Through this method, the information relating to the sample is projected to the population in order to determine whether the population belongs to one category or the other.

The Regulation does not make it obligatory to apply a statistical approach to sampling for control tests in the scope of a systems audit. Therefore, this chapter and the related annexes are included for general information and will not be developed further.

For further information and examples related to the sampling techniques applicable to system audits, please refer to the specialized audit sampling literature.

When applying attribute sampling in a system audit, the following generic six-step plan should be applied.

1. Define the test objectives: for instance, determine whether the error frequency in a population meets the criteria for a high assurance level;
2. Define the population and sampling unit: for instance the invoices allocated to a programme;

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[63] For further explanation or examples see “Audit Guide on Sampling, American Institute of Certified Public Accountants, 01/04/2001”.

3. Define the deviation condition: this is the attribute being assessed, e.g. the presence of a signature on the invoices allocated to an operation within a programme;
4. Determine the sample size, according to the formula below;
5. Select the sample and carry out the audit (the sample should be selected randomly);
6. Evaluate and document the results.

7.9.2 Sample size

Computing sample size n within the framework of attribute sampling relies on the following information:

- Confidence level and the related coefficient z from a normal distribution (see Section 5.3)
- Maximum tolerable deviation rate, **T**, determined by the auditor; the tolerable levels are set by the Member State audit authority (e.g. the number of missing signatures on invoices under which the auditor considers there is no issue);
- The anticipated population deviation rate, p, estimated or observed from a preliminary sample. Note that the tolerable deviation rate should be higher than the expected population deviation rate, as, if that is not the case, the test has no purpose (i.e. if you expect an error rate of 10%, setting a tolerable error rate of 5% is pointless because you expect to find more errors in the population than you are willing to tolerate).

The sample size is computed as follows[64]:

$$n = \frac{z^2 \times p \times (1-p)}{T^2}.$$

<u>Example</u>: assuming a confidence level of 95% ($z = 1.96$), a tolerable deviation rate (T) of 12% and an expected population deviation rate (p) of 6%, the minimum sample size would be

$$n = \frac{1.96^2 \times 0.06 \times (1-0.06)}{0.12^2} \approx 16.$$

Note that the population size has no impact on the sample size; the calculation above slightly overstates the required sample size for small populations, which is accepted.

[64] When dealing with a small population size, i.e. if the final sample size represents a large proportion of the population (as a rule of thumb more than 10% of the population) a more exact formula can be used leading to $n = \frac{z^2 \times p \times (1-p)}{T^2} \Big/ \left(1 + \frac{z^2 \times p \times (1-p)}{N.T^2}\right)$.

Ways to reduce the required sample size include reducing the confidence level (i.e. raising the risk of assessing the control risk too low) and raising the tolerable deviation rate.

7.9.3 Extrapolation

The number of deviations observed in the sample divided by the number of items in the sample (i.e. the sample size) is the sample deviation rate:

$$EDR = \frac{\#\ of\ deviations\ in\ the\ sample}{n}$$

This is also the best estimator of the extrapolated deviation rate (EDR) one can obtain from the sample.

7.9.4 Precision

Remember that precision (sampling error) is a measure of the uncertainty associated with the projection (extrapolation). The precision is given by the following formula

$$SE = z \times \frac{p_s \times (1 - p_s)}{\sqrt{n}}$$

where p_s is the ratio of number of deviations observed in the sample to the sample size, the sample deviation rate.

7.9.5 Evaluation

The achieved upper deviation limit is a theoretical figure based on the sample size and the number of errors encountered:

$$ULD = EDR + SE.$$

It represents the maximum error rate of the population at the defined confidence level and results from binomial tables (for instance, for sample size 150 and an observed amount of deviations of 3 (sample deviation rate of 2%), the maximum deviation rate (or achieved upper deviation limit) at a 95% confidence level is:

$$ULD = \frac{3}{150} + 1.96 \times \frac{\frac{3}{150} \times \left(1 - \frac{3}{150}\right)}{\sqrt{150}} = 0.023.$$

If this percentage is higher than the tolerable deviation rate, the sample does not support the assumed expected error rate of the population at that confidence level. The logical conclusion is therefore that the population does not meet the criterion set of high assurance level and must be classified as having an average or low assurance level. Note that the threshold at which low, average or high assurance is reached is defined by the AA.

7.9.6 Specialised methods of attribute sampling

Attribute sampling is a generic method, and therefore some variants have been designed for specific purposes. Among those, discovery sampling and stop-or-go sampling serve specialised needs.

Discovery sampling aims at auditing cases where a single error would be critical; it is therefore particularly geared towards the detection of cases of fraud or avoidance of controls. Based on attribute sampling, this method assumes a zero (or at least very low) rate of error and is not well suited for projecting the results to the population, should errors be found in the sample. Discovery sampling allows the auditor to conclude, based on a sample, whether the assumed very low or zero error rate in the population is a valid assumption. It is not a valid method for assessing the level of assurance of internal controls, and therefore is not applicable to system audits.

Stop-or-go sampling comes out of the frequent need to reduce the sample size as much as possible. This method aims at concluding that the error rate of the population is below a predefined level at a given confidence level by examining as few sample items as possible – the sampling stops as soon as the expected result is reached. This method is also not well-suited for projecting the results to the population, though it can be useful for assessing system audit conclusions. It can be used when the outcome of system audits is questioned, to check whether the criterion is indeed reached for the assurance level provided.

7.10 Proportional control arrangements under the programming period 2014-2020 – implications for sampling

7.10.1 Restrictions to sample selection imposed by Article 148(1) CPR

The proportional control arrangements established by Article 148(1) CPR intend to ease the administrative burden for beneficiaries and avoid that they are audited several times by different bodies and occasionally even on the same expenditure. These arrangements are summarized below and have implications for the AA's work:

a) In the case of operations for which the total eligible expenditure does not exceed **EUR 100 000 (EMFF), 150 000 (ESF) or 200 000 (ERDF and Cohesion Fund)**, only one audit by either the audit authority or the Commission can be carried out prior to the submission of the accounts for the accounting year in which the operation is completed;

b) In the case of operations for which the total eligible expenditure exceeds **EUR 100 000 (EMFF), 150 000 (ESF) or 200 000 (ERDF and Cohesion Fund)**, one audit per accounting year can be carried out by either the audit authority or the Commission prior to the submission of the accounts for the accounting year in which the operation is completed;

c) No audit can be carried out by the AA or the Commission in any year if there has already been an audit in that year by the European Court of Auditors, provided that the results of the audit work performed by the European Court of Auditors for such operations can be used by the audit authority or the Commission for the purpose of fulfilling their respective tasks.

To decide whether this Article applies, the assessment of the level of the "total eligible operation expenditure" is to be done on the basis of the amount in the grant agreement, as the exact expenditure that will be declared during the programming period is not known in advance.

Article 148(4) CPR foresees that the AA and the Commission may still audit the operations subject to the above-mentioned conditions (in the event that a risk assessment or an audit by the European Court of Auditors establishes a specific risk of irregularity or fraud or in the case of evidence of serious deficiencies in the effective functioning of the management and control system of the operational programme concerned during the period referred to in Article 140(1).) **In particular, for AA, this means that the provisions of Article 148(1) do not apply in the case of risk-based complementary audit samples.**

Article 148(1) CPR introduces some practical challenges for the AA's work, namely in regard to the strategy to be adopted for the sample selection, having in mind the general rule set out in Article 127(1) CPR. This provision states that the AA shall ensure that audits are carried out on "an appropriate sample of operations on the basis of the declared expenditure" and, in the case of the use of non-statistical sampling, a sufficient size of the sample to enable the AA to draw a valid audit opinion. Section 7.10.2 below provides clarification in regard to the adjustments to bring to the sampling methodology under Article 148 arrangements.

The AA could carry out its audit in relation to an accounting year either after the accounting year within one-period sampling procedure or in phases, using two- or multi-period sampling design.

In the context of one-period sampling, the fact that the AA (or the EC) audits in one year operations under the thresholds above mentioned implies that these operations cannot be audited by the AA in subsequent years prior to the submission of the accounts for the accounting year in which the operation is completed, unless Article 148(4) CPR applies.

In the context of multi-period sampling in relation to an accounting year and where expenditure for the same operation is selected more than once for that year, the AA may consider that the audit of an individual operation in two (or more) stages. This means that if any operation was selected for sampling in one sampling period of the accounting year, the AA would keep the operation in the population to be sampled and audited for the following sampling periods of the same accounting year. In this case replacement or exclusion of operations are not applicable since there is a single audit, which work is spread over different moments referring to the same year. As after the sample selection for the first sampling period the AA cannot predict whether the selected operations will be selected for audit of expenditure on any other sampling period of that accounting year, it is recommended that the AA informs the concerned beneficiaries on the fact that their operations have been selected for an audit concerning the relevant accounting year and on the possibility for the operation to be audited in different phases. This requires a clarification in the letter to the MA/beneficiary announcing that the operation has been selected for audit.[65]

Article 148(1) CPR specifies that one audit per accounting year can be carried out in regard to operations exceeding the relevant thresholds. This requirement is interpreted as one audit referring to the expenditure declared within an accounting year and not as one audit in the period of an accounting year.

In order to avoid the administrative burden for the beneficiary of more than one on-the-spot visit for the same operation, the AA may decide to continue the subsequent phases of the audit following the first verifications at the level of the Managing Authority/Intermediate Body, provided that the supporting documentation can be verified on the files kept by these bodies.

Operations audited by ECA:

In addition to the first two conditions set under Article 148(1) CPR, this provision goes on establishing that the AA cannot carry out an audit of an operation if this has been

[65] The AAs are recommended to introduce the following (or similar) text in letters announcing an audit in the framework of two- or multi-period sampling designs: "Your operation has been selected for an audit by the programme's audit authority related to expenditure declared by the national authorities to the European Commission in the accounting year July 20xx to June 20xx. You are informed that this audit may be spread over more than one audit phase, during the upcoming months. You will be informed at a later stage if the audit will be restricted to expenditure declared for the first semester *(other sampling period)* or will include also expenditure related to the second semester *(other sampling period)*."

audited in the same year by the ECA and the AA can use the conclusions drawn by this institution.

This provision also brings practical challenges to the AA, in particular when the ECA's conclusions on the audit of the selected operations are not available in time for the AA to assess those conclusions and to decide whether they can be used for the purposes of the AA's audit opinion. In addition, it may happen that the ECA's conclusions relate to a reference period for expenditure declared different from the one on which the AA needs to draw an audit opinion, thus meaning that the ECA's conclusions cannot be used by AA for that purpose.

If indeed there are ECA conclusions on the audit of the operation selected by the AA available in due time for the AA to draw the relevant audit opinion, the AA uses the results of the audit work performed by the ECA to determine the error for that operation, when it agreed with the conclusions and without the need to re-perform audit procedures.

7.10.2 Sampling methodology under proportionate control arrangements

Sample selection

As stated in Article 28(8) CDR: *"Where conditions for the proportional control provided for in Article 148(1) of Regulation (EU) No 1303/2013 apply, the audit authority may exclude the items referred to in that Article from the population to be sampled. If the operation concerned has already been selected in the sample, the audit authority shall replace it using an appropriate random selection."*

As follows from the provisions of this article, the AA could use for sample selection either the original positive population of expenditure declared or a reduced population, i.e. population from which sampling units subject to Article 148 CPR are excluded.

In the case of replacement of the operations/other sampling units at stake, these sampling units should be replaced in the sample by selecting an additional sample with a size equal to the number of the operations replaced. The "replacement units" should be selected using the same methodology as for the original sample. In particular, within PPS methods (i.e. MUS and PPS non-statistical sampling), the additional sampling units should be selected using probability proportional to size selection. Examples of selection are included in section 7.10.3.1.

In the case of both replacement and exclusion, the sample size is calculated based on the population parameters (such as book value, number of sampling units) corresponding to the original population (i.e. population including operations/other sampling units affected by Article 148(1) CPR). The standard respective formulas for sample size calculation (presented in section 6 of the guidance) are used.

The decision to use either exclusion or replacement of sampling units should be taken by the AA based on professional judgement. The AA could consider it more practical to apply replacement of operations for populations with small number of sampling units (simple random sampling) or small part of expenditure (MUS) affected by Article 148, as the probability of selection of such units (and related technical implications of replacement) is low. On the contrary, in the case of populations with large number of sampling units/expenditure subject to Article 148, replacement would be more frequent and sometimes needed to be repeated several times. Consequently, in such cases the AA could consider it more practical to apply exclusion of population units subject to Article 148 CPR from the population to be sampled, to avoid replacements of sampling units.

Projection of errors

The AA needs to draw an audit opinion on the total expenditure declared, as follows from Article 127(1) CPR. Hence, even if the population from which the sample has been drawn corresponds to the expenditure declared reduced by the expenditure relating to the operations affected by Article 148, there is still a need to calculate the total error for the expenditure declared, for the purposes of drawing-up the audit opinion on this expenditure.

This can be achieved in two different ways. Firstly, in the projection formulas, the population size $N_{(h)}$ and the population book value $BV_{(h)}$ are the ones corresponding to the original population (i.e. the population including the sampling units affected by Article 148). In such a case the projection of the error will be performed to the original population (by stratum) and no further action needs to be done. It is a recommended approach in particular in the case of replacement of operations/other sampling units.

Alternatively, this may be done in two stages: first, in the projection formulas, the population size $N_{(h)}$ and the population book value $BV_{(h)}$ are the ones related to the reduced population (i.e. obtained after deducting the population units affected by Article 148 CPR). After projecting the error in this way, this projected error would be multiplied by the ratio between expenditure declared in the original population and expenditure declared in the reduced population $\frac{BV_{(h)\, original\, population}}{BV_{(h)\, reduced\, population}}$ in order to obtain the total projected error of the original population (typically in MUS and in simple random sampling with ratio estimation). This projection from the reduced to the original population may also be performed by multiplying the error of the reduced population by the ratio between the population size of the original population and the population size of the reduced population $\frac{N_{(h)\, original\, population}}{N_{(h)\, reduced\, population}}$ (typically in simple random sampling with mean-per-unit estimation). This proceeding carried out in two stages is in particular a recommended approach in the case of exclusion of operations/other sampling units.

Similarly, the precision could also be calculated either in regard to the original population $SE_{(h)\ original}$ or to the reduced population $SE_{(h)\ reduced}$ (see however some restrictions presented in the tables below). In case the precision is calculated for the reduced population, it should be in the next stage adjusted to reflect the original population.

Similarly as in the case of projection of error, this adjustment is carried out by multiplying the precision for the reduced population by the ratio $\frac{BV_{(h)\ original\ population}}{BV_{(h)\ reduced\ population}}$ (in the case of MUS and simple random sampling with ratio estimation) or by the ratio $\frac{N_{(h)\ original\ population}}{N_{(h)\ reduced\ population}}$ (in the case of in simple random sampling with mean-per-unit estimation).

It is not possible to identify a methodology that is always more suitable than the others (for example projecting and calculating precision in regard to the original or to the reduced population) as some sampling methods could impose some technical restrictions in this regard.

The tables below include a summary of approaches to sample selection, projection of errors and calculation of sample precision under restrictions imposed by principles of proportional control arrangements.

a) MUS standard approach

Sampling design	**MUS standard: Exclusion of sampling units**	**MUS standard: Replacement of sampling units**
Parameters used for sample size calculation	Correspond to the original population.	Correspond to the original population.
Population used for sample selection	Reduced population	Original population
Recommended approach to projection of error and precision calculation	Projection of error and precision calculation for the reduced population, in the next stage adjusted to reflect the original population. The adjustment may be performed by multiplying the projected error and precision by the ratio between expenditure $BV_{(h)\ original}$ of the original population and the expenditure $BV_{(h)\ reduced}$ of the reduced population. In the case of units of high-value stratum affected by Article 148 (or any other exhaustive stratum), there could be a need to calculate the error for the high-value stratum and to project this error to the units which were not audited in this stratum using the formula $EE_e = EE_{e\ reduced} \times \frac{BV_{e\ original}}{BV_{e\ reduced}}$ (where $EE_{e\ reduced}$ represents the amount of error in the sampling units of the high-value stratum audited, $BV_{e\ original}$ refers to book value of the original high-value stratum and $BV_{e\ reduced}$ refers to the book value of items in	Projection of error and precision calculation for the original population. The units of high-value stratum (or units of any other exhaustive stratum), which are excluded from the audit procedures due to Article 148 provisions should be replaced by the sampling units of the low-value stratum. In such a case there could be a need to calculate the error for the high-value stratum and to project this error to the units which were not audited in this stratum using the formula $EE_e = EE_{e\ reduced} \times \frac{BV_{e\ original}}{BV_{e\ reduced}}$ (where $EE_{e\ reduced}$ represents the amount of error in the sampling units of the high-value stratum audited, $BV_{e\ original}$ refers to book value of the original high-value stratum and $BV_{e\ reduced}$ refers to the book value of items in the high-value stratum which were subject to audit).

	the high-value stratum which were subject to audit.)	

b) MUS conservative approach

Sampling design	**MUS conservative: Exclusion of sampling units**	**MUS conservative: Replacement of sampling units**
Parameters used for sample size calculation	NA (sample size will remain the same regardless whether calculated with original population or reduced population parameters)	NA (sample size will remain the same regardless whether calculated with original population or reduced population parameters)
Population used for sample selection	Reduced population	Original population
Recommended approach to projection of error and precision calculation	Projection of error and precision calculation for the reduced population, in the next stage adjusted to reflect the original population. The adjustment may be performed by multiplying the projected error and precision by the ratio between expenditure $BV_{(h)\ original}$ of the original population and the expenditure $BV_{(h)\ reduced}$ of the reduced population. In the case of units of high-value stratum affected by Article 148, there could be a need to calculate the error for the high-value stratum and to project this error to the units which were not audited in this stratum using the formula $EE_e = EE_{e\ reduced} \times \frac{BV_{e\ original}}{BV_{e\ reduced}}$ (where $EE_{e\ reduced}$ represents the amount of error in the sampling units of the high-value stratum audited, $BV_{e\ original}$ refers to book value of the original high-value stratum and $BV_{e\ reduced}$ refers to the book value of items in the high-value stratum which were subject to audit.)	In view of technical issues related to error projection and precision calculation in the case of replacement of sampling units in MUS conservative approach, it is recommended to use exclusion of sampling units if MUS conservative approach is applied.[66]

c) Simple Random Sampling

Sampling design	**Simple Random Sampling: Exclusion of sampling units**	**Simple Random Sampling: Replacement of sampling units**
Parameters used for sample size calculation	Correspond to the original population.	Correspond to the original population.
Population used for sample selection	Reduced population	Original population
Recommended approach to projection of error and precision calculation	Projection of error and precision calculation for the reduced population, in the next stage adjusted to reflect the original population. When using mean-per-unit estimation, the adjustment may be performed by multiplying	Projection of error to the original population (both in the case of ratio estimation and mean-per-unit estimation).

[66] In case the AA decided to apply replacement in MUS conservative approach, advice of the Commission could be sought to determine the specific formulas to be applied and to obtain technical information in regard to sample selection and projection.

Sampling design	**Simple Random Sampling: Exclusion of sampling units**	**Simple Random Sampling: Replacement of sampling units**
	the projected error and precision by the ratio between population size $N_{(h)\ original}$ of the original population and $N_{(h)\ reduced}$ of the reduced population. When using ratio estimation, the adjustment may be performed by multiplying the projected error and precision by the ratio between expenditure $BV_{(h)\ original}$ of the original population and the expenditure $BV_{(h)\ reduced}$ of the reduced population. Projection of error can also be performed directly for the original population both in ratio estimation and in mean-per-unit estimation. Precision should not be calculated directly for the original population in the case of ratio estimation; it is only possible for mean-per-unit estimation. The precision calculated for reduced population in ratio estimation should be adjusted for the original population by multiplying the precision of the reduced population by the ratio $\frac{BV\ (h)\ original\ population}{BV\ (h)\ reduced\ population}$. In the case of units of high-value stratum (or any other exhaustive stratum) subject to Article 148, there could be a need to calculate an error for the high-value stratum and to project this error to the units which were not audited in this stratum. In the case of ratio estimation it would be performed using the formula $EE_e = EE_{e\ reduced} \times \frac{BV_{e\ original}}{BV_{e\ reduced}}$, where $EE_{e\ reduced}$represents the amount of error in the sampling units of the high-value stratum audited, $BV_{e\ original}$ refers to book value of the original high-value stratum and $BV_{e\ reduced}$ refers to the book value of items in the high-value stratum which were subject to audit. In the case of mean-per unit estimation it would be performed using the formula $EE_e = EE_{e\ reduced} \times \frac{N_{e\ original}}{N_{e\ reduced}}$, where $EE_{e\ reduced}$ represents the amount of error in the sampling units of the high-value stratum audited, $N_{e\ original}$ refers to the number of sampling units of the original high-value stratum and $N_{e\ reduced}$ refers to the number of sampling units of the high-value stratum audited.	Precision is calculated for the original population in the case of mean-per-unit estimation. In the case of ratio estimation, the precision has to be calculated for the reduced population (population from which all sampling items subject to Article 148 were deducted). Subsequently, it should be in the next stage adjusted to reflect the original population. It may be performed by multiplying the precision of the reduced population by the ratio between expenditure $BV_{(h)\ original}$ of the original population and the expenditure $BV_{(h)\ reduced}$ of the reduced population. It should be also noted that even if the AA did not select any sampling items affected by Article 148 in its sample, the precision in the case of ratio estimation will also have to be calculated to the reduced population and subsequently adjusted using the above mentioned formula. In the case of units of high-value stratum (or any other exhaustive stratum) subject to Article 148, there could be a need to calculate an error for the high-value stratum and to project this error to the units which were not audited in this stratum. In the case of ratio estimation it would be performed using the formula $EE_e = EE_{e\ reduced} \times \frac{BV_{e\ original}}{BV_{e\ reduced}}$, where $EE_{e\ reduced}$represents the amount of error in the sampling units of the high-value stratum audited, $BV_{e\ original}$ refers to book value of the original high-value stratum and $BV_{e\ reduced}$ refers to the book value of items in the high-value stratum which were subject to audit. In the case of mean-per unit estimation it would be performed using the formula $EE_e = EE_{e\ reduced} \times \frac{N_{e\ original}}{N_{e\ reduced}}$, where $EE_{e\ reduced}$ represents the amount of error in the sampling units of the high-value stratum audited, $N_{e\ original}$ refers to the number of sampling units of the original high-value stratum and $N_{e\ reduced}$ refers to the number of sampling units of the high-value stratum

Sampling design	**Simple Random Sampling: Exclusion of sampling units**	**Simple Random Sampling: Replacement of sampling units**
		audited.

7.10.3 Examples

7.10.3.1 *Examples of replacement of sampling units in PPS methods (MUS and PPS non-statistical sampling)*

As clarified in the section above, in PPS methods (MUS and PPS non-statistical sampling) the sampling units subject to Article 148 should be replaced by selection of the new units using probability proportional to size selection.

It should be noted that the procedure for selection of new sampling units in PPS non-statistical sampling is the same as in the case of MUS standard approach, thus common examples illustrate replacement of sampling units in these 2 methods. The 2 examples presented below illustrate respectively:

a) Replacement of sampling units in low-value stratum in the case of MUS standard approach and PPS non-statistical sampling
b) Replacement of sampling units of high-value stratum in the case of MUS standard approach and PPS non-statistical sampling

a) Replacement of sampling units in low-value stratum – MUS standard approach and PPS non-statistical sampling

Let's assume a positive population of expenditure declared to the Commission in a given reference period for operations in a programme.

The population is summarised in the following table:

Population size (number of operations)	3,852
Book value (expenditure in the reference period)	4,199,882,024 €

The sample size is 30 operations (calculated for MUS standard on the basis of the relevant sample parameters or recommended coverage of operations for non-statistical PPS selection based on assurance level from the system audits). The high-value stratum includes 8 operations above the cut-off of 139,996,067.47 with a total value of 1,987,446,254 €. Accordingly, the sampling interval amounts to 100,565,262 €:

$$Sampling\ interval\ (SI) = \frac{BV_s}{n_s} = \frac{4{,}199{,}882{,}024 - 1{,}987{,}446{,}254}{22\ (i.e. 30 - 8)} = 100{,}565{,}262$$

The value of the 22 operations selected by the AA from the low-value stratum with application of the above interval is 65,550,000 €. This sample includes two operations audited by the EC services with 950,000 € of expenditure declared to the EC. The operations are replaced in view of provisions of Article 148 by selection of a replacement unit using probability proportional to size selection.

The new sampling units should be selected from the remaining population of the low-value stratum, that is a file containing 3,822 sampling units (3,852 operations in the population minus 30 operations originally selected)[67] using the interval of 1,073,442,885 €:

$$Sampling\ interval\ used\ for\ replacement\ (SI') = \frac{BV_{s\prime}}{n_{s\prime}} = \frac{4{,}199{,}882{,}024 - 1{,}987{,}446{,}254 - 65{,}550{,}000}{2} =$$
1,073,442,885

In the original sample, the operations affected by Article 148 are substituted by the 2 newly selected operations. The projection is done as usual using the population and sample parameters BV_s and n_s, i.e. we sum errors of the high value stratum and we project the errors of the low-value stratum using the formula:

$$EE_s = \frac{BV_s}{n_s} \sum_{i=1}^{n_s} \frac{E_i}{BV_i}$$

where BV_s = 2,212,435,770 (4,199,882,024 - 1,987,446,254) and n_s=22.

Assuming that the sum of the error rates over all the units in low value stratum ($\sum_{i=1}^{n_s} \frac{E_i}{BV_i}$) is 0.52, the extrapolated error for the low-value stratum amounts to 52,293,936 €.

The audit authority has detected errors of the total amount of 692 € in the high-value stratum. Thus, the projected error in our population amounts to 52,294,628 € (52,293,936 + 692), i.e. 1.25% of the population value.

In the case of application of PPS non-statistical sampling, the audit authority would assess that there is not sufficient evidence to conclude that the population contains material error. Nevertheless, the achieved precision cannot be determined and the confidence of the conclusion is unknown.

In the case of application of MUS standard approach, in order to assess the upper error limit the audit authority would calculate the precision using the standard formula:

[67] The AA could also decide to remove from the file all the other sampling units affected by Article 148 and select the new sampling units only from the population of the low-value stratum which is not affected by Article 148. This proceeding would avoid the risk of performing selection due to replacement several times which would be required if the newly selected items are also subject to Article 148.

$$SE = z \times \frac{BV_s}{\sqrt{n_s}} \times s_r$$

where BV_s = 2,212,435,770 (4,199,882,024 - 1,987,446,254) and n_s=22.

b) Replacement of sampling units in high-value stratum – MUS standard approach and PPS non-statistical sampling

Let's assume a positive population of expenditure declared to the Commission in a given reference period for operations in a programme.

The population is summarised in the following table:

Population size (number of operations)	3,852
Book value (expenditure in the reference period)	4,199,882,024 €

The sample size is 30 operations (calculated for MUS standard on the basis of the relevant sample parameters or recommended coverage of operations for non-statistical PPS selection based on assurance level from the system audits). The high-value stratum includes 8 operations above the cut-off of 139,996,067.47 with a total value of 1,987,446,254 €.

After determinations of the operations/sampling units belonging to the high-value stratum in MUS standard approach and PPS non-statistical sampling, it is recommended that before selection of the sample in the low-value stratum the AA verifies whether the high value stratum includes any sampling units affected by Article 148. If in our example the 8 operations of the high-value stratum include one operation affected by Article 148, the sample size to be allocated to the low-value stratum would be 23 (30 minus 7), ensuring audit of 30 operations. In such a case there is no need to carry out a specific selection of sampling units aimed at replacing the operation subject to Article 148 in the high-value stratum.

In case however the audit authority would establish after selection of the low value stratum of 22 operations (30 minus 8) that 1 operation in the high-value stratum is subject to article 148, the additional sampling unit of the low-value stratum aimed at replacing the sampling unit of the high-value stratum would be selected using probability proportional to size. (As there are no other units available for replacement in the high-value stratum, in order to avoid the artificial reduction of sample size by this restriction, an item of low-value stratum would be selected for replacement ensuring coverage of 30 operations).

Originally, the AA has selected the 22 operations with the total amount of 65,550,000 € from the low-value stratum using the interval of 100,565,262 €:

$$Sampling\ interval\ (SI) = \frac{BV_s}{n_s} = \frac{4{,}199{,}882{,}024 - 1{,}987{,}446{,}254}{22\ (i.e.\ 30 - 8)} = 100{,}565{,}262$$

The new sampling unit of the low-value stratum aimed at replacing the sampling unit of the high-value stratum should be selected from the remaining population of the low-value stratum, that is a file containing 3,822 sampling units (3,852 operations in the population minus 30 operations originally selected)[68] using the interval of 2,146,885,770.00 €:

$$Sampling\ interval\ used\ for\ replacement\ (SI') = \frac{BV_{s'}}{n_{s'}} = \frac{4{,}199{,}882{,}024 - 1{,}987{,}446{,}254 - 65{,}550{,}000}{1} = 2{,}146{,}885{,}770.00$$

Consequently, our audit covers 7 operations in the high-value stratum and 23 operations in the low-value stratum.

The projection of errors in the low-value stratum is based on the standard formula:

$$EE_s = \frac{BV_s}{n_s} \sum_{i=1}^{n_s} \frac{E_i}{BV_i}$$

where BV_s = 2,212,435,770 (4,199,882,024 - 1,987,446,254) and n_s = 23.

Assuming that the sum of the error rates over all the units in the low value stratum ($\sum_{i=1}^{n_s} \frac{E_i}{BV_i}$) is 0.52, the extrapolated error for the low-value stratum amounts to 50,020,287 €.

The audit authority has detected errors of the total amount of 420 € in the 7 operations of high-value stratum, which were subject to audit. The error of the high value stratum would need to be calculated using the following formula:

$$EE_{e\ original} = EE_{e\ reduced} \times \frac{BV_{e\ original}}{BV_{e\ reduced}}$$

where:

- $EE_{e\ reduced}$ refers to the amount of error detected in the operations of the high-value stratum which were subject to audit (excluding the operations affected by Article 148),
- $BV_{e\ original}$ refers to the total book value of the high-value stratum including the operations affected by Article 148, and

[68] See also footnote above clarifying that the AA could decide to select the new sampling units only from the population not affected by Article 148.

- $BV_{e\,reduced}$ refers to the book value of high-value stratum excluding operations affected by Article 148.

Assuming that in our example the amount of 290,309,600 € was declared for the operation subject to Article 148 in high-value stratum, the error of the high-value stratum would amount to 492 €:

$$EE_{e\ original} = 420 \times \frac{1,987,446,254}{1,697,136,654} = 492$$

Accordingly, the extrapolated error at the population level would be 50,020,779 (i.e. 1.19% of the population value):

$$EE = 50,020,287 + 492 = 50,020,779$$

In the case of application of PPS non-statistical sampling, the audit authority would assess that there is not sufficient evidence to conclude that the population contains material error. Nevertheless, the achieved precision cannot be determined and the confidence of the conclusion is unknown.

In the case of application of MUS standard approach, in order to assess the upper error limit the audit authority would calculate the precision using the standard formula:

$$SE = z \times \frac{BV_s}{\sqrt{n_s}} \times s_r$$

where BV_s = 2,212,435,770 (4,199,882,024 - 1,987,446,254) and n_s = 23.

7.10.3.2 Example of exclusion of operations at the stage of sample selection in MUS standard approach

Let's assume a population of expenditure declared to the Commission in a given reference period for operations in a programme. The system audits performed by the audit authority have yielded a low assurance level. Therefore, sampling for this programme should be done with a confidence level of 90%.

The population is summarised in the following table:

Population size (number of operations)	3,852
Book value (sum of the expenditure in the reference period)	4,199,882,024 €

There are 4 operations affected by provisions of Article 148(1) CPR; the total sum of their book values is 12,706,417 €. They will be excluded from the population to be sampled.

The sample size is computed as follows:

$$n = \left(\frac{z \times BV \times \sigma_r}{TE - AE}\right)^2$$

where σ_r is the standard-deviation of error rates resulting from a MUS sample and BV is the total expenditure in the reference year which includes the four previous operations. Based on a preliminary sample of 20 operations the AA estimates the standard deviation of error rates to be 0.0935.

Given this estimate for the standard deviation of error rates, the maximum tolerable error and the anticipated error, we can compute the sample size. Assuming a tolerable error which is 2% of the total book value, 2% x 4,199,882,024 = 83,997,640, (materiality value set by the regulation) and an anticipated error rate of 0.4%, 0.4% x 4,199,882,024 = 16,799,528,

$$n = \left(\frac{1.645 \times 4{,}199{,}882{,}024 \times 0.0935}{83{,}997{,}640 - 16{,}799{,}528}\right)^2 \approx 93$$

First, it is necessary to identify the high value population units (if any) that will belong to a high-value stratum to be submitted to a 100% audit work. The cut-off value for determining this top stratum is equal to the ratio between the book value (*BV*), excluding the four operations already referred (totalling 12,706,417 €) and the planned sample size (*n*). All items whose book value is higher than this cut-off (if $BV_i > BV/n$) will be placed in the 100% audit stratum. In this case the cut-off value is 4,187,175,607/93=45,023,394 €.

The AA puts in an isolated stratum all the operations with book value larger than 45,023,394, which corresponds to 6 operations, amounting to 586,837,081 €

The sampling interval for the remaining population is equal to the book value in the non-exhaustive stratum (BV_s) (the difference between the total book value from which the excluded operations were deducted and the book value of the 6 operations belonging to the top stratum, divided by the number of operations to be selected (93 minus the 6 operations in the top stratum).

$$Sampling\ interval = \frac{BV_s}{n_s} = \frac{4{,}187{,}175{,}607 - 586{,}837{,}081}{87} = 41{,}383{,}201$$

The AA has checked that there were no operations with book values higher than the interval, thus the top stratum includes only the 6 operations with book-value larger than the cut-off value. The sample is selected from a randomised list of operations, selecting each item containing the 41,383,201st monetary unit.

A file containing the remaining 3,842 operations (3,852 minus 4 excluded operations and 6 high value operations) of the population is randomly sorted and a sequential cumulative book value variable is created. A sample value of 87 operations (93 minus 6 high value operations) is drawn using systematic selection.

After auditing the 93 operations, the AA is able to project the error.

Out of the 6 high-value operations (total book value of 586,837,081 €), 3 operations contain error corresponding to an amount of error of 7,616,805 €.

For the remaining sample, the error has a different treatment. For these operations, we follow the following procedure:
1) for each unit in the sample calculate the error rate, i.e. the ratio between the error and the respective expenditure $\frac{E_i}{BV_i}$
2) sum these error rates over all units in the sample
3) multiply the previous result by the sampling interval (SI)

$$EE_s = \frac{BV_s}{n_s} \sum_{i=1}^{n_s} \frac{E_i}{BV_i}$$

where BV_s and n_s are, respectively, the book value used to compute the sampling interval (4,187,175,607 €-586,837,081 € = 3,600,338,526 €) and 87.

$$EE_s = 41{,}383{,}201 \times 1.026 = 42{,}459{,}164$$

To project the error (in euros) of the sampling stratum to the original positive population of expenditure declared to the EC, the projected error has to be multiplied by the ratio of the stratum original expenditure (without deducting the excluded units) and the stratum reduced expenditure (after deducting the excluded units)

$$EE_{s,original} = \frac{\mathrm{BV}_{s,original}}{\mathrm{BV}_{s,reduced}} \times EE_s = \frac{3{,}613{,}044{,}943}{3{,}600{,}338{,}526} \times 42{,}459{,}164 = 42{,}609{,}012$$

The error found in the high-value stratum does not need to be projected to the original population as the expenditure of the 4 excluded units is below the cut-off.

The projected error at the level of the original population is just the sum of the two components (high-value stratum and sampling stratum):

$$EE_{original} = 7{,}616{,}805 + 42{,}609{,}012 = 50{,}225{,}817$$

The projected error rate is the ratio between the projected error and the total expenditure of the original population:

$$r = \frac{50{,}225{,}817}{4{,}199{,}882{,}024} = 1.20\%$$

The standard deviation of error rates in the sampling stratum is 0.0832.

The precision is given by:

$$SE = z \times \frac{BV_s}{\sqrt{n_s}} \times s_r = 1.645 \times \frac{3{,}600{,}338{,}526}{\sqrt{87}} \times 0.0832 = 52{,}829{,}067$$

In order to project this precision to the original population (including the excluded units) the obtained value has to be multiplied by the ratio between the original expenditure of the sampling stratum and the reduced expenditure of the sampling stratum (from which the excluded units were deducted)

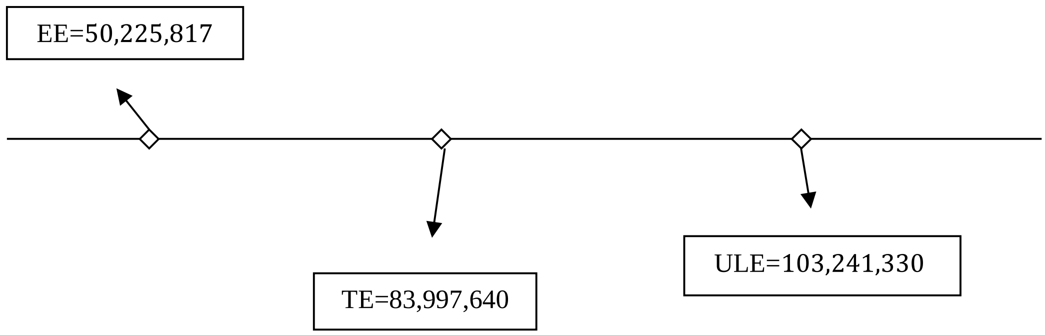
$$SE_{original} = \frac{BV_{s,original}}{BV_{s,reduced}} \times SE = \frac{3{,}613{,}044{,}943}{3{,}600{,}338{,}526} \times 52{,}829{,}067 = 53{,}015{,}513$$

To draw a conclusion about the materiality of the errors, the upper limit of error (ULE) should be calculated. This upper limit is equal to the sum of the projected error EE itself and the precision of the extrapolation

$$ULE = 50{,}225{,}817 + 53{,}015{,}513 = 103{,}241{,}330$$

Then the projected error and the upper limit should both be compared to the maximum tolerable error, 83,997,640 €, to draw audit conclusions.

Since the maximum tolerable error is larger than the projected error, but smaller than the upper limit of error, this means that the sampling results may be inconclusive. See further explanations in Section 4.12.



7.10.3.3 Example of exclusion of operations at the stage of sample selection in MUS conservative approach

Let’s assume a population of 3,857 operations with the total expenditure of 4,207,500,608€ declared to the Commission in a given reference period (population of positive amounts). The AA decided to use MUS conservative approach with the use of an operation as the sampling unit. Moreover, based on Article 28(8) CDR, the audit authority decided to exclude the operations referred to in Article 148(1) CPR from the population to be sampled.

5 operations of the population with a total amount of 7,618,584 € were affected by Article 148 CPR provisions and were excluded from the population before the sample selection. Thus, the sample was selected from the population of 3,852 operations with the total expenditure of 4,199,882,024 €.

The population excluding operation affected by Article 148 provisions is summarised in the following table:

Population size (number of operations)	3,852
Book value (expenditure in the reference period)	4,199,882,024 €

The sample size corresponding to 90% confidence level and 2% materiality threshold is 136 ($n = \frac{BV \times RF}{TE-(AE \times EF)} = \frac{4{,}207{,}500{,}608 \times 2.31}{0.02 \times 4{,}207{,}500{,}608 - (0.002 \times 4{,}207{,}500{,}608 \times 1.5)} \approx 136$).

The selection of the sample is made using probability proportional to size by application of the interval of 30,881,485 ($SI = \frac{BV}{n} = \frac{4{,}199{,}882{,}024}{136} = 30{,}881{,}485$)

In our population there are 24 operations whose book value is larger than the sampling interval. These 24 operations with the total book value of 1,375,130,377 € will constitute our high value stratum (accounting for 45 hits as some operations were hit more than once). The sample size of the low-value stratum is 91 operations, with the total amount of 301,656,001 €.

The projection of the error in the low-value stratum is done as usual using the formula

$$EE_s = SI \sum_{i=1}^{n_s} \frac{E_i}{BV_i}$$

where

$$SI = \frac{BV}{n}$$

refers to the interval used for sample selection, i.e. based on our reduced population value (BV = 4,199,882,024) and the sample size (number of hits n = 136).

Assuming that the sum of error rates in the low-value sample $\left(\sum_{i=1}^{n_s} \frac{E_i}{BV_i}\right)$ is 1.077, the projected error of the low-value stratum is 33,259,360:

$$EE_s = 30{,}881{,}485 \times 1.077 = 33{,}259{,}360$$

To project the error (in euros) of the sampling stratum to the original positive population of expenditure declared to the EC, the projected error has to be multiplied by the ratio of the stratum original expenditure (without deducting the excluded units) and the stratum reduced expenditure (after deducting the excluded units). In our example all the 5 operations affected by Article 148 are part of the low-value stratum.

$$EE_{s,original} = \frac{\mathrm{BV}_{s,original}}{\mathrm{BV}_{s,reduced}} \times EE_s = \frac{2{,}832{,}370{,}231}{2{,}824{,}751{,}647} \times 33{,}259{,}360 = 33{,}349{,}063$$

The error found in the high-value stratum does not need to be projected to the original population as the expenditure of the 5 excluded operations is below the cut-off.

The projected error at the level of the original population is just the sum of the detected error in the high-value stratum and the projected error in the low value stratum (corrected for the original population). Assuming that in the high-value stratum the audit authority has detected a total error of 7,843,574, the projected error at the level of the original population would be:

$$EE_{original} = 7{,}843{,}574 + 33{,}349{,}063 = 41{,}192{,}637$$

(corresponding to a projected error rate of 0.98%).

The global precision (SE) for the reduced population will be calculated as usual by summing two components: basic precision ($BP = SI \times RF$) and incremental allowance $\left(IA = \sum_{i=1}^{n_s} IA_i\right)$, where the incremental allowance is computed for every sampling unit belonging to the non-exhaustive stratum that contains an error using the following standard formula:

$$IA_i = (RF(n) - RF(n-1) - 1) \times SI \times \frac{E_i}{BV_i}$$

The basic precision in our example will be 71,336,231:
BP = 30,881,485 × 2.31 = 71,336,231

Assuming that *IA* amounts to 14,430,761 (calculated using the interval of 30,881,485 as *SI*), the global precision of the reduced population would amount to 85,766,992 (the sum of 71,336,231 and 14,430,761).

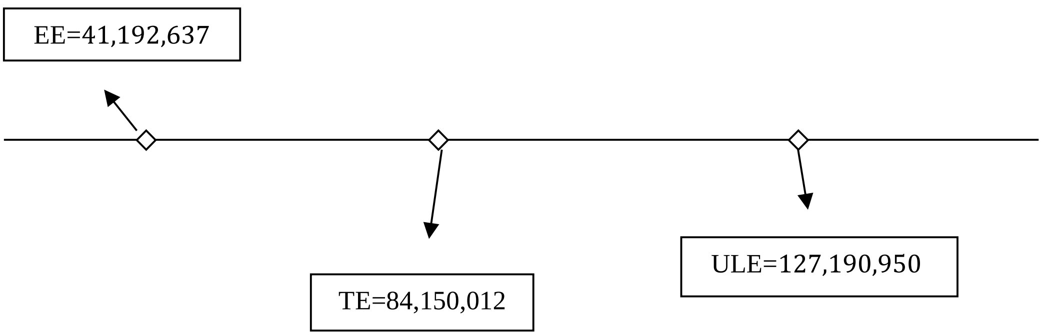
In order to project this precision to the original population (which includes the operations affected by Article 148), the obtained value has to be multiplied by the ratio between the original expenditure of the sampling stratum and the reduced expenditure of the sampling stratum (from which the operations affected by Article 148 were deducted)

$$SE_{original} = \frac{BV_{s,original}}{BV_{s,reduced}} \times SE_{reduced} = \frac{2{,}832{,}370{,}231}{2{,}824{,}751{,}647} \times 85{,}766{,}992 \approx 85{,}998{,}313$$

To draw a conclusion about the materiality of the errors, the upper limit of error (ULE) should be calculated. This upper limit is equal to the sum of the projected error EE itself and the precision of the extrapolation

$$ULE = 41{,}192{,}637 + 85{,}998{,}313 = 127{,}190{,}950$$

Then the projected error and the upper limit should both be compared to the maximum tolerable error, 84,150,012 € (2% of 4,207,500,608). In our example the maximum tolerable error is larger than the projected error, but smaller than the upper limit of error.



7.10.3.4 Example of exclusion of operations at the stage of sample selection in simple random sample (mean-per-unit and ratio estimation)

Let's assume a population of 3,520 operations with the total expenditure of 2,301,882,970€ declared to the Commission in a given reference period (population of positive amounts). The AA decided to apply a sampling design with the use of simple random sampling method combined with stratification by level of expenditure per operation, which will constitute our sampling unit. Moreover, based on Article 28(8) CDR, the audit authority decided to exclude the operations referred to in Article 148(1) CPR from the population to be sampled.

6 operations of the population with a total amount of 93,598,481 € were affected by Article 148 CPR provisions and were excluded from the population before the sample selection. Thus the sample was selected from the population of 3,514 operations with the total expenditure of 2,208,284,489 €.

Taking into account the population characteristics, the AA applied a cut-off of 3% of the (reduced) positive population (3% x 2,208,284,489 = 66,248,535). Two operations had expenditure above this threshold with a total amount of 203,577,481 €. Consequently, the stratum of low-value items included 3,512 operations with a total amount of 2,004,707,008 €.

The reduced positive population excluding 6 operations subject to Article 148 is summarised in the following table:

Population size without 6 operations subject to Article 148 (number of operations)	3,514
Total book value excluding 6 operations (positive population of expenditure in the reference period)	2,208,284,489 €
Cut-off (3% of the population value)	66,248,535 €
Top stratum (2 operations)	203,577,481 €
Stratum of low-value operations without 5 operations subject to Article 148 (3,512 operations)	2,004,707,008 €

The original positive population declared to the EC is summarised below:

Population size (number of operations)	3,520
Total book value (positive population of expenditure in the reference period)	2,301,882,970 €
Top stratum (3 operations)	295,006,242 €
Stratum of low-value operations (3,517 operations)	2,006,876,728 €

For the calculation of the sample size the AA applies the standard formula

$$n = \left(\frac{N \times z \times \sigma_e}{TE - AE}\right)^2$$

using, in line with explanation above, the sampling parameters corresponding to the full population (including operations excluded for sample selection in view of Article 148 provisions).

In particular, the calculation of the sample size was based on the following parameters:

1) z – 1.036
coefficient corresponding to a 70% confidence level determined on the basis of the system audits' work, during which it was evaluated that the assurance from the system is average (category 2)

2) AE - 13,811,297.82 €
The audit authority decided to use historical data for determination of the anticipated error. 0.6% was applied as an anticipated error rate (the error rate resulting from the last exercise of audit of operations), resulting in AE of 13,811,297.82 € (0.006 × 2,301,882,970 €, i.e. the total value of positive population – the total amount of top and low-value strata, which include operations excluded at a later stage in view of Article 148 provisions)

3) TE - 46,037,659.40 €
2% of the total population value, i.e. the maximum materiality level as provided for in Article 28(11) CDR

4) σ_e - 58,730
The audit authority decided to use historical data for determination of standard deviation of errors. Based on AA's professional judgement, it was decided to apply an average standard deviation resulting from 3 previous sampling exercises: accordingly 34,973; 97,654; 97,654 and 43,564:

$$\sigma_e = \frac{34{,}973+97{,}654+43{,}564}{3} \approx 58{,}730$$

5) N – 3,517
N = 3,512 + 5 (population size of the low-value stratum, including also operations subject to Article 148 of the low-value stratum, which were excluded from the sample selection procedure; in our case out of 6 excluded operation, 5 were below the cut-off value)

Based on the above listed parameters, it was established that the sample size of low-value stratum shall be 45 operations:

$$n = \left(\frac{3{,}517 \times 1.036 \times 58{,}730}{0.02 \times 2{,}301{,}882{,}970 - 0.006 \times 2{,}301{,}882{,}970}\right)^2 \approx 45$$

Thus, our sample will include together 47 operations, including 2 operations of the top stratum and 45 operations of the low-value stratum.

For the purpose of the sample selection in low-value stratum, the AA created a file of 3,512 operations excluding the operations affected by Article 148 from the population to be sampled and also excluding operations of the high-value stratum. Subsequently, a sample of 45 operations was selected at random from this population with the total amount of 23,424,898 €.

During the audit of operations of the top stratum, an error of 469,301 € was detected in one of the two operations audited. As no irregular expenditure was detected in the second audited operation of this stratum, the total amount of error in the audited high-value stratum was 469,301 €.

Within the audit of the remaining sample of 45 operations selected at random, a total error of 378,906 € was detected.

<u>Mean-per-unit estimation</u>

Taking into account the results obtained, the AA has established that mean-per-unit estimation will be applied to project the errors to the population. It was decided to project the error in the low-value stratum directly to the level of the original population.[69]

$$EE_{low-value\ stratum} = N_{low-value\ stratum\ of\ original\ population} \times \frac{\sum_{i=1}^{n} E_i}{n}$$

$$EE_{low-value\ stratum} = N \times \frac{\sum_{i=1}^{45} E_i}{n} = 3{,}517 \times \frac{378{,}906}{45} \approx\ 29{,}613{,}608.93\ €$$

To calculate the total error of the population in the standard SRS procedures, the AA needs to add this extrapolated error of the low-value stratum to the error of the top stratum. Please note, however, that in our case one operation of the top stratum was excluded from the audit procedure in view of Article 148 provisions. Consequently, the AA needs to extrapolate the error established in the top stratum which did not include one operation to the whole high-value stratum. In our case, we would calculate the error of top value stratum according to the following formula:

[69] The AA could also calculate the error for the reduced population and later adjust it for the original population. Such adjustment could be performed by multiplying the error of the reduced population by the ratio $\frac{N_{low-value\ stratum\ of\ original\ population}}{N_{low-value\ stratum\ of\ reduced\ population}}$. The final result of this calculation would be the same as in the case of calculation of the error by direct projection to the level of the original population, as presented in this example.

$EE_{original\ high-value\ stratum} = \frac{N_{high-value\ stratum\ of\ original\ population}}{N_{high-value\ stratum\ of\ reduced\ population}} \times \sum_{i=1}^{2} E_i = \frac{3}{2} \times 469{,}301$= 703,951.5

To calculate the total error of the original population, the AA needs to add the extrapolated error of the low-value stratum to the error of the original high-value stratum.

EE = 29,613,608.93 + 703,951.5 = 30,317,560.43

Thus, our most likely error of 30,317,560.43 constitutes 1.32% of the original population expenditure.

The precision for the original population can be calculated using the following standard formula[70]:

$$SE_{original} = N_{original} \times z \times \frac{s_e}{\sqrt{n}}$$

where $N_{original}$ = 3,517 (that is all low-value operations in the original population). Assuming that s_e would amount to 28,199, the precision at the level of the original population would be 15,316,501.38:

$SE_{original} = 3{,}517 \times 1.036 \times \frac{28{,}199}{\sqrt{45}} \approx$ 15,316,501.38

Based on this calculation, our upper error limit is 45,634,061.81 (30,317,560.43 +15,316,501.38), that is below the materiality threshold of 2% of the original population (46,037,659).

Ratio estimation

To illustrate calculation of the projected error for ratio estimation, let's assume that taking into account the results obtained, the AA has applied ratio estimation.

To obtain the error of the low-value stratum at the level of the reduced population the AA applies the standard formula:

$$EE_{low-value\ stratum\ of\ reduced\ population} = BV_{low-value\ stratum\ of\ reduced\ population} \times \frac{\sum_{i=1}^{n} E_i}{\sum_{i=1}^{n} BV_i}$$

[70] The AA could also calculate the precision for the reduced population and later adjust it for the original population. Such adjustment could be performed by multiplying the precision of the reduced population by the ratio $\frac{N_{low-value\ stratum\ of\ original\ population}}{N_{low-value\ stratum\ of\ reduced\ population}}$. The final result of this calculation would be the same as in the case of calculation of the precision directly at the level of the original population, as presented in this example.

In our example, we will use the following data for calculation of the projected error in the low-value stratum of the reduced population[71] based on the results as described above:

$BV_{low\ value\ stratum\ of\ reduced\ population}$ - 2,004,707,008

$\sum_{i=1}^{n} E_i$ – 378,906 (total amount of errors found in the low-value stratum)

$\sum_{i=1}^{n} BV_i$ - 23,424,898 (total amount of expenditure declared for 45 operations audited in the random sample of the low-value stratum)

$$EE_{low-value\ stratum\ of\ reduced\ population} = 2{,}004{,}707{,}008 \times \frac{378{,}906}{23{,}424{,}898} \approx 32{,}426{,}844.02$$

The projected error in low-value stratum of the original population can be obtained using the following formula:

$$EE_{original\ low-value\ stratum} = EE_{reduced\ low-value\ stratum} \times \frac{BV_{low-value\ stratum\ of\ original\ population}}{BV_{low-value\ stratum\ of\ reduced\ population}}$$

$$EE_{low\ value\ stratum\ of\ original\ population} = 32{,}426{,}844.02 \times \frac{2{,}006{,}876{,}728}{2{,}004{,}707{,}008} \approx 32{,}461{,}940.01$$

To calculate the total error of the population in standard SRS procedures, the AA needs to add this extrapolated error of the low-value stratum to the error of the top stratum. Please note, however, that in our case one operation of the top stratum was excluded from the audit procedure in view of Article 148 provisions. Consequently, the AA needs to extrapolate the error established in the top stratum which did not include one operation to the total value of the top stratum including this operation. In our case, we would calculate the error of top value stratum according to the following formula:

$$EE_{e\ original} = \sum_{i=1}^{2} E_i \times \frac{BV_{e\ original}}{BV_{e\ reduced}} = 469{,}301 \times \frac{295{,}006{,}242}{203{,}577{,}481} = 680{,}068.95$$

To calculate the total error of the original population, the AA needs to add the extrapolated error of the original low-value stratum to the error of the original high-value stratum.

EE = 32,461,940.01 + 680,068.95 = 33,142,008.96

[71] As clarified in section 7.10.2 above, the projected error in the stratum could be also directly calculated to the original population (leading to the same result). In this case the following formula could be used:

$$EE_{original\ low-value\ stratum} = BV_{original\ low-value\ stratum} \times \frac{\sum_{i=1}^{n} E_i}{\sum_{i=1}^{n} BV_i}$$

This extrapolated error of the original population constitutes 1.44% of the original population value.

The precision for the reduced population is calculated by use of the following standard formula (as clarified in the section 7.10.2 above, it is not possible to calculate the precision directly for the original population in the case of ration estimation):

$$SE_{reduced\ population} = N_{low-value\ stratum\ of\ reduced\ population} \times z \times \frac{s_q}{\sqrt{n}}$$

In our example, we would use the following data for calculation of the precision for the reduced population:

$N_{reduced\ population\ of\ the\ low\text{-}value\ stratum}$ – 3,512
z – 1.036
n - 45
s_q is the sample standard deviation of the variable q:

$$q_i = E_i - \frac{\sum_{i=1}^{n} E_i}{\sum_{i=1}^{n} BV_i} \times BV_i.$$

where:
$\sum_{i=1}^{n} E_i$ – 378,906 (total amount of errors found in the low-value stratum)

$\sum_{i=1}^{n} BV_i$ - 23,424,898 (total amount of expenditure declared for 45 operations audited in the random sample of the low-value stratum)

The precision for the original population would need to be adjusted based on the formula:

$$SE_{original\ population} = SE_{reduced\ population} \times \frac{BV_{low\ value\ stratum\ of\ original\ population}}{BV_{low\ value\ stratum\ of\ reduced\ population}} = SE_{reduced\ population} \times \frac{2{,}006{,}876{,}728}{2{,}004{,}707{,}008} = SE_{reduced\ population} \times 1.0011$$

To calculate the upper error limit, the audit authority should add the most likely error of the original population (33,142,008.96 in our case) and the precision calculated for the original population (that is $SE_{reduced\ population} \times 1.0011$ in our example). This upper error limit should be compared with the materiality threshold (46,037,659 which is 2% of the original population) to draw the audit conclusions.

Appendix 1 – Projection of random errors when systemic errors are identified

1. Introduction

The purpose of this appendix is to clarify the calculation of the projected random errors when systemic errors are identified. The identification of a potential systemic error implies carrying out the complementary work necessary for the identification of its total extent and subsequent quantification. This means that all the situations susceptible of containing an error of the same type as the one detected in the sample should be identified, thus allowing the delimitation of its total effect in the population. If such delimitation is not done before the ACR is submitted, the systemic errors are to be treated as random for the purposes of the calculation of the projected random error.

The total error rate (TER) corresponds to the sum of the following errors: projected random errors, systemic errors and uncorrected anomalous errors.

In this context, when extrapolating the random errors found in the sample to the population, the Audit Authority should deduct the amount of systemic error from the book value (total expenditure declared in the reference period) whenever this value is part of the projection formula, as explained below.

As regard mean-per-unit estimation[72] and difference estimation, there is no change in the formulas presented in the guidance for the projection of random errors. For monetary unit sampling this appendix sets out two possible approaches (one approach that does not change the formula and another approach that requires formulas that are more complex in order to obtain better precision). For ratio estimation, the projection of the random errors and the calculation of the precision (SE) requires the use of the total book value from which systemic errors are deducted.

In all statistical sampling methods, when systemic errors or anomalous non-corrected errors exist, the upper limit of error (ULE) corresponds to the sum of the TER plus the precision (SE). When only random errors exist, the ULE is the sum of the projected random errors plus the precision.

In the following sections a more detailed explanation about the extrapolation of random errors in the presence of systemic errors for the most important sampling techniques is offered.

[72] cf. section on "simple random sampling" in the guidance.

2. Simple random sampling

2.2 Mean-per-unit estimation

The projection of random errors and the calculation of precision are as usual:

$$EE_1 = N \times \frac{\sum_{i=1}^{n} E_i}{n}.$$

$$SE_1 = N \times z \times \frac{s_e}{\sqrt{n}}$$

where E_i represents the amount of random error found in each sampling unit and s_e is, as usual, the standard-deviation of random errors in the sample.

The total projected error is the sum of random projected errors, systemic errors and anomalous non-corrected errors.

The upper limit of error (ULE) is equal to the summation of the total projected error, TPE, and the precision of the extrapolation

$$ULE = TPE + SE$$

2.3 Ratio estimation

The projection of the random error is:

$$EE_2 = BV' \times \frac{\sum_{i=1}^{n} E_i}{\sum_{i=1}^{n} BV'_i}$$

where BV' represents the total book value of the population from which systemic errors are deducted that were previously delimited, $BV' = BV - systemic\ errors$. BV'_i is the book value of unit *i* deducted by the amount of systemic error affecting that unit.

The sample error rate in the above formula is just the division of the total amount of random error in the sample by the total amount of expenditure (from which systemic errors are deducted) of units in the sample (expenditure audited).

The precision is given by the formula

$$SE_2 = N \times z \times \frac{s_{q'}}{\sqrt{n}}$$

where $s_{q'}$ is the sample standard deviation of the variable q':

$$q'_i = E_i - \frac{\sum_{i=1}^{n} E_i}{\sum_{i=1}^{n} BV'_i} \times BV'_i.$$

This variable is for each unit in the sample computed as the difference between its random error and the product between its book value (from which systemic errors are deducted) and the error rate in the sample.

The total projected error is the sum of random projected errors, systemic errors and anomalous non-corrected errors.

The upper limit of error (ULE) is equal to the summation of the total projected error, TPE, and the precision of the extrapolation

$$ULE = TPE + SE$$

3. Difference estimation

The projected random error at the level of the population can be computed as usual by multiplying the average random error observed per operation in the sample by the number of operations in the population, yielding the projected error

$$EE = N \times \frac{\sum_{i=1}^{n} E_i}{n}.$$ [73]

In a second step, the total error rate (TER) should be computed adding the amount of systemic error and anomalous non corrected errors to the random projected error (EE).

The correct book value (the correct expenditure that would be found if all the operations in the population were audited) can be projected subtracting the TER from the book value (BV) in the population (declared expenditure without deducting the systemic errors). The projection for the correct book value (CBV) is

$$CBV = BV - TER$$

[73] Alternatively the projected random error can be obtained using the formula proposed under ratio estimation $EE_2 = BV' \times \frac{\sum_{i=1}^{n} E_i}{\sum_{i=1}^{n} BV'_i}$.

The precision of the projection is, as usual, given by

$$SE = N \times z \times \frac{s_e}{\sqrt{n}}$$

where s_e is the standard-deviation of random errors in the sample.

To conclude about the materiality of the errors the lower limit for the corrected book value should firstly be calculated. This lower limit is, as usual, equal to

$$LL = CBV - SE$$

The projection for the correct book value and the upper limit should both be compared to the difference between the book value (declared expenditure) and the maximum tolerable error (TE), which corresponds to the materiality level times the book value:

$$BV - TE = BV - 2\% \times BV = 98\% \times BV$$

The evaluation of the error should be done in accordance with section 6.2.1.5 of the guidance.

4. Monetary unit sampling

There are two possible approaches to project random errors and calculate precision under monetary unit sampling in the presence of systemic errors. They will be referred as *MUS standard approach* and *MUS ratio estimation*. The second method is based on a more complex calculation. Although, they can both be used in any scenario, the second method will generally produce more precise results when the random errors are more correlated with the book values corrected from the systemic error than with the original book values. When the level of systemic errors in the population is small, the precision gain originated by the second method will usually be very modest and the first method may be a preferable choice due to its simplicity of application.

4.1 MUS standard approach

The projection random errors and the calculation of precision are performed as usual.

The projection of the random errors to the population should be made differently for the units in the exhaustive stratum and for the items in the non-exhaustive stratum.

For the exhaustive stratum, that is, for the stratum containing the sampling items with book value larger than the cut-off ($BV_i > \frac{BV}{n}$) the projected error is just the summation of the errors found in the items belonging to the stratum:

$$EE_e = \sum_{i=1}^{n_e} E_i$$

For the non-exhaustive stratum, i.e. the stratum containing the sampling items with book value smaller or equal to the cut-off value ($BV_i \leq \frac{BV}{n}$) the projected random error is

$$EE_s = \frac{BV_s}{n_s} \sum_{i=1}^{n_s} \frac{E_i}{BV_i}$$

Note that the book values mentioned in the above formula refer to the expenditure **without** subtracting the amount of systemic error. This means that the error rates, $\frac{E_i}{BV_i}$, should be calculated using the total expenditure of the sample units despite a systemic error was or not found in each unit.

The precision is also given by the usual formula:

$$SE = z \times \frac{BV_s}{\sqrt{n_s}} \times s_r$$

where s_r is the standard-deviation of random error rates in the sample of the non-exhaustive stratum. Again this error rates should be calculated using the original book values, BV_i, **without** subtracting the amount of systemic error.

The total projected error is the sum of random projected errors, systemic errors and anomalous non-corrected errors.

The upper limit of error (ULE) is equal to the summation of the total projected error, TPE, and the precision of the extrapolation

$$ULE = TPE + SE$$

4.2 MUS ratio estimation

The projection of the random errors to the population should again be made differently for the items in the exhaustive stratum and for the items in the non-exhaustive stratum.

For the exhaustive stratum, that is, for the stratum containing the sampling units with book value larger than the cut-off ($BV_i > \frac{BV}{n}$) the projected error is just the summation of the random errors found in the items belonging to the stratum:

$$EE_e = \sum_{i=1}^{n_e} E_i$$

For the non-exhaustive stratum, i.e. the stratum containing the sampling units with book value smaller or equal to the cut-off value ($BV_i \leq \frac{BV}{n}$) the projected random error is

$$EE_s = BV'_s \times \frac{\sum_{i=1}^{n_s} \frac{E_i}{BV_i}}{\sum_{i=1}^{n_s} \frac{BV'_i}{BV_i}}$$

where BV'_s represents the total book value of the low-value stratum from which systemic errors are deducted that were previously delimited in the same stratum, $BV'_s = BV_s - systemic\ errors\ in\ the\ sampling\ stratum.$ BV'_i is the book value of unit *i* reduced by the amount of systemic error affecting that unit.

The precision is given by the formula:

$$SE = z \times \frac{BV_s}{\sqrt{n_s}} \times s_{rq}$$

where s_{rq} is the standard-deviation of the error rates for the **transformed error** q'. To calculate this formula, it is first necessary to calculate the values of the **transformed errors** for all units in the sample:

$$q'_i = E_i - \frac{\sum_{i=1}^{n_s} \frac{E_i}{BV_i}}{\sum_{i=1}^{n_s} \frac{BV'_i}{BV_i}} \times BV'_i.$$

Finally, the standard-deviation of error rates in the sample of the non-exhaustive stratum (s_{rq}), for the transformed error q', is obtained as:

$$s_{rq} = \sqrt{\frac{1}{n_s - 1}\sum_{i=1}^{n_s}\left(\frac{q'_i}{BV_{i_i}} - \overline{rq}_s\right)^2}$$

having $\overline{rq}_s$ equal to the simple average of the transformed error rates in the sample of the stratum

$$\bar{r}q_s = \frac{\sum_{i=1}^{n_s}\frac{q'_i}{BV_i}}{n_s}$$

The total projected error is the sum of random projected errors, systemic errors and anomalous non-corrected errors.

The upper limit of error (ULE) is equal to the summation of the total projected error (TPE), and the precision of the extrapolation

$$ULE = TPE + SE$$

4.3 MUS conservative approach

In the context of MUS conservative approach the use of ratio estimation is not advisable as it is not possible to take account of its effects over the precision of estimation. Therefore it is recommended to project the errors and calculate the projected error and the precision using the usual formulas (without deducting from the expenditure the amount affected by systemic errors).

5. Non-statistical sampling

If the projection is based on mean-per-unit estimation, the projection is performed as usual.

If an exhaustive stratum exists, that is, a stratum containing the sampling units with book value larger cut-off value, the projected error is just the sum of random errors found in this group:

$$EE_e = \sum_{i=1}^{n_e} E_i$$

For the sampling stratum, if units were selected with equal probabilities, the projected random error is as usual

$$EE_s = N_s \frac{\sum_{i=1}^{n_s} E_i}{n_s}.$$

where N_s is the population size and n_s the sample size in the low value stratum.

If ratio estimation is used (associated with equal probability random selection), the projection of the random error is the same as presented in the context of simple random sampling:

$$EE_{s2} = BV_s' \times \frac{\sum_{i=1}^{n_s} E_i}{\sum_{i=1}^{n_s} BV'_i}$$

where BV_s' represents the total book value of the population of the sampling stratum from which the systemic errors are deducted. BV'_i is the book value of unit *i* from which the amount of systemic error affecting that unit is deducted.

If units were selected with probabilities proportional to the value of expenditure, the projected random error for the low-value stratum is

$$EE_s = \frac{BV_s}{n_s} \sum_{i=1}^{n_s} \frac{E_i}{BV_i}$$

where BV_s is the total book value (**without** deducting the amount of systemic error), BV_i the book value of sample unit *i* (**without** deducting the amount of systemic error and n_s the sample size in the low value stratum.

Similarly to what has been presented for MUS method, the ratio estimation formula,

$$EE_s = BV'_s \times \frac{\sum_{i=1}^{n_s} \frac{E_i}{BV_i}}{\sum_{i=1}^{n_s} \frac{BV'_i}{BV_i}}$$

can alternatively be used. Again BV'_s represents the total book value of the low-value stratum from which systemic errors were deducted that were previously delimited in the same stratum, $BV'_s = BV_s - systemic\ errors\ in\ the\ sampling\ stratum.$ BV'_i is the book value of unit *i* reduced by the amount of systemic error affecting that unit.

The total error rate (TER) is the sum of random projected errors, systemic errors and anomalous non-corrected errors.

Appendix 2 – Formulas for multi-period sampling

1. Simple random sampling

1.1 Three periods

1.1.1 Sample size

First period

$$n_{1+2+3} = \frac{\left(z \times N_{1+2+3} \times \sigma_{ew1+2+3}\right)^2}{(TE - AE)^2}$$

where

$$\sigma^2_{ew1+2+3} = \frac{N_1}{N_{1+2+3}}\sigma^2_{e1} + \frac{N_2}{N_{1+2+3}}\sigma^2_{e2} + \frac{N_3}{N_{1+2+3}}\sigma^2_{e3}$$

$$N_{1+2+3} = N_1 + N_2 + N_3$$

$$n_t = \frac{N_t}{N_{1+2+3}} n_{1+2+3}$$

Second period

$$n_{2+3} = \frac{\left(z \times N_{2+3} \times \sigma_{ew2+3}\right)^2}{(TE - AE)^2 - z^2 \times \frac{N_1^2}{n_1} \times s^2_{e1}}$$

where

$$\sigma^2_{ew2+3} = \frac{N_2}{N_{2+3}}\sigma^2_{e2} + \frac{N_3}{N_{2+3}}\sigma^2_{e3}$$

$$N_{2+3} = N_2 + N_3$$

$$n_t = \frac{N_t}{N_{2+3}} n_{2+3}$$

Third period

$$n_3 = \frac{\left(z \times N_3 \ \ \times \sigma_{e3}\right)^2}{(TE - AE)^2 - z^2 \times \frac{N_1^2}{n_1} \times s_{e1}^2 - z^2 \times \frac{N_2^2}{n_2} \times s_{e2}^2}$$

Notes:

In each period all the population parameters must be updated with the most accurate information available.

Whenever different approximations for the standard-deviations of each period cannot be obtained/are not applicable, the same value of standard deviation may be applied to all periods. In such a case $\sigma_{ew1+2+3}$ is just equal to the single standard-deviation of errors σ_e .

The parameter σ refers to the standard-deviation obtained from auxiliary data (e.g. historical data) and *s* refers to the standard-deviation obtained from the audited sample. In the formulas, whenever *s* is not available, it may be substituted by σ.

1.1.2 Projection and precision

Mean-per-unit estimation

$$EE_1 = \frac{N_1}{n_1}\sum_{i=1}^{n_1} E_{1i} + \frac{N_2}{n_2}\sum_{i=1}^{n_2} E_{2i} + \frac{N_3}{n_3}\sum_{i=1}^{n_3} E_{3i}$$

$$SE \quad = z \times \sqrt{\left(N_1^2 \times \frac{s_{e1}^2}{n_1} + N_2^2 \times \frac{s_{e2}^2}{n_2} + N_3^2 \times \frac{s_{e3}^2}{n_3}\right)}$$

Ratio estimation

$$EE_2 = BV_1 \times \frac{\sum_{i=1}^{n_1} E_{1i}}{\sum_{i=1}^{n_1} BV_{1i}} + BV_2 \times \frac{\sum_{i=1}^{n_2} E_{2i}}{\sum_{i=1}^{n_2} BV_{2i}} + BV_3 \times \frac{\sum_{i=1}^{n_3} E_{3i}}{\sum_{i=1}^{n_3} BV_{3i}}$$

$$SE \quad = z \times \sqrt{\left(N_1^2 \times \frac{s_{q1}^2}{n_1} + N_2^2 \times \frac{s_{q2}^2}{n_2} + N_3^2 \times \frac{s_{q3}^2}{n_3}\right)}$$

$$q_{ti} = E_{ti} - \frac{\sum_{i=1}^{n_t} E_{ti}}{\sum_{i=1}^{n_t} BV_{ti}} \times BV_{ti}.$$

1.2 Four periods

1.2.1 Sample size

First period

$$n_{1+2+3+4} = \frac{\left(z \times N_{1+2+3+4} \times \sigma_{ew1+2+3+4}\right)^2}{(TE - AE)^2}$$

where

$$\sigma_{ew1+2+3+4}^2 = \frac{N_1}{N_{1+2+3+4}}\sigma_{e1}^2 + \frac{N_2}{N_{1+2+3+4}}\sigma_{e2}^2 + \frac{N_3}{N_{1+2+3+4}}\sigma_{e3}^2 + \frac{N_4}{N_{1+2+3+4}}\sigma_{e4}^2$$

$$N_{1+2+3+4} = N_1 + N_2 + N_3 + N_4$$

$$n_t = \frac{N_t}{N_{1+2+3+4}} n_{1+2+3+4}$$

Second period

$$n_{2+3+4} = \frac{\left(z \times N_{2+3+4} \times \sigma_{ew2+3+4}\right)^2}{(TE - AE)^2 - z^2 \times \frac{N_1^2}{n_1} \times s_{e1}^2}$$

where

$$\sigma_{ew2+3+4}^2 = \frac{N_2}{N_{2+3+4}}\sigma_{e2}^2 + \frac{N_3}{N_{2+3+4}}\sigma_{e3}^2 + \frac{N_4}{N_{2+3+4}}\sigma_{e4}^2$$

$$N_{2+3+4} = N_2 + N_3 + N_4$$

$$n_t = \frac{N_t}{N_{2+3+4}} n_{2+3+4}$$

Third period

$$n_{3+4} = \frac{\left(z \times N_{3+4} \times \sigma_{ew3+4}\right)^2}{(TE - AE)^2 - z^2 \times \frac{N_1^2}{n_1} \times s_{e1}^2 - z^2 \times \frac{N_2^2}{n_2} \times s_{e2}^2}$$

where

$$\sigma_{ew3+4}^2 = \frac{N_3}{N_{3+4}}\sigma_{e3}^2 + \frac{N_4}{N_{3+4}}\sigma_{e4}^2$$

$$N_{3+4} = N_3 + N_4$$

$$n_t = \frac{N_t}{N_{3+4}} n_{3+4}$$

Fourth period

$$n_4 = \frac{\left(z \times N_4 \ \times \sigma_{e4}\right)^2}{(TE - AE)^2 - z^2 \times \frac{N_1^2}{n_1} \times s_{e1}^2 - z^2 \times \frac{N_2^2}{n_2} \times s_{e2}^2 - z^2 \times \frac{N_3^2}{n_3} \times s_{e3}^2}$$

Notes:
In each period all the population parameters must be updated with the most accurate information available.
Whenever different approximations for the standard-deviations of each period cannot be obtained/are not applicable, the same value of standard deviation may be applied to all periods. In such a case $\sigma_{ew1+2+3+4}$ is just equal to the single standard-deviation of errors σ_e .
The parameter σ refers to the standard-deviation obtained from auxiliary data (e.g. historical data) and s refers to the standard-deviation obtained from the audited sample. In the formulas, whenever s is not available, it may be substituted by σ.

1.2.2 Projection and precision

Mean-per-unit estimation

$$EE_1 = \frac{N_1}{n_1}\sum_{i=1}^{n_1} E_{1i} + \frac{N_2}{n_2}\sum_{i=1}^{n_2} E_{2i} + \frac{N_3}{n_3}\sum_{i=1}^{n_3} E_{3i} + \frac{N_4}{n_4}\sum_{i=1}^{n_4} E_{4i}$$

$$SE \quad = z \times \sqrt{\left(N_1^2 \times \frac{s_{e1}^2}{n_1} + N_2^2 \times \frac{s_{e2}^2}{n_2} + N_3^2 \times \frac{s_{e3}^2}{n_3} + N_4^2 \times \frac{s_{e4}^2}{n_4}\right)}$$

Ratio estimation

$$EE_2 = BV_1 \times \frac{\sum_{i=1}^{n_1} E_{1i}}{\sum_{i=1}^{n_1} BV_{1i}} + BV_2 \times \frac{\sum_{i=1}^{n_2} E_{2i}}{\sum_{i=1}^{n_2} BV_{2i}} + BV_3 \times \frac{\sum_{i=1}^{n_3} E_{3i}}{\sum_{i=1}^{n_3} BV_{3i}} + BV_4 \times \frac{\sum_{i=1}^{n_4} E_{4i}}{\sum_{i=1}^{n_4} BV_{4i}}$$

$$SE \quad = z \times \sqrt{\left(N_1^2 \times \frac{s_{q1}^2}{n_1} + N_2^2 \times \frac{s_{q2}^2}{n_2} + N_3^2 \times \frac{s_{q3}^2}{n_3} + N_4^2 \times \frac{s_{q4}^2}{n_4}\right)}$$

$$q_{ti} = E_{ti} - \frac{\sum_{i=1}^{n_t} E_{ti}}{\sum_{i=1}^{n_t} BV_{ti}} \times BV_{ti}.$$

2. Monetary unit sampling

2.1 Three periods

2.1.1 Sample size

First period

$$n_{1+2+3} = \frac{\left(z \times BV_{1+2+3} \times \sigma_{rw1+2+3}\right)^2}{(TE - AE)^2}$$

where

$$\sigma^2_{rw1+2+3} = \frac{BV_1}{BV_{1+2+3}}\sigma^2_{r1} + \frac{BV_2}{BV_{1+2+3}}\sigma^2_{r2} + \frac{BV_3}{BV_{1+2+3}}\sigma^2_{r3}$$

$$BV_{1+2+3} = BV_1 + BV_2 + BV_3$$

$$n_t = \frac{BV_t}{BV_{1+2+3}} n_{1+2+3}$$

Second period

$$n_{2+3} = \frac{\left(z \times BV_{2+3} \times \sigma_{rw2+3}\right)^2}{(TE - AE)^2 - z^2 \times \frac{BV_1^2}{n_1} \times s^2_{r1}}$$

where

$$\sigma^2_{rw2+3} = \frac{BV_2}{BV_{2+3}}\sigma^2_{r2} + \frac{BV_3}{BV_{2+3}}\sigma^2_{r3}$$

$$BV_{2+3} = BV_2 + BV_3$$

$$n_t = \frac{BV_t}{BV_{2+3}} n_{2+3}$$

Third period

$$n_3 = \frac{\left(z \times BV_3 \times \sigma_{r3}\right)^2}{(TE - AE)^2 - z^2 \times \frac{BV_1^2}{n_1} \times s^2_{r1} - z^2 \times \frac{BV_2^2}{n_2} \times s^2_{r2}}$$

Notes:

In each period all the population parameters must be updated with the most accurate information available.

Whenever different approximations for the standard-deviations of each period cannot be obtained/are not applicable, the same value of standard deviation may be applied to all periods. In such a case $\sigma_{rw1+2+3}$ is just equal to the single standard-deviation of error rates σ_r .

The parameter σ refers to the standard-deviation obtained from auxiliary data (e.g. historical data) and s refers to the standard-deviation obtained from the audited sample. In the formulas, whenever s is not available, it may be substituted by σ.

2.1.2 Projection and precision

$$EE_e = \sum_{i=1}^{n_1} E_{1i} + \sum_{i=1}^{n_2} E_{2i} + \sum_{i=1}^{n_3} E_{3i}$$

$$EE_s = \frac{BV_{1s}}{n_{1s}} \times \sum_{i=1}^{n_{1s}} \frac{E_{1i}}{BV_{1i}} + \frac{BV_{2s}}{n_{2s}} \times \sum_{i=1}^{n_{2s}} \frac{E_{2i}}{BV_{2i}} + \frac{BV_{3s}}{n_{3s}} \times \sum_{i=1}^{n_{3s}} \frac{E_{3i}}{BV_{3i}}$$

$$SE = z \times \sqrt{\frac{BV_{1s}^2}{n_{1s}} \times s_{r1s}^2 + \frac{BV_{2s}^2}{n_{2s}} \times s_{r2s}^2 + \frac{BV_{3s}^2}{n_{3s}} \times s_{r3s}^2}$$

2.2 Four periods

2.2.1 Sample size

First period

$$n_{1+2+3+4} = \frac{\left(z \times BV_{1+2+3+4} \times \sigma_{rw1+2+3+4}\right)^2}{(TE - AE)^2}$$

where

$$\sigma^2_{rw1+2+3+4} = \frac{BV_1}{BV_{1+2+3+4}}\sigma^2_{r1} + \frac{BV_2}{BV_{1+2+3+4}}\sigma^2_{r2} + \frac{BV_3}{BV_{1+2+3+4}}\sigma^2_{r3} + \frac{BV_4}{BV_{1+2+3+4}}\sigma^2_{r4}$$

$$BV_{1+2+3+4} = BV_1 + BV_2 + BV_3 + BV_4$$

$$n_t = \frac{BV_t}{BV_{1+2+3+4}} n_{1+2+3+4}$$

Second period

$$n_{2+3+4} = \frac{\left(z \times BV_{2+3+4} \times \sigma_{rw2+3+4}\right)^2}{(TE - AE)^2 - z^2 \times \frac{BV_1^2}{n_1} \times s^2_{r1}}$$

where

$$\sigma^2_{rw2+3+4} = \frac{BV_2}{BV_{2+3+4}}\sigma^2_{r2} + \frac{BV_3}{BV_{2+3+4}}\sigma^2_{r3} + \frac{BV_4}{BV_{2+3+4}}\sigma^2_{r4}$$

$$BV_{2+3+4} = BV_2 + BV_3 + BV_4$$

$$n_t = \frac{BV_t}{BV_{2+3+4}} n_{2+3+4}$$

Third period

$$n_{3+4} = \frac{\left(z \times BV_{3+4} \times \sigma_{rw3+4}\right)^2}{(TE - AE)^2 - z^2 \times \frac{BV_1^2}{n_1} \times s^2_{r1} - z^2 \times \frac{BV_2^2}{n_2} \times s^2_{r2}}$$

where

$$\sigma_{rw3+4}^2 = \frac{BV_3}{BV_{3+4}}\sigma_{r3}^2 + \frac{BV_4}{BV_{3+4}}\sigma_{r4}^2$$

$$BV_{3+4} = BV_3 + BV_4$$

$$n_t = \frac{BV_t}{BV_{3+4}} n_{3+4}$$

Fourth period

$$n_4 = \frac{\left(z \times BV_4 \ \times \sigma_{r4}\right)^2}{(TE - AE)^2 - z^2 \times \frac{BV_1^2}{n_1} \times s_{r1}^2 - z^2 \times \frac{BV_2^2}{n_2} \times s_{r2}^2 - z^2 \times \frac{BV_3^2}{n_3} \times s_{r3}^2}$$

Notes:
In each period all the population parameters must be updated with the most accurate information available.
Whenever different approximations for the standard-deviations of each period cannot be obtained/are not applicable, the same value of standard deviation may be applied to all periods. In such a case $\sigma_{rw1+2+3+4}$ is just equal to the single standard-deviation of error rates σ_r .
The parameter σ refers to the standard-deviation obtained from auxiliary data (e.g. historical data) and s refers to the standard-deviation obtained from the audited sample. In the formulas, whenever s is not available, it may be substituted by σ.

2.2.2 Projection and precision

$$EE_e = \sum_{i=1}^{n_1} E_{1i} + \sum_{i=1}^{n_2} E_{2i} + \sum_{i=1}^{n_3} E_{3i} + \sum_{i=1}^{n_4} E_{4i}$$

$$EE_s = \frac{BV_{1s}}{n_{1s}} \times \sum_{i=1}^{n_{1s}} \frac{E_{1i}}{BV_{1i}} + \frac{BV_{2s}}{n_{2s}} \times \sum_{i=1}^{n_{2s}} \frac{E_{2i}}{BV_{2i}} + \frac{BV_{3s}}{n_{3s}} \times \sum_{i=1}^{n_{3s}} \frac{E_{3i}}{BV_{3i}} + \frac{BV_{4s}}{n_{4s}} \times \sum_{i=1}^{n_{4s}} \frac{E_{4i}}{BV_{4i}}$$

$$SE = z \times \sqrt{\frac{BV_{1s}^2}{n_{1s}} \times s_{r1s}^2 + \frac{BV_{2s}^2}{n_{2s}} \times s_{r2s}^2 + \frac{BV_{3s}^2}{n_{3s}} \times s_{r3s}^2 + \frac{BV_{4s}^2}{n_{4s}} \times s_{r4s}^2}$$

Appendix 3 – Reliability factors for MUS

	Risk of incorrect acceptation									
Number of errors	1%	5%	10%	15%	20%	25%	30%	37%	40%	50%
0	4.61	3.00	2.30	1.90	1.61	1.39	1.20	0.99	0.92	0.69
1	6.64	4.74	3.89	3.37	2.99	2.69	2.44	2.14	2.02	1.68
2	8.41	6.30	5.32	4.72	4.28	3.92	3.62	3.25	3.11	2.67
3	10.05	7.75	6.68	6.01	5.52	5.11	4.76	4.34	4.18	3.67
4	11.60	9.15	7.99	7.27	6.72	6.27	5.89	5.42	5.24	4.67
5	13.11	10.51	9.27	8.49	7.91	7.42	7.01	6.49	6.29	5.67
6	14.57	11.84	10.53	9.70	9.08	8.56	8.11	7.56	7.34	6.67
7	16.00	13.15	11.77	10.90	10.23	9.68	9.21	8.62	8.39	7.67
8	17.40	14.43	12.99	12.08	11.38	10.80	10.30	9.68	9.43	8.67
9	18.78	15.71	14.21	13.25	12.52	11.91	11.39	10.73	10.48	9.67
10	20.14	16.96	15.41	14.41	13.65	13.02	12.47	11.79	11.52	10.67
11	21.49	18.21	16.60	15.57	14.78	14.12	13.55	12.84	12.55	11.67
12	22.82	19.44	17.78	16.71	15.90	15.22	14.62	13.88	13.59	12.67
13	24.14	20.67	18.96	17.86	17.01	16.31	15.70	14.93	14.62	13.67
14	25.45	21.89	20.13	19.00	18.13	17.40	16.77	15.97	15.66	14.67
15	26.74	23.10	21.29	20.13	19.23	18.49	17.83	17.02	16.69	15.67
16	28.03	24.30	22.45	21.26	20.34	19.57	18.90	18.06	17.72	16.67
17	29.31	25.50	23.61	22.38	21.44	20.65	19.96	19.10	18.75	17.67
18	30.58	26.69	24.76	23.50	22.54	21.73	21.02	20.14	19.78	18.67
19	31.85	27.88	25.90	24.62	23.63	22.81	22.08	21.17	20.81	19.67
20	33.10	29.06	27.05	25.74	24.73	23.88	23.14	22.21	21.84	20.67
21	34.35	30.24	28.18	26.85	25.82	24.96	24.20	23.25	22.87	21.67
22	35.60	31.41	29.32	27.96	26.91	26.03	25.25	24.28	23.89	22.67
23	36.84	32.59	30.45	29.07	28.00	27.10	26.31	25.32	24.92	23.67
24	38.08	33.75	31.58	30.17	29.08	28.17	27.36	26.35	25.95	24.67
25	39.31	34.92	32.71	31.28	30.17	29.23	28.41	27.38	26.97	25.67
26	40.53	36.08	33.84	32.38	31.25	30.30	29.46	28.42	28.00	26.67
27	41.76	37.23	34.96	33.48	32.33	31.36	30.52	29.45	29.02	27.67
28	42.98	38.39	36.08	34.57	33.41	32.43	31.56	30.48	30.04	28.67
29	44.19	39.54	37.20	35.67	34.49	33.49	32.61	31.51	31.07	29.67
30	45.40	40.69	38.32	36.76	35.56	34.55	33.66	32.54	32.09	30.67
31	46.61	41.84	39.43	37.86	36.64	35.61	34.71	33.57	33.11	31.67
32	47.81	42.98	40.54	38.95	37.71	36.67	35.75	34.60	34.14	32.67
33	49.01	44.13	41.65	40.04	38.79	37.73	36.80	35.63	35.16	33.67
34	50.21	45.27	42.76	41.13	39.86	38.79	37.84	36.66	36.18	34.67
35	51.41	46.40	43.87	42.22	40.93	39.85	38.89	37.68	37.20	35.67
36	52.60	47.54	44.98	43.30	42.00	40.90	39.93	38.71	38.22	36.67
37	53.79	48.68	46.08	44.39	43.07	41.96	40.98	39.74	39.24	37.67
38	54.98	49.81	47.19	45.47	44.14	43.01	42.02	40.77	40.26	38.67
39	56.16	50.94	48.29	46.55	45.20	44.07	43.06	41.79	41.28	39.67
40	57.35	52.07	49.39	47.63	46.27	45.12	44.10	42.82	42.30	40.67
41	58.53	53.20	50.49	48.72	47.33	46.17	45.14	43.84	43.32	41.67
42	59.71	54.32	51.59	49.80	48.40	47.22	46.18	44.87	44.34	42.67
43	60.88	55.45	52.69	50.87	49.46	48.27	47.22	45.90	45.36	43.67
44	62.06	56.57	53.78	51.95	50.53	49.32	48.26	46.92	46.38	44.67
45	63.23	57.69	54.88	53.03	51.59	50.38	49.30	47.95	47.40	45.67
46	64.40	58.82	55.97	54.11	52.65	51.42	50.34	48.97	48.42	46.67
47	65.57	59.94	57.07	55.18	53.71	52.47	51.38	49.99	49.44	47.67
48	66.74	61.05	58.16	56.26	54.77	53.52	52.42	51.02	50.45	48.67
49	67.90	62.17	59.25	57.33	55.83	54.57	53.45	52.04	51.47	49.67
50	69.07	63.29	60.34	58.40	56.89	55.62	54.49	53.06	52.49	50.67

Appendix 4 – Values for the standardized normal distribution (z)

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.998650	0.998694	0.998736	0.998777	0.998817	0.998856	0.998893	0.998930	0.998965	0.998999
3.1	0.999032	0.999064	0.999096	0.999126	0.999155	0.999184	0.999211	0.999238	0.999264	0.999289
3.2	0.999313	0.999336	0.999359	0.999381	0.999402	0.999423	0.999443	0.999462	0.999481	0.999499
3.3	0.999517	0.999533	0.999550	0.999566	0.999581	0.999596	0.999610	0.999624	0.999638	0.999650
3.4	0.999663	0.999675	0.999687	0.999698	0.999709	0.999720	0.999730	0.999740	0.999749	0.999758
3.5	0.999767	0.999776	0.999784	0.999792	0.999800	0.999807	0.999815	0.999821	0.999828	0.999835
3.6	0.999841	0.999847	0.999853	0.999858	0.999864	0.999869	0.999874	0.999879	0.999883	0.999888
3.7	0.999892	0.999896	0.999900	0.999904	0.999908	0.999912	0.999915	0.999918	0.999922	0.999925
3.8	0.999928	0.999930	0.999933	0.999936	0.999938	0.999941	0.999943	0.999946	0.999948	0.999950
3.9	0.999952	0.999954	0.999956	0.999958	0.999959	0.999961	0.999963	0.999964	0.999966	0.999967
4.0	0.999968	0.999970	0.999971	0.999972	0.999973	0.999974	0.999975	0.999976	0.999977	0.999978

Appendix 5 – MS Excel formulas to assist in sampling methods

The formulas listed below can be used in MS Excel to assist in computing the various parameters required by the methods and concepts detailed in this guidance. For further information on the way these formulas work, you can refer to the Excel "help" file that provides the details of the underlying mathematical formulas.

In the above formulas (.) means a vector containing the address of the cells with the values of the sample or population.

=AVERAGE(.) : mean of a data set
=VAR.S(.) : variance of a sample data set
=VAR.P(.) : variance of a population data set
=STDEV.S(.) : standard deviation of a sample data set
=STDEV.P(.) : standard deviation of a population data set
=COVARIANCE.S(.) : covariance between two variables in a sample
=COVARIANCE.P(.) : covariance between two sample variables in a population
=RAND() : random number between 0 and 1, taken from a uniform distribution
=SUM(.) : sum of a data set

Appendix 6 – Glossary

Term	Definition
Anomalous error	An error/misstatement that is demonstrably not representative of the population. A statistical sample is representative for the population and therefore anomalous errors should only be accepted in very exceptional, well-motivated circumstances.
Anticipated error (AE)	The anticipated error is the amount of error the auditor expects to find in the population (after performing the audit). For sample size planning purposes the anticipated error rate is set to a maximum of 4.0% of the book value of the population.
Attribute sampling	Is a statistical approach to determine the level of assurance of the system and to assess the rate at which errors appear in a sample. Its most common use in auditing is to test the rate of deviation from a prescribed control to support the auditor's assessed level of control risk.
Audit assurance	The assurance model is the opposite of the risk model. If the audit risk is considered to be 5%, the audit assurance is considered to be 95%. The use of the audit assurance model relates to the planning and the underlying resource allocation for a particular programme or group of programmes.
Audit risk *(AR)*	Is the risk that the auditor issues an unqualified opinion, when the declaration of expenditure contains material errors.
Basic precision (*BP*)	Is used in Conservative MUS and corresponds the product between sampling interval and the reliability factor (RF) (already used for calculating sample size).
Book value *(BV)*	The expenditure declared to the Commission of an item (operation/payment claim), $BV_i, i = 1,2,\dots,N$. The total book value of a population comprises the sum of item book values in the population.
Confidence interval	The interval that contains the true (unknown) population value (in general the amount of error or the error rate) with a certain probability (called confidence level).

Term	Definition
Confidence level	The probability that a confidence interval produced by sample data contains the true population error (unknown).
Control risk *(CR)*	Is the perceived level of risk that a material error in the client's financial statements, or underlying levels of aggregation, will not be prevented, detected and corrected by the management's internal control procedures.
Correct book value *(CBV)*	The correct expenditure that would be found if all the operations/payments claims in the population were audited and no errors exist in the population.
Detection risk	Is the perceived level of risk that a material error in the client's financial statements, or underlying levels of aggregation, will not be detected by the auditor. Detection risks are related to performing audits of operations.
Difference estimation	Is a statistical sampling method based on selection with equal probabilities. The method relies on extrapolating the error in the sample. The extrapolated error is subtracted from the total declared expenditure in the population in order to assess the correct expenditure in the population (i.e. the expenditure that would be obtained if all the operations in the population were audited).
Error *(E)*	For the purposes of this guidance, an error is a quantifiable overstatement of the expenditure declared to the Commission. Is defined as the difference between the book value of the *i*-th item included in sample and the respective correct book value, $E_i = BV_i - CBV_i, i = 1,2,\dots,N$. If the population is stratified, an index h is used to denote the respective stratum: $E_{hi} = BV_{hi} - CBV_{hi}$, where $i = 1,2,\dots; N_h, h = 1,2,\dots,H$ and H is the number of strata.

Term	Definition
Expansion factor (*EF*)	Is a factor used in the calculation of conservative MUS when errors are expected, which is based upon the risk of incorrect acceptance. It reduces the sampling error. If no errors are expected, the anticipated error (AE) will be zero and the expansion factor is not used. Values for the expansion factor are found in section 6.3.4.2 of this guidance.
Incremental allowance (*IA*)	The incremental allowance measures the increment in the level of precision introduced by each error found in the sample. This allowance is used in the conservative approach to MUS and should be added to the basic precision value whenever errors are found in the sample (cf. section 6.3.4.5 of this guidance).
Inherent risk *(IR)*	Is the perceived level of risk that a material error may occur in the declared statements of expenditure to the Commission or underlying levels of aggregation, in the absence of internal control procedures. The inherent risk needs to be assessed before starting detailed audit procedures through interviews with management and key personnel, reviewing contextual information such as organisation charts, manuals and internal/external documents.
Irregularity	Same meaning as error.
Known error	An error found in the sample can lead the auditor to detect one or more errors outside that sample. These errors identified outside the sample are classified as "known errors". The error found in the sample is considered as random and included in the projection. This sample error that led to the identification of the known errors should therefore be extrapolated to the whole population as any other random error.

Term	Definition
Materiality	Errors are material if they exceed a certain level of error that is above what would be considered to be tolerable. A materiality level of 2% maximum is applicable to the expenditure declared to the Commission in the reference period. The audit authority can consider reducing the materiality for planning purposes (tolerable error). The materiality is used as a threshold to compare the projected error in expenditure;
Maximum tolerable error (*TE*)	The maximum acceptable error that can be found in the population for a certain year, i.e. the level of above which the population is considered materially misstated. With a 2% materiality level this maximum tolerable error is therefore 2% of the expenditure declared to the Commission for that reference period.
Misstatement	Same meaning as error.
Monetary Unit Sampling (MUS)	Is a statistical sampling method that uses the monetary unit as an auxiliary variable for sampling. This approach is usually based on systematic sampling with probability proportional to size (PPS), i.e. proportional to the monetary value of the sampling unit (high value items have larger probability of selection).
Multi-stage sampling	A sample which is selected by stages, the sampling units at each stage being sub-sampled from the (larger) units chosen at the previous stage. The sampling units pertaining to the first stage are called primary or first stage units; and similarly for second stage units, etc.
Population	The population for sampling purposes includes the expenditure declared to the Commission for operations within a programme or group of programmes in the reference period, except for negative sampling units (as explained below in section 4.6) and where the proportional control arrangements set out by Article 148(1) CPR and Article 28(8) of the Delegated Regulation (EU) No 480/2014 apply in the context of the sampling carried out for the programming period 2014-2020.

Term	Definition
Population size (N)	Is the number of operations or payment claims included in the expenditure declared to the Commission in reference period. If the population is stratified, an index h is used to denote the respective stratum, $N_h, h = 1,2, \ldots, H$ where H is the number of strata.
Planned precision	The maximum planned sampling error for sample size determination, i.e. the maximum deviation between the true population value and the estimate produced from sample data. Usually is the difference between maximum tolerable error and the anticipated error and it should be set to a value lower than the materiality level (or equal to).
(Effective) Precision (SE)	This is the error that arises because we are not observing the whole population. In fact, sampling always implies an estimation (extrapolation) error as the auditor relies on sample data to extrapolate to the whole population. This effective sampling error is an indication of the difference between the sample projection (estimate) and the true (unknown) population parameter (value of error). It represents the uncertainty in the projection of results to the population.
Projected/Extrapolated error (EE)	The projected/extrapolated error represents the estimated effect of random errors at population level.
Projected random error	The projected random error is the result of extrapolating the random errors found in the sample (in the audit of operations) to the total population. The extrapolation/projection procedure is dependent on the sampling method used.
Random error	The errors which are not considered systemic, known or anomalous are classified as random errors. This concept presumes the probability that random errors found in the audited sample are also present in the non-audited population. These errors are to be included in the calculation of the projection of errors.

Term	Definition
Reference period	This term corresponds to the period on which the AA needs to provide assurance. For the programming period 2007-2013, the reference period corresponds to the year N, to which the ACR submitted by end of year N+1 refers to; exceptions to this rule are applicable to the first ACR and to the final control report to be submitted by 31/03/2017 (cf. guidance on closure). For the programming period 2014-2020, the reference period corresponds to the accounting year that goes from 01/07/N till 30/06/N+1, to which the ACR submitted by 15 February of year N+2 refers to.
Reliability factor (RF)	The reliability factor RF is a constant from the Poisson distribution for an expected zero error. It is dependent on the confidence level and the values to apply in each situation can be found in section 6.3.4.2 of this guidance.
Risk of material error	Is the product of inherent and control risk. The risk of material error is related to the result of the system audits.
Sample error rate	The sample error rate corresponds to the amount of irregularities detected by the audits of operations divided by the expenditure audited.
Sample size (n)	Is the number of units/items included in the sample. If the population is stratified, an index h is used to denote the respective stratum, $\mathrm{n_h}, \mathrm{h} = 1,2, \ldots, \mathrm{H}$ and H is the number of strata.
Sampling error	The same as precision.
Sampling interval *(SI)*	Sampling interval is the selection step used in sampling methods based on systematic selection. For methods using selection probability proportional to expenditure (as the MUS method) the sampling interval is the ratio of the total book value in the population and the sample size.

Term	Definition
Sampling method	Sampling method encompasses two elements: the sampling design (e.g. equal probability, probability proportional to size) and the projection (estimation) procedure. Together, these two elements provide the framework to calculate sample size and project the error.
Sampling period	In the context of two-period sampling or multi-period sampling, the sampling period(s) refers to a part of the reference period (normally a trimester, four-months period or a semester). The sampling period may also be the same as the reference period.
Sampling unit	A sampling unit is one of the units into which a population is divided for the purpose of sampling. The sampling unit may be an operation, a project within an operation or a payment claim by a beneficiary.
Simple random sampling	Simple random sampling is a statistical sampling method. The statistical unit to be sampled is the operation (or payment claim, as explained above). Units in the sample are selected randomly with equal probabilities.
Standard-deviation (σ or s)	It is a measure of the variability of the population around its mean. It can be calculated using errors or book-values. When calculated over the population is usually represented by σ and when calculated over the sample is represented by s. The larger the standard-deviation the more heterogeneous is the population (sample).

Term	Definition
Stratification	Consists of partitioning a population into several groups (strata) according to the value of an auxiliary variable (usually the variable being audited, that is, the value of expenditure per operation within the audited programme). In stratified sampling independent samples are drawn from each stratum. The main goal of stratification is two-folded: on one hand usually allows an improvement of precision (for the same sample size) or a reduction of sample size (for the same level of precision); on the other hand ensures that the subpopulations corresponding to each stratum are represented in the sample.
Systemic error	The systemic errors are errors found in the sample audited that have an impact in the non-audited population and occur in well-defined and similar circumstances. These errors generally have a common feature, e.g. type of operation, location or period of time. They are in general associated with ineffective control procedures within (part of) the management and control systems.
Tolerable error	The tolerable error is the maximum acceptable error rate that can be found in the population. With a 2% materiality level, the tolerable error is therefore 2% of the expenditure declared to the Commission for the reference period.
Tolerable misstatement	Same meaning as tolerable error.
Total Book value	Total expenditure declared to the Commission for a programme or group of programmes, corresponding to the population from which the sample is drawn.
Total Error Rate (*TER*)	The total error rate corresponds to the sum of the following errors: projected random errors, systemic errors and uncorrected anomalous errors. All errors should be quantified by the audit authority and included in the TER, with the exception of corrected anomalous errors. Same meaning as total projected error rate (TPER) or total projected misstatement.

Term	Definition
Two-stage sampling	A sample which is selected by 2 stages, in which the sampling units of the second stage (sub-sampling units) are chosen from the sampling units of the main sample. In the case of ESI Funds audits, a typical example of two-stage sampling design is related to the use of operation at the first stage and the use of invoice as the sub-sampling unit at the second stage.
Upper limit of error (*ULE*)	This upper limit is equal to the summation of the projected error and the precision of the extrapolation. Same meaning as upper limit of confidence interval, upper limit for population misstatement and upper misstatement limit.
Variance (σ^2)	The square of the standard deviation
z	Is a parameter from the normal distribution related to the confidence level determined from system audits. The possible values of z are presented in section 5.3 of this guidance.