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Bi-unitary multiperfect numbers, I

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Dedicated to the memory of Prof. D. Suryanarayana

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Abstract: A divisor d of a positive integer n is called a unitary divisor if $\gcd(d, n/d) = 1$; and d is called a bi-unitary divisor of n if the greatest common unitary divisor of d and n/d is unity. The concept of a bi-unitary divisor is due to D. Surynarayana [12]. Let $\sigma^{**}(n)$ denote the sum of the bi-unitary divisors of n. A positive integer n is called a bi-unitary perfect number if $\sigma^{**}(n) = 2n$. This concept was introduced by C. R. Wall in 1972 [15], and he proved that there are only three bi-unitary perfect numbers, namely 6, 60 and 90.

In 1987, Peter Hagis [6] introduced the notion of bi-unitary multi k-perfect numbers as solutions n of the equation $\sigma^{**}(n) = kn$. A bi-unitary multi 3-perfect number is called a bi-unitary triperfect number. A bi-unitary multiperfect number means a bi-unitary multi k-perfect number with $k \geq 3$. Hagis [6] proved that there are no odd bi-unitary multiperfect numbers. We aim to publish a series of papers on bi-unitary multiperfect numbers focusing on multiperfect numbers of the form $n = 2^a u$, where u is odd. In this paper—part I of the series—we investigate bi-unitary triperfect numbers of the form $n = 2^a u$, where $1 \leq a \leq 3$. It appears that $n = 120 = 2^3 15$ is the only such number. Hagis [6] found by computer the bi-unitary multiperfect numbers less than 10^7. We have found 31 such numbers up to 8.10^{10}. The first 13 are due to Hagis. After completing this paper we noticed that further numbers are already listed in *The On-Line Encyclopedia of Integer Sequences* (sequence A189000 Bi-unitary multiperfect numbers). The numbers listed there have been found by direct computer calculations. Our purpose is to present a mathematical search and treatment of bi-unitary multiperfect numbers.

Keywords: Perfect numbers, Triperfect numbers, Multiperfect numbers, Bi-unitary analogues.
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1 Introduction

Throughout this paper, all lower case letters denote positive integers; p and q denote primes. The letters u, v and w are reserved for odd numbers.

Let $\sigma(n)$ denote the sum of the positive divisors of n. It is well known that a positive integer n is called a *perfect number* if $\sigma(n) = 2n$; a concept introduced by Euclid (300 BC). He demonstrated that numbers of the form $n = 2^{p-1}(2^p - 1)$ are perfect numbers, where p and $2^p - 1$ are primes.

The converse that these numbers are the only even perfect numbers was established by Euler two thousand years later. Primes of the form $2^p - 1$, where p is a prime number, are called *Mersenne primes* and are denoted by M_p. Each Mersenne prime gives rise to an even perfect number. The first few examples of even perfect numbers are $6, 28, 496, 8128$ and 33550336 which correspond to the primes $p = 2, 3, 5, 7$ and 13. One does not know whether this list of even perfect numbers can be continued indefinitely as the answer depends on whether there exist infinitely many Mersenne primes. Till now, 51 Mersenne primes are known (consequently 51 even perfect numbers). The internet information says that the 51-st Mersenne prime namely $2^{82,589,933} - 1$ was found by Patrick Laroche on December 7, 2018. This appears to be the largest known prime with 24,862,048 digits.

Till to date, we do not know an example of an odd perfect number. Euler proved that if n is an odd perfect number, then $n = p^a k^2$, where p is a prime not dividing the positive integer k and $p \equiv a \equiv 1 \pmod 4$. In 2012, Pascal Ochem and Michael Rao (cf. [7]) proved that there are no odd perfect numbers less than 10^{1500} and an odd perfect number should have at least 101 distinct prime factors. These appear to be the latest results of this type on odd perfect numbers.

A positive integer n is called *multiply perfect* if $\sigma(n) = kn$ for some positive integer k and the integer k is called the multiplicity of the number n; or equivalently n is multiply perfect if and only if $n|\sigma(n)$. Multiply perfect numbers with multiplicity k are called *multi k-perfect numbers* or *k-fold perfect numbers*. Multiply perfect numbers with multiplicity 2 are of course perfect numbers. Multi 3-perfect numbers are called *triperfect numbers*. For example, the smallest positive integers that are solutions of $\sigma(n) = kn$ for $k = 1, 2, 3, 4, 5$ are, respectively, $1, 6, 120$, 30240 and $14182439040 = 2^7.3^4.5.7.11^2 17.19$.

Examples of multi k-perfect numbers are known for $1 \le k \le 11$ (see Wikipedia). The number $n = 2^{468}.3^{140}.5^{66}.11^{40}.17^{11}.19^{12} \ldots$, a multiply perfect number of multiplicity 11 and with 1907 digits appears to have been found by G.F. Woltman on March 13, 2001.

Till now, the only known odd multiply perfect number is 1. It is yet to be known whether there are finitely many multiply perfect numbers for each fixed multiplicity > 2 (see Wikipedia).

An open question is whether each multi k-perfect number is divisible by $k!$. Note that L. E. Dickson's book [2] contains a good account of work on multiply perfect numbers (see also [3, 8]).

A divisor d of n is called a *unitary divisor* if $\gcd(d, n/d) = 1$. If d is a unitary divisor of n, we write $d\|n$. The concept of unitary divisor was originally due to R. Vaidyanathaswamy [13] who called such a divisor as block factor. The present terminology is due to E. Cohen [1]. Let $\sigma^*(n)$ denote the sum of the unitary divisors of n. It is known (cf. [1]) that σ^* is a multiplicative function, $\sigma^*(1) = 1$ and $\sigma^*(p^a) = p^a + 1$ whenever p is a prime and a is a positive integer. M. V. Subbarao and L. J. Warren [11] introduced the notion of a *unitary perfect number* as follows: a positive integer n is called a unitary perfect number if $\sigma^*(n) = 2n$. They proved that there are no odd unitary perfect numbers. The only five unitary perfect numbers known till now are $6, 60, 90, 87360$ and

$$146361946186458562560000 = 2^{18}.3.5^4.7.11.13.19.37.79.109.157.313.$$

The first four examples were due to M. V. Subbarao and L. J. Warren [11] and the last being due to C. R. Wall [14, 16]. It is not known whether there exists a unitary perfect number not divisible by 3 (see M. V. Subbarao and L. J. Warren [11]) and also whether there are infinitely many unitary perfect numbers (see M. V. Subbarao [10]).

A positive integer m is called unitary multiply perfect if $m|\sigma^*(m)$; that is, it is solution of $\sigma^*(m) = km$ for some positive integer k. In the case $k > 2$, no example of a unitary multiply perfect number is known till now. It is known that if such a number exists, it must be even and greater than 10^{102} and must have more than forty four odd prime factors (see [4,5]). This problem is probably very difficult to settle.

It is easily seen that if p is a prime, a and n are positive integers such that

$$p^a|\sigma^*(n) \text{ and } n|\sigma^*(p^a), \tag{1.1}$$

then

$$m = p^a n \tag{1.2}$$

is unitary multiply perfect number. However it has been proved in [9] that the only solutions of (1.1) are $p = 2$, $a = 1$, $n = 3$; $p = 3$, $a = 1$, $n = 2$; and $p = 3$, $a = 2$, $n = 10$. The only solutions of (1.1) in the form (1.2) are $m = 6$ and $m = 90$, which are unitary perfect numbers. Thus if $k > 3$, there are no solutions to the equation $\sigma^*(m) = km$ of the form (1.2).

A divisor d of n is called a *bi-unitary* divisor if $(d, n/d)^{**} = 1$, where the symbol $(a, b)^{**}$ denotes the greatest common unitary divisor of a and b. The concept of a bi-unitary divisor is due to D. Suryanarayana [12].

Let $\sigma^{**}(n)$ denote the sum of bi-unitary divisors of n. The function $\sigma^{**}(n)$ is multiplicative, that is, $\sigma^{**}(1) = 1$ and $\sigma^{**}(mn) = \sigma^{**}(m)\sigma^{**}(n)$ whenever $(m, n) = 1$. If p^α is a prime power and α is odd then every divisor of p^α is a bi-unitary divisor; if α is even, each divisor of p^α is a bi-unitary divisor except for $p^{\alpha/2}$. Hence

$$\sigma^{**}(p^\alpha) = \begin{cases} \sigma(p^\alpha) = \frac{p^{\alpha+1}-1}{p-1} & \text{if } \alpha \text{ is odd,} \\ \sigma(p^\alpha) - p^{\alpha/2} & \text{if } \alpha \text{ is even.} \end{cases} \tag{1.3}$$

If α is even, say $\alpha = 2k$, then $\sigma^{**}(p^\alpha)$ can be simplified to

$$\sigma^{**}(p^\alpha) = \left(\frac{p^k - 1}{p - 1}\right).(p^{k+1} + 1). \tag{1.4}$$

From (1.3), it is not difficult to observe that $\sigma^{**}(n)$ is odd only when $n = 1$ or $n = 2^\alpha$.

The concept of a bi-unitary perfect number was introduced by C. R. Wall in 1972 [15]; a positive integer n is called a bi-unitary perfect number if $\sigma^{**}(n) = 2n$. In [15], C. R. Wall proved that there are only three bi-unitary perfect numbers, namely $6, 60$ and 90.

In 1987, Peter Hagis [6] introduced the notion of bi-unitary multi k-perfect numbers; n is called a bi-unitary multi k-perfect number if $\sigma^{**}(n) = kn$. A bi-unitary multiply perfect number is simply a bi-unitary multi k-perfect number for some positive integer k. Equivalently, bi-unitary multiply perfect numbers n are solutions of $n|\sigma^{**}(n)$. A bi-unitary multi 3-perfect number is called a bi-unitary triperfect number.

Since $\sigma^{**}(p^\alpha) = \sigma(p^\alpha)$ when α is odd, it follows that if $n = \prod_i p_i^{\alpha_i}$ is the canonical representation of n with each α_i odd then $\sigma^{**}(n) = \sigma(n)$. Hence, if n has all odd exponents in its canonical representation, then n is multiply perfect if and only if it is bi-unitary multiply perfect. We may also note that if k and n are relatively prime and n is bi-unitary multi k-perfect, then kn is bi-unitary multi m-perfect, where $m = \sigma^{**}(k)$.

Hereafter, a bi-unitary multiperfect number means a bi-unitary multi k-perfect number with $k \geq 3$. Peter Hagis [6] proved that there are no odd bi-unitary multiperfect numbers. This motivates us to examine even bi-unitary multiperfect numbers.

We are going to publish a series of papers on bi-unitary multiperfect numbers focusing on multiperfect numbers of the form $n = 2^a u$, where u is odd. In this paper, that is the first part of the series, we investigate bi-unitary triperfect numbers of the form $n = 2^a u$, where $1 \leq a \leq 3$. In Section 3, we prove (see Theorem 3.1) that if $1 \leq a \leq 3$ and $n = 2^a u$ is a bi-unitary triperfect number, then $a = 3$ and $n = 120 = 2^3.3.5$. This shows that there exists only one bi-unitary triperfect number of this type.

Peter Hagis [6] found by computer all bi-unitary multiperfect numbers less than 10^7. We have found these numbers from 10^7 to 8.10^{10}. A list of these numbers (up to 8.10^{10}) is given in Appendix A. This list contains 31 bi-unitary multiperfect numbers. The first 13 are due to Hagis [6].

Note. After completing this paper we found that even more bi-unitary multiperfect numbers are already listed in *The On-Line Encyclopedia of Integer Sequences* (sequence A189000 Bi-unitary multiperfect numbers). The numbers listed there have been found by direct computer calculations. On the contrary, our purpose is to present a mathematical search and treatment of bi-unitary multiperfect numbers.

Section 2 contains preliminaries needed in the main topic. Appendix A contains our list of bi-unitary multiperfect numbers less than 8.10^{10}. Appendix B/Appendix C contain tables helpful in finding the divisibility of ($5^t - 1$ and $5^t + 1)/(7^t - 1$ and $7^t + 1$) by primes in the interval $[3, 2501]$. Appendices D and E contain factorizations of numbers of the form $5^t - 1$ and $5^t + 1$, respectively, and Appendices F and G contain factorizations of numbers of the form $7^t - 1$ and $7^t + 1$, respectively. These are needed here and in the future papers.

2 Preliminaries

Lemma 2.1. (I) *If α is odd, then*

$$\frac{\sigma^{**}(p^\alpha)}{p^\alpha} > \frac{\sigma^{**}(p^{\alpha+1})}{p^{\alpha+1}}$$

for any prime p.

(II) *For any $\alpha \geq 2\ell - 1$ and any prime p,*

$$\frac{\sigma^{**}(p^\alpha)}{p^\alpha} \geq \left(\frac{1}{p-1}\right)\left(p - \frac{1}{p^{2\ell}}\right) - \frac{1}{p^\ell} = \frac{1}{p^{2\ell}}\left(\frac{p^{2\ell+1}-1}{p-1} - p^\ell\right).$$

(III) *If p is any prime and α is a positive integer, then*

$$\frac{\sigma^{**}(p^\alpha)}{p^\alpha} < \frac{p}{p-1}.$$

Proof. **(I)** Since α is odd,

$$\sigma^{**}(p^\alpha) = \sigma(p^\alpha) = 1 + p + p^2 + \cdots + p^\alpha$$

and

$$\sigma^{**}(p^{\alpha+1}) = \sigma(p^{\alpha+1}) - p^{\frac{\alpha+1}{2}} = 1 + p + \cdots + p^\alpha + p^{\alpha+1} - p^{\frac{\alpha+1}{2}}.$$

Hence

$$\begin{aligned}
\frac{\sigma^{**}(p^\alpha)}{p^\alpha} &> \frac{\sigma^{**}(p^{\alpha+1})}{p^{\alpha+1}} \\
\text{if and only if } \ p\sigma^{**}(p^\alpha) &> \sigma^{**}(p^{\alpha+1}) \\
\text{if and only if } \ p[1 + p + p^2 + \cdots + p^\alpha] &> [1 + p + \cdots + p^\alpha + p^{\alpha+1} - p^{\frac{\alpha+1}{2}}] \\
\text{if and only if } \ [p + p^2 + \cdots + p^{\alpha+1}] &> [1 + p + \cdots + p^\alpha + p^{\alpha+1} - p^{\frac{\alpha+1}{2}}] \\
\text{if and only if } \ 0 &> 1 - p^{\frac{\alpha+1}{2}}.
\end{aligned}$$

(II) Let $\alpha \geq 2\ell - 1$.

Case 1. Let α be odd. If $\alpha + 1 = 2k$, then $k \geq \ell$. By (I),

$$\begin{aligned}
\frac{\sigma^{**}(p^\alpha)}{p^\alpha} &> \frac{\sigma^{**}(p^{\alpha+1})}{p^{\alpha+1}} \\
&= \frac{\sigma(p^{2k}) - p^k}{p^{2k}} = \frac{\left(\frac{p^{2k+1}-1}{p-1} - p^k\right)}{p^{2k}} = \left(\frac{1}{p-1}\right)\left(p - \frac{1}{p^{2k}}\right) - \frac{1}{p^k} \\
&\geq \left(\frac{1}{p-1}\right)\left(p - \frac{1}{p^{2\ell}}\right) - \frac{1}{p^\ell}.
\end{aligned}$$

Case 2. Let α be even and $\alpha = 2k$. Then $\alpha \geq 2\ell - 1$ implies that $k \geq \ell$. Hence

$$\frac{\sigma^{**}(p^\alpha)}{p^\alpha} = \left(\frac{1}{p-1}\right)\left(p - \frac{1}{p^{2k}}\right) - \frac{1}{p^k} \geq \left(\frac{1}{p-1}\right)\left(p - \frac{1}{p^{2\ell}}\right) - \frac{1}{p^\ell}.$$

The proof of (II) is complete.

(III) We have

$$\frac{\sigma^{**}(p^\alpha)}{p^\alpha} \leq \frac{\sigma(p^\alpha)}{p^\alpha} = \frac{p^{\alpha+1}-1}{p^\alpha(p-1)} = \frac{1}{p-1}.\left[p - \frac{1}{p^\alpha}\right] < \frac{p}{p-1}.$$ □

Remark 2.1. (I) and (III) of Lemma 2.1 are mentioned in C. R. Wall [15]; (II) of Lemma 2.1 has been used by him [15] without explicitly stating it.

Lemma 2.2. *Let $a > 1$ be an integer not divisible by an odd prime p and let α be a positive integer. Let r denote the least positive integer such that $a^r \equiv 1\,(mod\ p^\alpha)$; then r is usually denoted by $ord_{p^\alpha}\, a$. We have the following properties:*

(i) *If r is even, then $s = r/2$ is the least positive integer such that $a^s \equiv -1\,(mod\ p^\alpha)$. Also, $a^t \equiv -1\,(mod\ p^\alpha)$ for a positive integer t if and only if $t = su$, where u is odd.*

(ii) *If r is odd, then $p^\alpha \nmid a^t + 1$ for any positive integer t.*

The proof is straightforward.

Remark 2.2. Let a, p, r and $s = r/2$ be as in Lemma 2.2 ($\alpha = 1$). Then, $p|a^t - 1$ if and only if $r|t$. If t is odd and r is even, then $r \nmid t$. Hence, $p \nmid a^t - 1$. Also, $p|a^t + 1$ if and only if $t = su$, where u is odd. In particular, if t is even and s is odd, then $p \nmid a^t + 1$. In order to check the divisibility of $a^t - 1$ (when t is odd) by an odd prime p, we can confine to those p for which $ord_p\, a$ is odd. Similarly, for examining the divisibility of $a^t + 1$ by p when t is even we need to consider primes p with $s = ord_p\, a/2$ even.

Lemma 2.3. *Let k be odd and $k \geq 3$. Let $p \neq 5$.*

(a) *If $p \in [3, 2520] - \{11, 19, 31, 71, 181, 829, 1741\}$, $ord_p 5$ is odd and $p|5^k - 1$, then we can find a prime p' (depending on p) such that $p'|\frac{5^k-1}{4}$ and $p' \geq 2521$.*

(b) *If $q \in [3, 2520] - \{13, 313, 601\}$, $s = \frac{1}{2} ord_q 5$ is even and $q|5^{k+1} + 1$ then we can find a prime q' (depending on q) such that $q'|\frac{5^{k+1}+1}{2}$ and $q' \geq 2521$.*

Proof. (a) Let $p|5^k - 1$. If $r = ord_p 5$, that is, r is the least positive integer such that $5^r \equiv 1 \pmod{p}$, then $r|k$. Since k is odd, r must be odd. Also, $5^r - 1|5^k - 1$. Let

$$S_5 = \{(p, r)\ : p \neq 5, p \in [3, 2520] \text{ and } r = ord_p 5 \text{ odd}\}.$$

From Appendix B, we have

$$\begin{aligned} S_5 = \{&(11, 5), (19, 9), (31, 3), (59, 29), (71, 5), (79, 39), (101, 25), (109, 27), (131, 65), (139, 69),\\ &(149, 37), (151, 75), (179, 89), (181, 15), (191, 19), (199, 33), (211, 35), (239, 119),\\ &(251, 25), (269, 67), (271, 27), (311, 155), (331, 165), (359, 179), (379, 21), (389, 97),\\ &(401, 25), (409, 17), (419, 209), (431, 215)(439, 219), (461, 115), (479, 239), (491, 245),\\ &(499, 249), (541, 135), (569, 71), (571, 285), (599, 299), (619, 309)(631, 35), (659, 329),\\ &(691, 115), (719, 359), (739, 123), (751, 375), (811, 405), (829, 9), (839, 419), (859, 429),\\ &(911, 455), (919, 459), (941, 235), (971, 485), (991, 495), (1019, 509), (1021, 255),\\ &(1031, 515), (1039, 173), (1051, 525), (1061, 265), (1069, 267), (1091, 109), (1151, 575),\\ &(1171, 45), (1231, 615), (1259, 629), (1279, 639), (1291, 215), (1319, 659), (1399, 699),\\ &(1429, 119), (1439, 719), (1451, 725), (1459, 243), (1471, 735), (1499, 749), (1511, 755),\\ &(1531, 85), (1559, 779), (1571, 785), (1579, 789), (1609, 67), (1619, 809), (1621, 405),\\ &(1699, 283), (1741, 15), (1759, 293), (1811, 905), (1831, 305), (1861, 31), (1871, 935),\\ &(1879, 939), (1931, 965), (1949, 487), (1951, 975), (1979, 989), (1999, 999), (2011, 1005),\\ &(2039, 1019), (2099, 1049), (2111, 1055), (2131, 1065), (2179, 1089), (2251, 1125),\\ &(2239, 373), (2269, 567), (2309, 577), (2311, 165), (2339, 1169), (2351, 1175), (2371, 1185),\\ &(2239, 373), (2399, 1199), (2411, 1205), (2441, 305), (2459, 1229)\}. \end{aligned}$$

Let $p|5^k-1$ and $p \in [3, 2520] - \{11, 19, 31, 71, 181, 829, 1741\}$. Then

$$(p, r) \in S_5 - \{(11, 5), (19, 9), (31, 3), (71, 5), (181, 15), (829, 9), (1741, 15)\},$$

where $r = ord_p 5$. Also, $5^r - 1|5^k - 1$. To prove (a), it is enough to show that $\frac{5^r-1}{4}$ is divisible by a prime $p' \geq 2521$. From Appendix D, we can know the factors of $5^r - 1$. By examining the factors of $5^r - 1$ for $r \notin \{5, 9, 3, 5, 15, 9, 15\}$ which correspond to the primes $11, 19, 31, 71, 181, 829$ and 1741, respectively, we infer that we can a find a prime $p'|\frac{5^r-1}{4}|\frac{5^k-1}{4}$ satisfying $p' \geq 2521$. This proves (a).

For example, if $p = 59$, then $r = 29$. From Appendix D,

$$5^{29} - 1 = \{\{2, 2\}, \{59, 1\}, \{35671, 1\}, \{22125996444329, 1\}\}.$$

Thus, if $59|5^k - 1$, then $p' = 35671|\frac{5^k-1}{4}$ and trivially $p' > 2521$.

(b) Let $q|5^{k+1}+1$ and $q \in [3, 2520] - \{13, 313, 601\}$. Let $r = ord_q 5$. If r is odd, then $q \nmid 5^{k+1}+1$ (See Remark 2.2 ($a = 5$)). We may assume that r is even. Let $s = r/2$. Then s is the least positive integer such that $q|5^s + 1$. Again from Remark 2.2 ($a = 5$), $q \nmid 5^{k+1} + 1$ if s is odd. Since $q|5^{k+1} + 1$, we have that s is even. Also, $k + 1 = su$, where u is odd. This implies that $5^s + 1|5^{k+1} + 1$. Let

$$T_5 = \{(q, s) : q \neq 5, q \in [3, 2520] \text{ and } s = \frac{1}{2} ord_q 5 \text{ even}\}.$$

From Appendix B, we have

$$\begin{aligned} T_5 =\{&(13, 2), (17, 8), (37, 18), (41, 10), (53, 26), (73, 36), (89, 22), (97, 48), (113, 56), (137, 68),\\ &(157, 78), (173, 86), (193, 96), (197, 98), (233, 116), (241, 20), (257, 128), (277, 138),\\ &(281, 70), (293, 146), (313, 4), (317, 158), (337, 56), (353, 176), (373, 186), (397, 198),\\ &(433, 216), (457, 76), (557, 278), (577, 288), (593, 296), (601, 6), (613, 306), (617, 308),\\ &(641, 32), (653, 326), (673, 336), (677, 338), (733, 122), (757, 378), (769, 64), (773, 386),\\ &(797, 398), (809, 202), (853, 142), (857, 428), (877, 438), (881, 220), (929, 116), (937, 468),\\ &(953, 476), (977, 488), (997, 166), (1009, 252), (1013, 252), (1033, 506), (1049, 262),\\ &(1093, 546), (1097, 548), (1117, 186), (1153, 576), (1193, 596), (1201, 300), (1213, 606),\\ &(1217, 608), (1237, 618), (1249, 312), (1277, 638), (1289, 322), (1297, 72), (1321, 330),\\ &(1361, 170), (1373, 686), (1409, 352), (1433, 716), (1453, 66), (1481, 370), (1489, 186),\\ &(1493, 746), (1553, 776), (1597, 266), (1601, 200), (1613, 806), (1637, 818), (1657, 276),\\ &(1693, 846), (1697, 848), (1733, 866), (1753, 292), (1777, 888), (1801, 450), (1873, 312),\\ &(1877, 938), (1889, 236)(1913, 956), (1933, 966), (1973, 34), (1993, 996), (1997, 998),\\ &(2017, 1008), (2053, 1026), (2081, 260), (2089, 522), (2113, 1056), (2137, 356),\\ &(2153, 1076), (2161, 540), (2213, 1106), (2237, 1118), (2273, 1136), (2281, 30),\\ &(2293, 1146), (2297, 1148), (2333, 1166), (2357, 1178), (2377, 1188), (2393, 1196),\\ &(2417, 1208), (2473, 1236), (2477, 1238).\end{aligned}$$

Let $q|5^{k+1}+1$ and $q \in [3, 2520]-\{13, 313, 601\}$. Then $(q, s) \in T-\{(13, 2), (313, 4), (601, 6)\}$, where $s = \frac{1}{2}ord_q 5$. To prove (b), it is enough to show that $\frac{5^s+1}{2}$ is divisible by a prime $q' \geq 2521$ for all $s \in T' = \{s : (q, s) \in T - \{(13, 2), (313, 4), (601, 6)\}\}$. This follows by examining the factors of $5^t + 1$ given in Appendix E.

For example, if $q = 53$, then $s = 26$. Also,

$$5^{26} + 1 = \{\{2, 1\}, \{13, 2\}, \{53, 1\}, \{83181652304609, 1\}\}.$$

We can take $q' = 83181652304609$.

The proof of Lemma 2.3 is complete. □

Lemma 2.4. *Let k be odd and $k \geq 3$. Let $p \neq 7$.*

(a) *If $p \in [3, 2520] - \{3, 19, 37, 1063\}$, $r = ord_p 7$ is odd and $p|7^k - 1$, then we can find a prime p' (depending on p) such that $p'|\frac{7^k-1}{6}$ and $p' \geq 2521$.*

(b) *If $q \in [3, 1193] - \{5, 13, 181, 193, 409\}$, $s = \frac{1}{2}ord_q 7$ is even and $q|7^{k+1} + 1$, then we can find a prime q' (depending on q) such that $q'|\frac{7^{k+1}+1}{2}$ and $q' > 1193$.*

Proof. (a) Let $p|7^k-1$. If $r = ord_p 7$ that is, r is the least positive integer such that $7^r \equiv 1 \pmod{p}$, then $r|k$. Since k is odd, r must be odd. Also, $7^r - 1|7^k - 1$. Let

$$S_7 = \{(p, r) : p \neq 7, p \in [3, 2520] \text{ and } r = ord_p 7 \text{ odd}\}.$$

From Appendix C, we have

$$\begin{aligned} S_7 = \{&(3,1),(19,3),(29,7),(31,15),(37,9),(47,23),(59,29),(83,41),(103,51),(109,27),\\ &(131,65),(139,69),(167,83),(199,99),(223,37),(227,113),(251,125),(271,135),\\ &(283,141),(307,153),(311,31),(367,61),(383,191),(389,97),(419,19),(439,73),\\ &(467,233),(479,239),(503,251),(523,261),(563,281),(587,293),(607,101),(613,153),\\ &(619,309),(643,321),(647,323),(653,163),(691,345),(701,175),(709,177),(719,359),\\ &(727,363),(757,189),(787,393),(809,101),(811,27),(839,419),(859,(429),(877,219),\\ &(887,443),(971,97),(983,491),(1039,519),(1061,265),(1063,9),(1091,545),(1093,273),\\ &(1123,11),(1151,115),(1213,303),(1223,611),(1231,615),(1259,629),(1279,639),\\ &(1291,645),(1307,653),(1319,659),(1373,343),(1381,345),(1399,699),(1427,713),\\ &(1429,357),(1447,241),(1453,121),(1459,243),(1481,185),(1483,741),(1487,743),\\ &(1511,755),(1531,85),(1543,257),(1559,779),(1567,783),(1571,785),(1621,81),\\ &(1627,813),(1699,849),(1733,433),(1783,891),(1811,181),(1823,911),(1847,923),\\ &(1867,933),(1873,117),(1879,939),(1907,953),(1931,965),(1951,975),(1979,989),\\ &(1987,993),(1997,499),(2063,1031),(2069,517),(2099,517),(2131,355),(2153,269),\\ &(2203,1101),(2213,553),(2237,559),(2239,373),(2243,1121),(2267,1133),(2269,567),\\ &(2287,1143),(2333,583),(2351,1175),(2371,237),(2377,27),(2383,397),(2399,1199),\\ &(2411,1205),(2467,1233)\}. \end{aligned}$$

Let $p|7^k-1$ and $p \in [3,2520]-\{3,19,37,1063\}$. Then

$$(p,r) \in S_7 - \{(3,1),(19,3),(37,9),(1063,9)\},$$

where $r = ord_p 7$. Also, $7^r-1|7^k-1$. To prove (a), it is enough to show that $\frac{7^r-1}{6}$ is divisible by a prime $p' \geq 2521$. From Appendix F, we know the factors of 7^r-1. By examining the factors of 7^r-1 for $r \notin \{1,3,9,9\}$ which correspond to the primes $3,19,37,1063$, respectively, we infer that we can a find a prime $p'|\frac{7^r-1}{6}|\frac{7^k-1}{6}$ satisfying $p' \geq 2521$. This proves (a).

For example, if $p=47$, then $r=23$. Also,

$$7^{23}-1 = \{\{2,1\},\{3,1\},\{47,1\},\{3083,1\},\{31479823396757,1\}\}.$$

We can take $p'=3083$.

(b) Let $q|7^{k+1}+1$ and $q \in [3,1193]-\{5,13,181,193,409\}$. Let $r = ord_q 7$. If r is odd, then $q \nmid 7^{k+1}+1$ (See Remark 2.2 ($a=7$)). We may assume that r is even. Let $s=r/2$. Then s is the least positive integer such that $q|7^s+1$. Again from Remark 2.2 ($a=7$), $q \nmid 7^{k+1}+1$ if s is odd. Since $q|7^{k+1}+1$, we have that s is even. Also, $k+1=su$, where u is odd. This implies that $7^s+1|7^{k+1}+1$. Let

$$T_7 = \{(q,s) : q \neq 7, q \in [3,1193] \text{ and } s = \frac{1}{2} ord_q 7 \text{ even}\}.$$

From Appendix C, we have

$$\begin{aligned}T_7 =&\{(5,2),(13,6),(17,8),(41,20),(61,30),(73,12),(89,44),(97,48),(101,50),(137,34),\\&(157,26),(173,86),(181,6),(193,12)(229,114),(233,58),(241,120),(257,128),\\&(269,134),(281,10),(293,146),(313,52),(337,28),(349,174),(353,16),(397,198),\\&(401,100),(409,12),(433,216),(449,56),(461,230),(509,254),(521,260),(569,142),\\&(577,288),(593,296),(601,300),(617,154),(641,160),(661,22),(673,56),(677,38),\\&(733,366),(761,380),(769,128),(773,386),(797,398),(829,138),(853,142),(857,428),\\&(881,40),(929,464),(937,468),(941,470),(977,244),(997,498),(1009,126),\\&(1013,506),(1021,170),(1033,86),(1049,524),(1069,534),(1097,548),(1109,554),\\&(1129,282),(1153,192),(1181,590),(1193,596)\}.\end{aligned}$$

Let $q|7^{k+1}+1$ and $q \in [3,1193]-\{5,13,181,193,409\}$. Then

$$(q,s) \in T_7 - \{(5,2),(13,6),(181,6),(193,12),(409,12)\},$$

where $s = \frac{1}{2} ord_q 7$. To prove (b), it is enough to show that $\frac{7^s+1}{2}$ is divisible by a prime $q' > 1193$ for all $s \in T_7' = \{s : (q,s) \in T_7 - \{(5,2),(13,6),(181,6),(193,12),(409,12)\}$. This follows by examining the factors of 7^s+1 given in Appendix G.

For example if $q=41$ then $s=20$. Also,

$$7^{20}+1 = \{\{2,1\},\{41,1\},\{1201,1\},\{810221830361,1\}\}.$$

We can take $q'=1201$. $\square$

3 Bi-unitary triperfect numbers of the form $n = 2^a u$ with $1 \leq a \leq 3$

In this section, we find all bi-unitary triperfect numbers n such that $2^a \| n$, where $1 \leq a \leq 3$.

Theorem 3.1. *Let $1 \leq a \leq 3$. If $n = 2^a u$ (where u is odd) is a bi-unitary triperfect number, then $a = 3$ and $n = 120 = 2^3.3.5$.*

Proof. Let $n = 2^a u$ be a bi-unitary triperfect number so that n is a solution of

$$\sigma^{**}(n) = 3n. \tag{3.1}$$

(i) Let $a = 1$. Then $n = 2u$. From (3.1), we have $6u = 3n = \sigma^{**}(2u) = \sigma^{**}(2).\sigma^{**}(u) = 3\sigma^{**}(u)$ so that $\sigma^{**}(u) = 2u$. Hence u is an odd bi-unitary perfect number. But such a number does not exist (C. R. Wall [15]). Hence $a = 1$ is not possible.
(ii) Let $a = 2$ so that $n = 4u$. From (3.1), we have

$$12u = 3n = \sigma^{**}(n) = \sigma^{**}(4).\sigma^{**}(u) = 5.\sigma^{**}(u). \tag{3.1a}$$

and so $5|u$. Hence we can write $u = 5^b.v$, where $b \geq 1$ and v is prime to 2.5. Thus we have

$$n = 4u = 2^2.5^b.v, \quad (v, 2.5) = 1, \tag{3.1b}$$

and from (3.1a), we obtain $12.5^b.v = 5.\sigma^{**}(5^b).\sigma^{**}(v)$, or

$$2^2.3.5^{b-1}.v = \sigma^{**}(5^b).\sigma^{**}(v). \tag{3.1c}$$

Considering the parity of the values of the function σ^{**} in (3.1c), it is clear that v cannot have more than one odd prime factor. Hence $v = 1$ or $v = p^c$, where p is an odd prime. In any case $\frac{\sigma^{**}(v)}{v} < \frac{p}{p-1}$. Hence from (3.1b), we have by Lemma 2.1,

$$3 = \frac{\sigma^{**}(n)}{n} < \frac{5}{4}.\frac{5}{4}.\frac{p}{p-1} \leq \frac{5}{4}.\frac{5}{4}.\frac{3}{2} = 2.34375 < 3,$$

a contradiction. Thus $a = 2$ is not possible.
(iii) Let $a = 3$, so that $n = 2^3.u$. Since $\sigma^{**}(2^3) = 15$, from (3.1), we obtain $3.2^3.u = 3n = \sigma^{**}(n) = 15.\sigma^{**}(u)$ and after simplification,

$$2^3.u = 5.\sigma^{**}(u). \tag{3.2}$$

From (3.2) we have $5|u$ and so $u = 5^b.v$, where $(v, 2.5) = 1$. Hence

$$n = 2^3.5^b.v, \tag{3.2a}$$

and from (3.2), after simplification we obtain

$$2^3.5^{b-1}.v = \sigma^{**}(5^b).\sigma^{**}(v); \tag{3.2b}$$

$$v \text{ cannot have more than two odd prime factors.} \tag{3.2c}$$

The remaining proof of (iii) depends on the following lemmas:

Lemma 3.1. *Let n be as given in (3.2a). If $b = 1$ and n is a bi-unitary triperfect number, then $n = 120 = 2^3.3.5$.*

Proof. Let $b = 1$ and n be a bi-unitary triperfect number. Hence n satisfies (3.2b) $(b = 1)$. Taking $b = 1$ in (3.2b), we get $2^3.v = 6.\sigma^{**}(v)$ or

$$2^2.v = 3.\sigma^{**}(v); \tag{3.2d}$$

hence $3|v$. Let $v = 3^c.w$, where $(w, 2.3.5) = 1$. From (3.2a) $(b = 1)$, we get

$$n = 2^3.5.3^c.w, \tag{3.3a}$$

and from (3.2d),

$$2^2.3^{c-1}.w = \sigma^{**}(3^c).\sigma^{**}(w), \tag{3.3b}$$

where w can have at most one odd prime factor and $(w, 2.3.5) = 1$.

Let $c = 1$. From (3.3b), we get $2^2.w = 4.\sigma^{**}(w)$ or $w = \sigma^{**}(w)$; hence $w = 1$. Thus when $c = 1$, (3.3b) is satisfied. Hence $n = 2^3.5.3 = 120$ is a bi-unitary triperfect number.

Let $c = 2$. From (3.3b) $(c = 2)$, we get $2^2.3.w = 10.\sigma^{**}(w)$ and this implies that $5|w$. This is false since w is prime to 5.

So we may assume that $c \geq 3$. By Lemma 2.1 ($\ell = 2$),

$$\frac{\sigma^{**}(3^c)}{3^c} \geq \frac{1}{3^4}.\left(\frac{3^5-1}{2} - 3^2\right) = \frac{112}{81} \quad (c \geq 3). \tag{3.3c}$$

Hence from (3.3a), we have

$$3 = \frac{\sigma^{**}(n)}{n} \geq \frac{15}{8}.\frac{6}{5}.\frac{112}{81} = 3.11 > 3,$$

a contradiction.

The proof of Lemma 3.1 is complete. □

Lemma 3.2. *Let n be as in (3.2a). If $b \geq 2$ and $3|n$, then n cannot be a bi-unitary triperfect number.*

Proof. Let n be a bi-unitary triperfect number with $b \geq 2$. The relevant equations are (3.2a) and (3.2b). We assume that $3|n$. Hence $v = 3^c.w$, where $(w, 2.3.5) = 1$. In this case we have from (3.2a) and (3.2b),

$$n = 2^3.5^b.3^c.w, \tag{3.4a}$$

and

$$2^3.5^{b-1}.3^c.w = \sigma^{**}(5^b).\sigma^{**}(3^c).\sigma^{**}(w), \tag{3.4b}$$

where

$$w \text{ has no more than one odd prime factor.} \tag{3.4c}$$

Let $b = 2$. Since $\sigma^{**}(5^2) = 26$, taking $b = 2$ in (3.2b), after simplification we get

$$2^2.5.3^c.w = 13.\sigma^{**}(3^c).\sigma^{**}(w). \tag{3.4d}$$

From (3.4d), $13|w$. Let $w = 13^d$. Hence from (3.4a),

$$n = 2^3.5^2.3^c.13^d, \tag{3.5a}$$

and (3.4d) reduces to

$$2^2.5.3^c.13^{d-1} = \sigma^{**}(3^c).\sigma^{**}(13^d). \tag{3.5b}$$

If $d = 1$, (3.5b) ($d = 1$) implies that 7 divides the left-hand side of it. This is false. Similarly, taking $d = 2$ in (3.5b), we see that 17 divides the left-hand side of it. Again this is false.

Hence we may assume that $d \geq 3$.

Taking $c = 1$ in (3.5b), we find that 2^3 divides the right-hand side of it while 2^2 is a unitary divisor of its left-hand side. Also, taking $c = 2$ in (3.5b), we obtain $2.3^2.13^{d-1} = \sigma^{**}(13^d)$; this implies that $13|\sigma^{**}(13^d)$ which is false. Since $\sigma^{**}(3^3) = \frac{3^4-1}{2} = 40 = 2^3.5$ and $\sigma^{**}(3^4) = \left(\frac{3^2-1}{2}\right).(3^3+1) = 2^4.7$, it follows that $2^3|\sigma^{**}(3^c)$ when $c = 3$ or $c = 4$. Hence if $c = 3$ or $c = 4$ in (3.5b), we find an imbalance between its two sides in powers of two. Hence we may assume that $c \geq 5$.

Since $c \geq 5$, by Lemma 2.1 ($\ell = 3$), we have

$$\frac{\sigma^{**}(3^c)}{3^c} \geq \frac{1}{3^6}\left(\frac{3^7-1}{2} - 3^3\right) = \frac{1066}{729} \tag{3.5c},$$

and for $d \geq 3$ ($\ell = 2$),

$$\frac{\sigma^{**}(13^d)}{13^d} \geq \frac{1}{13^4}\left(\frac{13^5-1}{12} - 13^2\right) = \frac{30772}{28561}. \tag{3.5d}$$

Since we can assume that $c \geq 5$ and $d \geq 3$, we have from (3.5a),

$$3 = \frac{\sigma^{**}(n)}{n} \geq \frac{15}{8}.\frac{26}{25}.\frac{1066}{729}.\frac{30772}{28561} = 3.072179609 > 3,$$

a contradiction.

Thus $b = 2$ cannot occur. We may assume that $b \geq 3$. The relevant equations are (3.4a) and (3.4b). Using the results $\frac{\sigma^{**}(5^b)}{5^b} \geq \frac{756}{625}$ when $b \geq 3$, and $\frac{\sigma^{**}(3^c)}{3^c} \geq \frac{112}{81}$, ($c \geq 3$) we have from (3.4a) for $c \geq 3$,

$$3 = \frac{\sigma^{**}(n)}{n} \geq \frac{15}{8}.\frac{756}{625}.\frac{112}{81} = 3.136 > 3,$$

a contradiction. Hence $c = 1$ or $c = 2$.

Taking $c = 1$ in (3.4b), we get $2.5^{b-1}.3.w = \sigma^{**}(5^b).\sigma^{**}(w)$ which implies $w = 1$ and hence it follows that $5|\sigma^{**}(5^b)$; but this is false. Hence $c = 1$ is not possible.

Let $c = 2$. Taking $c = 2$ in (3.4a) and (3.4b), we obtain,

$$n = 2^3.5^b.3^2.w, \quad (b \geq 3) \tag{3.6a}$$

and

$$2^2.5^{b-2}.3^2.w = \sigma^{**}(5^b).\sigma^{**}(w), \tag{3.6b}$$

where

$$w \text{ has not more than one odd prime factor.} \tag{3.6c}$$

We obtain a contradiction by examining the factors of $\sigma^{**}(5^b)$.

Case I. Let b be odd. Then

$$\sigma^{**}(5^b) = \frac{5^{b+1}-1}{4} = \frac{(5^t-1)(5^t+1)}{4}. \quad \left(t = \frac{b+1}{2}\right)$$

Since $b \geq 3$, we have $t \geq 2$.

(i) Let t be even. Then $8|5^t-1$ and for any t, $2|5^t+1$. Hence $4|\frac{(5^t-1)(5^t+1)}{4} = \sigma^{**}(5^b)$. From (3.6b), we must have $w = 1$. With this, (3.6b) reduces to $2^2.5^{b-2}.3^2 = \sigma^{**}(5^b)$; this implies that $5|\sigma^{**}(5^b)$, which is false.

(ii) Let t be odd. We prove that $\frac{5^t-1}{4}$ is odd, > 1 and divisible by a prime $p \geq 29$.

(A) Clearly, $4\|5^t-1$. Hence $\frac{5^t-1}{4}$ is odd; it is > 1, since $t \geq 2$. Also, since t is odd, $\frac{5^t-1}{4}$ is not divisible by $3, 7, 13, 17$ or 23; trivially not divisible by 5.

(B) Suppose that $11|5^t-1$. This holds if and only if $5|t$. This implies $5^5-1|5^t-1$. Also, $5^5-1 = 2^2.11.71$. Hence 11 and 71 are factors of $\frac{5^t-1}{4}|\sigma^{**}(5^b)$. From (3.6b) it follows that 11 and 71 divide w. This contradicts (3.6c). Thus $11 \nmid 5^t-1$.

(C) Suppose that $19|5^t-1$. This is if and only if $9|t$. So, $5^9-1 = 2^2.19.31.829|5^t-1$ and so $\frac{5^t-1}{4}|\sigma^{**}(5^b)$ is divisible by three prime factors $19, 31$ and 829. It follows form (3.6b) that these three prime factors divide w contradicting $3.6c$. Thus $19 \nmid 5^t-1$.

(D) From (A)–(C), we conclude that $\frac{5^t-1}{4}$ is not divisible by any prime in $[3, 23]$; hence it must be divisible by a prime $p \geq 29$. Since $p|\sigma^{**}(5^b)$, from(3.6b), $p|w$. From (3.6c), $w = p^d$. Substituting this into (3.6a), we obtain $n = 2^3.5^b.3^2.p^d$. Hence

$$3 = \frac{\sigma^{**}(n)}{n} < \frac{15}{8}.\frac{5}{4}.\frac{10}{9}.\frac{29}{28} = 2.697172619 < 3,$$

a contradiction.

Case I is complete.

Case II. Let b be even and $b = 2k$ say. Then

$$\sigma^{**}(5^b) = \left(\frac{5^k-1}{4}\right).(5^{k+1}+1).$$

(iii) Let k be even. Then as in (i) of Case I, $4|\sigma^{**}(5^b)$. This leads to a contradiction from (3.6b) as before.

(iv) Let k be odd. Since $b \geq 3$, we have $k \geq 3$; $\frac{5^k-1}{4} > 1$, odd and must be divisible by a prime $p \geq 29$ and this gives a contradiction as in (ii)(D) of Case I.

Case II and thus the case $c = 2$ is complete.

With this we have completed the proof of Lemma 3.2. □

Lemma 3.3. *Let n be as in (3.2a). If $b \geq 2$ and $3 \nmid n$, then n cannot be a bi-unitary triperfect number.*

Proof. Let $b \geq 2$ and $3 \nmid n$. Assume that n is a bi-unitary triperfect number. We return to the equations (3.2a) and (3.2b). Again we consider $\sigma^{**}(5^b)$ for obtaining a contradiction.

(i) If b is odd or $4|b$, then $3|\sigma^{**}(5^b)$. From (3.2b), it follows that $3|v$ and so $3|n$. But we have assumed that $3 \nmid n$.

(ii) Let $b = 2k$ and k be odd. $b \geq 2$ implies that $k \geq 3$; $\frac{5^k-1}{4} > 1$, odd and not divisible by $3, 5, 7, 13, 17$ and 23 as in (ii)(A) of Case I in Lemma 3.2.

(iii) Suppose $11|5^k - 1$. This is if and only if $5|k$ and this implies that $5^5 - 1 = 2^2.11.71|5^k - 1$. Hence 11 and 71 divide $\frac{5^k-1}{4}|\sigma^{**}(5^b)$. From (3.2b), we infer that v is divisible by 11 and 71 so that $v = 11^c.71^d$. Hence from (3.2a), $n = 2^3.5^b.11^c.71^d$ so that by Lemma 2.1,

$$3 = \frac{\sigma^{**}(n)}{n} < \frac{15}{8}.\frac{5}{4}.\frac{11}{10}.\frac{71}{70} = 2.614955357 < 3,$$

a contradiction. Hence $11 \nmid 5^k - 1$.

(iv) That $19 \nmid 5^k - 1$ can be proved exactly as in (ii)(C) of Case I in Lemma 3.2.

Thus $\frac{5^k-1}{4}$ is not divisible by any prime in $[3, 23]$; being > 1 and odd, it must be divisible by a prime $p \geq 29$. By (3.2b), $p|v$. Since v has no more than two odd prime factors, let q be the other possible odd prime factor of v; we may assume that $q \geq 7$. We can assume that $v = p^c.q^d$ and so from (3.2a), $n = 2^a.5^b.p^c.q^d$. This implies

$$3 = \frac{\sigma^{**}(n)}{n} < \frac{15}{8}.\frac{5}{4}.\frac{29}{28}.\frac{7}{6} = 2.83203125 < 3, \tag{3.7}$$

a contradiction.

This completes the proof of Lemma 3.3. □

Completion of proof of Theorem 3.1. Follows from Lemmas 3.1, 3.2 and 3.3. □

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Appendix A Bi-unitary multiperfect numbers $< 8.10^{10}$

SL.No	$k \geq 3$	n	Factorization
1	3	120	$2^3.3.5$
2	3	672	$2^5.3.7$
3	3	2160	$2^4.3^3.5$
4	3	10080	$2^5.3^2.5.7$
5	3	22848	$2^6.3.7.17$
6	4	30240	$2^5.3^3.5.7$
7	3	342720	$2^6.3^2.5.7.17$
8	3	523776	$2^9.3.11.31$
9	4	1028160	$2^6.3^3.5.7.17$
10	3	1528800	$2^5.3.5^2.7^2.13$
11	4	6168960	$2^7.3^4.5.7.17$
12	3	7856640	$2^9.3^2.5.11.31$
13	4	7983360	$2^8.3^4.5.7.11$
14	3	14443520	$2^{10}.5.7.13.31$
15	3	22932000	$2^5.3^2.5^3.7^2.13$
16	4	23569920	$2^9.3^3.5.11.31$
17	4	43330560	$2^{10}.3.5.7.13.31$
18	3	44553600	$2^7.3^2.5^2.7.13.17$
19	3	51979200	$2^6.3.5^2.7^2.13.17$
20	3	57657600	$2^8.3^2.5^2.7.11.13$
21	4	68796000	$2^5.3^3.5^3.7^2.13$
22	4	133660800	$2^7.3^3.5^2.7.13.17$
23	4	172972800	$2^8.3^3.5^2.7.11.13$
24	3	779688000	$2^6.3^2.5^3.7^2.13.17$
25	3	1476304896	$2^{13}.3.11.43.127$
26	4	2339064000	$2^6.3^3.5^3.7^2.13.17$
27	3	6840038400	$2^{10}.3.5^2.13^2.17.31$
28	4	14034384000	$2^7.3^4.5^3.7^2.13.17$
29	4	18162144000	$2^8.3^4.5^3.7^2.11.13$
30	3	22144573440	$2^{13}.3^2.5.11.43.127$
31	4	66433720320	$2^{13}.3^3.5.11.43.127$

Note. SL.No 1 to 13 are due to Peter Hagis (Hagis, P., Jr. (1987). Bi-unitary amicable and multiperfect numbers, *Fibonacci Quart.*, 25 (2), 144–150.).

Appendix B ord_p5

Let p be an odd prime $\neq 5$. In the following table, r denotes the smallest positive integer such that $5^r \equiv 1(\bmod p)$; that is, $r = ord_p5$; s denotes the smallest positive integer such that $5^s \equiv -1(\bmod p)$ if s exists; if s does not exist, that is, if $5^t + 1$ is not divisible by p for any positive integer t, the entry in column s will be filled up by dash sign. If r is even, then $s = r/2$, and if r is odd, s does not exist.

Table-I

SL.No	p	r	s	SL.No	p	r	s	SL.No	p	r	s
1	3	2	1	11	37	36	18	21	79	39	-
2	5	-	-	12	41	20	10	22	83	82	41
3	7	6	3	13	43	42	21	23	89	44	22
4	11	5	-	14	47	46	23	24	97	96	48
5	13	4	2	15	53	52	26	25	101	25	-
6	17	16	8	16	59	29	-	26	103	102	51
7	19	9	-	17	61	30	15	27	107	106	53
8	23	22	11	18	67	22	11	28	109	27	-
9	29	14	7	19	71	5	-	29	113	112	56
10	31	3	-	20	73	72	36	30	127	42	21

Table-II

SL.No	p	r	s	SL.No	p	r	s	SL.No	p	r	s
31	131	65	-	57	271	27	-	83	433	432	216
32	137	136	68	58	277	276	138	84	439	219	-
33	139	69	-	59	281	140	70	85	443	442	221
34	149	37	-	60	283	282	141	86	449	14	7
35	151	75	-	61	293	292	146	87	457	152	76
36	157	156	78	62	307	306	153	88	461	115	-
37	163	54	27	63	311	155	-	89	463	462	231
38	167	166	83	64	313	8	4	90	467	466	233
39	173	172	86	65	317	316	158	91	479	239	-
40	179	89	-	66	331	165	-	92	487	54	27
41	181	15	-	67	337	112	56	93	491	245	-
42	191	19	-	68	347	346	173	94	499	249	-
43	193	192	96	69	349	174	87	95	503	502	-
44	197	196	98	70	353	352	176	96	509	254	-
45	199	33	-	71	359	179	-	97	521	10	5
46	211	35	-	72	367	122	61	98	523	522	261
47	223	222	111	73	373	372	186	99	541	135	-
48	227	226	113	74	379	21	-	100	547	546	273
49	229	114	57	75	383	382	191	101	557	556	278
50	233	232	116	76	389	97	-	102	563	562	-
51	239	119	-	77	397	396	198	103	569	71	-
52	241	40	20	78	401	25	-	104	571	285	-
53	251	25	-	79	409	17	-	105	577	576	288
54	257	256	128	80	419	209	-	106	587	586	293
55	263	262	-	81	421	210	105				
56	269	67	-	82	431	215	-				

Table-III

SL.No	p	r	s	SL.No	p	r	s	SL.No	p	r	s
107	593	592	296	132	751	375	-	157	929	232	116
108	599	299	-	133	757	756	378	158	937	936	468
109	601	12	6	134	761	38	19	159	941	235	-
110	607	606	303	135	769	128	64	160	947	946	473
111	613	612	306	136	773	772	386	161	953	952	476
112	617	616	308	137	787	786	393	162	967	966	483
113	619	309	-	138	797	796	398	163	971	485	-
114	631	35	-	139	809	404	202	164	977	976	488
115	641	64	32	140	811	405	-	165	983	982	491
116	643	214	107	141	821	410	205	166	991	495	-
117	647	646	323	142	823	274	137	167	997	332	166
118	653	652	326	143	827	118	59	168	1009	504	252
119	659	329	-	144	829	9	-	169	1013	1012	506
120	661	330	165	145	839	419	-	170	1019	509	-
121	673	672	336	146	853	284	142	171	1021	255	-
122	677	676	338	147	857	856	428	172	1031	515	-
123	683	682	341	148	859	429	-	173	1033	1032	516
124	691	115	-	149	863	862	431	174	1039	173	-
125	701	350	175	150	877	876	438	175	1049	524	262
126	709	354	177	151	881	440	220	176	1051	525	-
127	719	359	-	152	883	126	73	177	1061	265	-
128	727	726	363	153	887	886	443	178	1063	354	177
129	733	244	122	154	907	906	453	179	1069	267	-
130	739	123	-	155	911	455		180	1087	362	181
131	743	742	371	156	919	459	-	181	1091	545	-

Table-IV

SL.No	p	r	s	SL.No	p	r	s	SL.No	p	r	s
182	1093	1092	546	207	1283	1282	641	232	1471	735	-
183	1097	1096	548	208	1289	644	322	233	1481	740	370
184	1103	1102	551	209	1291	215	-	234	1483	1482	741 -
185	1109	554	277	210	1297	144	62	235	1487	1486	743
186	1117	372	186	211	1301	650	325	236	1489	372	186
187	1123	374	187	212	1303	62	31	237	1493	1492	746
188	1129	282	141	213	1307	1306	653	238	1499	749	-
189	1151	575	-	214	1319	659	-	239	1511	755	-
190	1153	1152	576	215	1321	660	330	240	1523	1522	761
191	1163	1162	581	216	1327	340	170	241	1531	85	-
192	1171	45	-	217	1361	1360	680	242	1543	1542	771
193	1181	590	295	218	1367	1366	683	243	1549	86	43
194	1187	1186	593	219	1373	1372	686	244	1553	1552	776
195	1193	1192	596	220	1381	690	345	245	1559	779	-
196	1201	600	300	221	1399	699	-	246	1567	1566	783
197	1213	1212	606	222	1409	44	22	247	1571	785	-
198	1217	1216	608	223	1423	704	352	248	1579	789	-
199	1223	1222	611	224	1427	1426	713	249	1583	1582	791
200	1229	614	307	225	1429	119	-	250	1597	532	266
201	1231	615	-	226	1433	1432	716	251	1601	400	200
202	1237	1236	618	227	1439	719	-	252	1607	1606	803
203	1249	624	312	228	1447	482	241	253	1609	67	-
204	1259	629	-	229	1451	725	-	254	1613	1612	806
205	1277	1276	638	230	1453	132	61	255	1619	809	-
206	1279	639	-	231	1459	243	-	256	1621	405	-

Table-V

SL.No	p	r	s	SL.No	p	r	s	SL.No	p	r	s
257	1627	542	271	282	1847	1846	923	307	2029	1014	507
258	1637	1636	818	283	1861	31	-	308	2039	1019	-
259	1657	552	276	284	1867	622	311	309	2053	2052	1026
260	1663	1662	831	285	1871	935	-	310	2063	2062	1031
261	1667	1666	833	286	1873	624	312	311	2069	94	37
262	1669	834	417	287	1877	1876	938	312	2081	520	260
263	1693	1692	846	288	1879	939	-	313	2083	2082	1041
264	1697	1696	848	289	1889	472	236	314	2087	2086	1043
265	1699	283	-	290	1901	50	25	315	2089	1044	522
266	1709	854	427	291	1907	1906	953	316	2099	1049	-
267	1721	430	860	292	1913	965	-	317	2111	1055	-
268	1723	574	287	293	1931	965	-	318	2113	2112	1056
269	1733	1732	866	294	1933	1932	966	319	2129	266	133
270	1741	15	-	295	1949	487	-	320	2131	1065	-
271	1747	1746	873	296	1951	975	-	321	2137	712	356
272	1753	584	292	297	1973	68	34	322	2141	1070	535
273	1759	293	-	298	1979	989	-	323	2143	2142	1071
274	1777	1776	888	299	1987	1986	993	324	2153	2152	1076
275	1783	162	81	300	1993	1992	996	325	2161	1080	540
276	1787	1786	893	301	1997	1996	998	326	2179	1089	-
277	1789	894	447	302	1999	999	-	327	2203	2202	1101
278	1801	900	450	303	2003	2002	1001	328	2207	2206	1103
279	1811	905	-	304	2011	1005	-	329	2213	2212	1106
280	1823	1822	911	305	2017	2016	1008	330	2221	370	185
281	1831	305	-	306	2027	2026	1013	331	2237	2236	1118

Table-VI

SL.No	p	r	s	SL.No	p	r	s
332	2239	373	-	351	2377	2376	1188
333	2243	2242	1121	352	2381	238	119
334	2251	1125	-	353	2383	2382	1191
335	2267	2266	1133	354	2389	398	194
336	2269	567	-	355	2393	2392	1196
337	2273	2272	1136	356	2399	1199	-
338	2281	60	30	357	2411	1205	-
339	2287	254	127	358	2417	2416	1208
340	2293	2292	1146	359	2423	2422	1211
341	2297	2296	1148	360	2437	2436	1218
342	2309	577	-	361	2441	305	-
343	2311	165	-	362	2447	2446	1223
344	2333	1169	-	363	2459	1229	-
345	2339	2338	1169	364	2467	2466	1233
346	2341	1170	585	365	2473	2472	1236
347	2347	2346	1173	366	2477	2476	1238
348	2351	1175	-	367	2503	2502	1251
349	2357	2356	1178				
350	2371	1185	-				

Appendix C ord_p7

Let p be an odd prime $\neq 7$. In the following table, r denotes the smallest positive integer such that $7^r \equiv 1(\bmod p)$; that is, $r = ord_p7$; s denotes the smallest positive integer such that $7^s \equiv -1(\bmod p)$ if s exists; if s does not exist, that is, if $7^t + 1$ is not divisible by p for any positive integer t, the entry in column s will be filled up by dash sign. If r is even, then $s = r/2$, and if r is odd, s does not exist.

Table-I

SL.No	p	r	s	SL.No	p	r	s	SL.No	p	r	s
1	3	1	-	11	37	9	-	21	79	78	39
2	5	4	2	12	41	40	20	22	83	41	-
3	7	-	-	13	43	6	3	23	89	88	44
4	11	10	5	14	47	23	-	24	97	96	48
5	13	12	6	15	53	26	13	25	101	100	50
6	17	16	8	16	59	29	-	26	103	51	-
7	19	3	-	17	61	60	30	27	107	106	53
8	23	22	11	18	67	66	33	28	109	27	-
9	29	7	-	19	71	70	35	29	113	14	7
10	31	15	-	20	73	24	12	30	127	126	63

Table-II

SL.No	p	r	s	SL.No	p	r	s	SL.No	p	r	s
31	131	65	-	57	271	135	-	83	433	432	216
32	137	68	34	58	277	138	69	84	439	73	-
33	139	69	-	59	281	20	10	85	443	442	221
34	149	74	37	60	283	141	-	86	449	112	56
35	151	150	75	61	293	292	146	87	457	114	57
36	157	52	26	62	307	153	-	88	461	460	230
37	163	162	81	63	311	31	-	89	463	154	77
38	167	83	-	64	313	104	52	90	467	233	-
39	173	172	86	65	317	158	79	91	479	239	-
40	179	178	89	66	331	110	55	92	487	162	81
41	181	12	6	67	337	56	28	93	491	490	245
42	191	10	5	68	347	346	173	94	499	498	249
43	193	24	12	69	349	348	174	95	503	251	-
44	197	98	49	70	353	32	16	96	509	508	254
45	199	99	-	71	359	358	179	97	521	520	260
46	211	210	105	72	367	61	-	98	523	261	-
47	223	37	-	73	373	62	31	99	541	90	45
48	227	113	-	74	379	378	189	100	547	546	273
49	229	228	114	75	383	191	-	101	557	278	139
50	233	116	58	76	389	97	-	102	563	281	-
51	239	238	119	77	397	396	198	103	569	284	142
52	241	240	120	78	401	200	100	104	571	190	95
53	251	125	-	79	409	24	12	105	577	576	288
54	257	256	128	80	419	19	-	106	587	293	-
55	263	262	131	81	421	70	35				
56	269	268	134	82	431	430	215				

Table-III

SL.No	p	r	s	SL.No	p	r	s	SL.No	p	r	s
107	593	592	296	132	751	250	125	157	929	928	464
108	599	598	299	133	757	189	-	158	937	936	468
109	601	600	300	134	761	760	380	159	941	940	470
110	607	101	-	135	769	256	128	160	947	86	43
111	613	153	-	136	773	772	386	161	953	238	119
112	617	308	154	137	787	393	-	162	967	966	483
113	619	309	-	138	797	796	398	163	971	97	-
114	631	630	315	139	809	101	-	164	977	488	244
115	641	320	160	140	811	27	-	165	983	491	-
116	643	321	-	141	821	410	205	166	991	990	495
117	647	323	-	142	823	822	411	167	997	996	498
118	653	163	-	143	827	826	413	168	1009	252	126
119	659	658	329	144	829	276	138	169	1013	1012	506
120	661	44	22	145	839	419	-	170	1019	1018	509
121	673	112	56	146	853	284	142	171	1021	340	170
122	677	676	338	147	857	856	428	172	1031	206	103
123	683	682	341	148	859	429	-	173	1033	172	86
124	691	345	-	149	863	862	431	174	1039	519	-
125	701	175	-	150	877	219	-	175	1049	1048	524
126	709	177	-	151	881	80	40	176	1051	1050	525
127	719	359	-	152	883	98	49	177	1061	265	-
128	727	363	-	153	887	443	-	178	1063	9	-
129	733	732	366	154	907	906	453	179	1069	1068	534
130	739	738	369	155	911	14	7	180	1087	362	181
131	743	742	371	156	919	918	459	181	1091	545	-

Table-IV

SL.No	p	r	s	SL.No	p	r	s	SL.No	p	r	s
182	1093	273	-	207	1283	1282	641	232	1471	1470	735
183	1097	1096	548	208	1289	322	161	233	1481	185	-
184	1103	1102	551	209	1291	645	-	234	1483	741	-
185	1109	1108	554	210	1297	648	324	235	1487	743	-
186	1117	558	279	211	1301	1300	650	236	1489	496	248
187	1123	11	-	212	1303	1302	651	237	1493	746	373
188	1129	564	282	213	1307	653	-	238	1499	214	107
189	1151	115	-	214	1319	659	-	239	1511	755	-
190	1153	384	192	215	1321	264	132	240	1523	1522	761
191	1163	1162	581	216	1327	442	221	241	1531	85	-
192	1171	234	117	217	1361	1360	680	242	1543	257	-
193	1181	1180	590	218	1367	1366	683	243	1549	774	387
194	1187	1186	593	219	1373	343	-	244	1553	1552	776
195	1193	1192	596	220	1381	345	-	245	1559	779	-
196	1201	8	4	221	1399	699	-	246	1567	783	-
197	1213	303	-	222	1409	44	22	247	1571	785	-
198	1217	1216	608	223	1423	474	237	248	1579	526	263
199	1223	611	-	224	1427	713	-	249	1583	226	113
200	1229	614	307	225	1429	357	-	250	1597	798	399
201	1231	615	-	226	1433	1432	716	251	1601	1600	800
202	1237	1236	618	227	1439	1438	719	252	1607	1606	803
203	1249	1248	624	228	1447	241	-	253	1609	1608	804
204	1259	629	-	229	1451	1450	725	254	1613	1612	806
205	1277	1276	638	230	1453	121	-	255	1619	1618	809
206	1279	639	-	231	1459	243	-	256	1621	81	-

Table-V

SL.No	p	r	s SL.No	p	r	s	SL.No	p	r	s	
257	1627	813	-	282	1847	923	-	307	2029	676	338
258	1637	1636	818	283	1861	372	186	308	2039	2038	1019
259	1657	184	92	284	1867	933	-	309	2053	342	171
260	1663	554	277	285	1871	374	187	310	2063	1031	-
261	1667	1666	833	286	1873	117	-	311	2069	517	-
262	1669	1668	834	287	1877	938	469	312	2081	1040	520
263	1693	1692	846	288	1879	939	-	313	2083	2082	1041
264	1697	1696	848	289	1889	1888	944	314	2087	2086	1043
265	1699	849	-	290	1901	950	475	315	2089	2088	1044
266	1709	854	427	291	1907	953	-	316	2099	1049	-
267	1721	1720	860	292	1913	956	478	317	2111	2110	1055
268	1723	574	287	293	1931	965	-	318	2113	192	96
269	1733	433	-	294	1933	966	483	319	2129	56	28
270	1741	580	290	295	1949	1948	974	320	2131	355	-
271	1747	1746	873	296	1951	975	-	321	2137	356	178
272	1753	1752	876	297	1973	1972	986	322	2141	2140	1070
273	1759	1758	879	298	1979	989	-	323	2143	2142	1071
274	1777	592	296	299	1987	993	-	324	2153	269	-
275	1783	891	-	300	1993	1992	996	325	2161	432	216
276	1787	1786	893	301	1997	499	-	326	2179	2178	1089
277	1789	894	447	302	1999	666	333	327	2203	1101	
278	1801	450	225	303	2003	2002	1001	328	2207	2206	1103
279	1811	181	-	304	2011	2010	1005	329	2213	553	-
280	1823	911	-	305	2017	1008	504	330	2221	1110	555
281	1831	1830	915	306	2027	2026	1013	331	2237	559	-

Table-VI

SL.No	p	r	s	SL.No	p	r	s
332	2239	373	-	351	2377	27	-
333	2243	1121	-	352	2381	238	119
334	2251	2250	1125	353	2383	397	-
335	2267	1133	-	354	2389	194	97
336	2269	567	-	355	2393	2392	1196
337	2273	2272	1136	356	2399	1199	-
338	2281	2280	1140	357	2411	1205	-
339	2287	1143	-	358	2417	604	302
340	2293	1146	573	359	2423	346	173
341	2297	1148	574	360	2437	1218	609
342	2309	2308	1154	361	2441	488	244
343	2311	110	55	362	2447	2446	1223
344	2333	583	-	363	2459	2458	1229
345	2339	2338	1169	364	2467	1233	-
346	2341	2340	1170	365	2473	1236	618
347	2347	782	391	366	2477	2476	1238
348	2351	1175	-	367	2503	2502	1251
349	2357	2356	1178				
350	2371	237	-				

Appendix D Factors of $5^t - 1$

$5^3 - 1 = \{\{2,2\},\{31,1\}\}.$

$5^5 - 1 = \{\{2,2\},\{11,1\},\{71,1\}\}.$

$5^9 - 1 = \{\{2,2\},\{19,1\},\{31,1\},\{829,1\}\}.$

$5^{15} - 1 = \{\{2,2\},\{11,1\},\{31,1\},\{71,1\},\{181,1\},\{1741,1\}\}.$

$5^{17} - 1 = \{\{2,2\},\{409,1\},\{466344409,1\}\}.$

$5^{19} - 1 = \{\{2,2\},\{191,1\},\{6271,1\},\{3981071,1\}\}.$

$5^{21} - 1 = \{\{2,2\},\{31,1\},\{379,1\},\{19531,1\},\{519499,1\}\}.$

$5^{25} - 1 = \{\{2,2\},\{11,1\},\{71,1\},\{101,1\},\{251,1\},\{401,1\},\{9384251,1\}\}.$

$5^{27} - 1 = \{\{2,2\},\{19,1\},\{31,1\},\{109,1\},\{271,1\},\{829,1\},\{4159,1\},\{31051,1\}\}.$

$5^{29} - 1 = \{\{2,2\},\{59,1\},\{35671,1\},\{22125996444329,1\}\}.$

$5^{31} - 1 = \{\{2,2\},\{1861,1\},\{625552508473588471,1\}\}.$

$5^{33} - 1 = \{\{2,2\},\{31,1\},\{199,1\},\{12207031,1\},\{386478495679,1\}\}.$

$5^{35} - 1 = \{\{2,2\},\{11,1\},\{71,1\},\{211,1\},\{631,1\},\{4201,1\},\{19531,1\},\{85280581,1\}\}.$

$5^{37} - 1 = \{\{2,2\},\{149,1\},\{13971969971,1\},\{8737481256739,1\}\}.$

$5^{39} - 1 = \{\{2,2\},\{31,1\},\{79,1\},\{305175781,1\},\{608459012088799,1\}\}.$

$5^{45} - 1 = \{\{2,2\},\{11,1\},\{19,1\},\{31,1\},\{71,1\},\{181,1\},\{829,1\},$
$\{1171,1\},\{1741,1\},\{169831,1\},\{297315901,1\}\}.$

$5^{65} - 1 = \{\{2,2\},\{11,1\},\{71,1\},\{131,1\},\{305175781,1\},$
$\{1034150930241911,1\},\{20986207825565581,1\}\}.$

$5^{67} - 1 = \{\{2,2\},\{269,1\},\{1609,1\},\{26399,1\},\{2454335007529,1\},$
$\{604088623657497125653141,1\}\}.$

$5^{69} - 1 = \{\{2,2\},\{31,1\},\{139,1\},\{6211,1\},\{8971,1\},\{332207361361,1\},$
$\{598761682261,1\},\{8868050880709,1\}\}.$

$5^{71} - 1 = \{\{2,2\},\{569,1\},\{1860792942122803908322325352986911164436273 2899,1\}\}.$

$5^{75} - 1 = \{\{2,2\},\{11,1\},\{31,1\},\{71,1\},\{101,1\},\{151,1\},\{181,1\},\{251,1\},\{401,1\},$
$\{1741,1\},\{3301,1\},\{1989151,1\},\{9384251,1\},\{49892851,1\},\{183794551,1\}\}.$

$5^{85} - 1 = \{\{2,2\},\{11,1\},\{71,1\},\{409,1\},\{1531,1\},\{466344409,1\},$
$\{34563155350221618511,1\},\{81982411129696268155 81,1\}\}.$

$5^{89} - 1 = \{\{2,2\},\{179,1\},\{9807089,1\},\{14597959,1\},$
$\{834019001,1\},\{8157179360521,1\},\{2316696543636831300959 09,1\}\}.$

$5^{97} - 1 = \{\{2,2\},\{389,1\},\{264811,1\},$
$\{153159805660301568024613754993807288151489686913246436306439,1\}\}.$

$5^{109} - 1 = \{\{2,2\},\{1091,1\},\{1007161,1\},\{1528399,1\},$
$\{2293559892404294400692399012979688850301546383400890423075969,1\}\}.$

$5^{115} - 1 = \{\{2,2\},\{11,1\},\{71,1\},\{461,1\},\{691,1\},\{8971,1\},$
$\{689081,1\},\{2855911,1\},\{29028071,1\},\{824480311,1\},$
$\{17223586571,1\},\{332207361361,1\},\{1000629701666403 31,1\}\}.$

$5^{119} - 1 = \{\{2,2\},\{239,1\},\{409,1\},\{1429,1\},\{4999,1\},\{19531,1\},\{466344409,1\},$
$\{59142910887838137123340053063843958969838096575237472651 09,1\}\}.$

$5^{123} - 1 = \{\{2,2\},\{31,1\},\{739,1\},\{960139,1\},\{1785961,1\},$

$\{2238236249, 1\}, \{9025583299, 1\},$

$\{29624915776321538277133753064212177050469622587571, 1\}\}.$

$5^{129} - 1 = \{\{2, 2\}, \{31, 1\}, \{18471511, 1\}, \{1644512641, 1\},$

$\{17282755219881588879 1, 1\},$

$\{2257128352803905294127694161479357934346533799518511, 1\}\}.$

$5^{135} - 1 = \{\{2, 2\}, \{11, 1\}, \{19, 1\}, \{31, 1\}, \{71, 1\}, \{109, 1\}, \{181, 1\}, \{271, 1\}, \{541, 1\},$

$\{829, 1\}, \{1171, 1\}, \{1741, 1\}, \{4159, 1\}, \{11071, 1\}, \{31051, 1\},$

$\{169831, 1\}, \{297315901, 1\},$

$\{131231569444974868833 1, 1\}, \{269412443730606502245 61, 1\}\}.$

$5^{155} - 1 = \{\{2, 2\}, \{11, 1\}, \{71, 1\}, \{311, 1\}, \{1861, 1\}, \{11161, 1\},$

$\{257611, 1\}, \{5624951, 1\}, \{31473312961, 1\},$

$\{23790532849140489709580350017078211374045899760678601809756955 1$

$7755589901, 1\}\}.$

$5^{165} - 1 = \{\{2, 2\}, \{11, 2\}, \{31, 1\}, \{71, 1\}, \{181, 1\}, \{199, 1\}, \{331, 1\},$

$\{1741, 1\}, \{2311, 1\}, \{103511, 1\}, \{511831, 1\}, \{12207031, 1\},$

$\{65628751, 1\}, \{190295821, 1\}, \{386478495679, 1\},$

$\{13404637917598844203623689836522709902429768876074 1, 1\}\}.$

$5^{175} - 1 = \{\{2, 2\}, \{11, 1\}, \{71, 1\}, \{101, 1\}, \{211, 1\}, \{251, 1\}, \{401, 1\}, \{631, 1\},$

$\{4201, 1\}, \{12601, 1\}, \{19531, 1\}, \{28001, 1\}, \{50051, 1\}, \{200201, 1\},$

$\{8894201, 1\}, \{9384251, 1\}, \{22661801, 1\}, \{85280581, 1\}, \{1657309151, 1\},$

$\{38105263380318401, 1\}, \{167114491486518752903881 01, 1\}\}.$

$5^{173} - 1 = \{\{2, 2\}, \{1039, 1\}, \{3461, 1\}, \{1708272343265311, 1\},$

$\{33991983102018763096034795719942109934243438662912992480772489 9$

$548171905079254114627192212342747249, 1\}\}.$

$5^{179} - 1 = \{\{2, 2\}, \{359, 1\}, \{3581, 1\}, \{75539, 1\}, \{17315029, 1\},$

$\{19403387668217339007272944543996712196870540760746870406322933 7$

$00153368084799315563561578033131860319559619, 1\}\}.$

$5^{185} - 1 = \{\{2, 2\}, \{11, 1\}, \{71, 1\}, \{149, 1\}, \{2591, 1\}, \{30810641, 1\},$

$\{13971969971, 1\}, \{8737481256739, 1\}, \{21829321837586441, 1\},$

$\{20592127439275593882406240824780062417908685074495330881599746 2$

$39870280151, 1\}\}.$

$5^{209} - 1 = \{\{2, 2\}, \{191, 1\}, \{419, 1\}, \{6271, 1\}, \{56431, 1\}, \{3981071, 1\}, \{12207031, 1\},$

$\{22077963297690156110122595623297583216190476696803907377343492 3$

$37000542711435241254425456657076461909793693026506147389, 1\}\}.$

$5^{215} - 1 = \{\{2, 2\}, \{11, 1\}, \{71, 1\}, \{431, 1\}, \{1291, 1\}, \{144982241, 1\}, \{1644512641, 1\},$

$\{45823089824758049069347482532092563162983045470038622578218304 2$

$2052712517500006897081991082494828288526639138845788438012251, 1\}\}.$

$5^{219} - 1 = \{\{2, 2\}, \{31, 1\}, \{439, 1\}, \{429241, 1\}, \{2183431, 1\}, \{4853479, 1\}, \{5729041, 1\},$

$\{83669849729535948393106586038292142289805834212423034310800133 2$

$2864101127722950816380947142641046870840077993693681442463 11, 1\}\}.$

$5^{235} - 1 = \{\{2, 2\}, \{11, 1\}, \{71, 1\}, \{941, 1\}, \{4231, 1\},$

$\{20885861, 1\}, \{68308861, 1\}, \{96766610646500911, 1\},$

$\{105475693151619035529055506571643366314164278037417148495742900$
$661283586936266520114117025199330220448787532328441826879351, 1\}\}.$

$5^{239} - 1 = \{\{2, 2\}, \{479, 1\}, \{40093613041379, 1\},$
$\{147353459691520632244555607717478117134030802606181953708010371$
$9298166168947549642538525464219037490151421698581086237756602026720817$
$75623592620984339, 1\}\}.$

$5^{243} - 1 = \{\{2, 2\}, \{19, 1\}, \{31, 1\}, \{109, 1\}, \{271, 1\}, \{829, 1\}, \{1459, 1\}, \{4159, 1\}, \{4861, 1\},$
$\{31051, 1\}, \{14001661, 1\},$
$\{956225817178767724216036727013172355771961587713861458223950326071325$
$32203177149599172957989777513262960247318785445596060912523871806859, 1\}\}.$

$5^{245} - 1 = \{\{2, 2\}, \{11, 1\}, \{71, 1\}, \{211, 1\}, \{491, 1\}, \{631, 1\}, \{4201, 1\}, \{8821, 1\},$
$\{10781, 1\}, \{19531, 1\}, \{85280581, 1\}, \{16650328910366149531471, 1\},$
$\{781663320776776898423953184448429313182200602330122956263346853$
$362609732059029620242463194363111463695774602408391, 1\}\}.$

$5^{249} - 1 = \{\{2, 2\}, \{31, 1\}, \{499, 1\}, \{12451, 1\}, \{169321, 1\}, \{20515111, 1\},$
$\{413066447794038087591111753737021242470889068026169507364672570$
$9170070986722837017231782732467782049914630406727484039932117694033845$
$10804794807777014929, 1\}\}.$

$5^{255} - 1 = \{\{2, 2\}, \{11, 1\}, \{31, 1\}, \{71, 1\}, \{181, 1\}, \{409, 1\}, \{1021, 1\}, \{1531, 1\}, \{1741, 1\},$
$\{90271, 1\}, \{236641, 1\}, \{317731, 1\}, \{466344409, 1\}, \{654652168021, 1\},$
$\{782302186051, 1\}, \{38385602257801, 1\}, \{263420722813531, 1\},$
$\{1445377105917001, 1\}, \{373669429273223813320308794167606428171892018391$
$7268996709350346031, 1\}\}.$

$5^{265} - 1 = \{\{2, 2\}, \{11, 1\}, \{71, 1\}, \{1061, 1\}, \{5960555749, 1\},$
$\{17154094481, 1\}, \{488094322309591, 1\},$
$\{1335618937071191, 1\}, \{27145365052629449, 1\},$
$\{281247180648443474201460266274425491641129732331700920080143804$
$12159116372499099715505705982461473940474528773181, 1\}\}.$

$5^{267} - 1 = \{\{2, 2\}, \{31, 1\}, \{179, 1\}, \{1069, 1\}, \{1133149, 1\}, \{1652731, 1\}, \{9807089, 1\},$
$\{14597959, 1\}, \{834019001, 1\}, \{8157179360521, 1\},$
$\{18484574880511, 1\}, \{369718501929859, 1\},$
$\{142567547569335926285313687359043351519843476238292654496637148$
$00094573110287775614743544597716721806051, 1\}\}.$

$5^{283} - 1 = \{\{2, 2\}, \{1699, 1\}, \{451669, 1\}, \{113334709, 1\}, \{416742821994932569, 1\},$
$\{443819633770294750370320744731630318561750138440580012523002611$
$2644708672382627319041587277968263957097948226651971357369795069152786$
$800319216626055584292591152381, 1\}\}.$

$5^{285} - 1 = \{\{2, 2\}, \{11, 1\}, \{31, 1\}, \{71, 1\}, \{181, 1\}, \{191, 1\}, \{571, 1\},$
$\{1741, 1\}, \{2851, 1\}, \{6271, 1\}, \{30211, 1\}, \{3981071, 1\},$
$\{4113691, 1\}, \{260930841421, 1\}, \{1245576402371959291, 1\},$
$\{168114510382313038536111985312939538121149394255181710985204428$
$0971975267257975994205963362124950242977896917525215729402771639991, 1\}\}.$

$$
\begin{aligned}
5^{293}-1 = {} & \{\{2,2\},\{1759,1\},\{520369,1\},\{224108738217451,1\}, \\
& \{765799772250459652078486518598847908914504333769682537713741665 \\
& 228778530924007490154252173099437927707219767275696226697021641547009 4\\
& 18209583175797179082256696281339893010020297156 1,1\}\}. \\
5^{299}-1 = {} & \{\{2,2\},\{599,1\},\{8971,1\},\{71761,1\}, \\
& \{1076401,1\},\{305175781,1\},\{332207361361,1\}, \\
& \{583286245014928896997069463108849349471727749367145398239679731 \\
& 7763508566250778043531079556086836650759856202647136537481564407558143 \\
& 3010149400697841827427269482981509003 9,1\}\}. \\
5^{305}-1 = {} & \{\{2,2\},\{11,1\},\{71,1\},\{1831,1\},\{2441,1\},\{8419,1\}, \\
& \{1884901,1\},\{918585913061,1\},\{1786560207910631,1\}, \\
& \{421888494254324998134324156849956550451596341562788247181999872 \\
& 1855141676009769858093534852494261536271460739996629671895241616825215 \\
& 88451579788853621174433924146443 9,1\}\}. \\
5^{309}-1 = {} & \{\{2,2\},\{31,1\},\{619,1\},\{3709,1\},\{28429,1\},\{8934148519,1\}, \\
& \{132601189834417563132816953286709466900740029888895604890397852 \\
& 2218406991430334953446196714481673022578230211786117461245564212271823 \\
& 71474604878102688673816111999698295055179737773756725117826 31,1\}\}. \\
5^{329}-1 = {} & \{\{2,2\},\{659,1\},\{6581,1\},\{19531,1\}, \\
& \{269879346467620727922073210910971549063284721356541870587135089 \\
& 6075844938230532472285390603019435957842291965934878140411372929812643 \\
& 9270054834295684389112752450891990796332912038187667802688926949422734 \\
& 218260851779046 9,1\}\}. \\
5^{359}-1 = {} & \{\{2,2\},\{719,1\}, \\
& \{296102909597228840049377866220257917124241775682244895752785124 \\
& 6068434512340439594078607012327786440854027013332931519736574813418608 \\
& 2540344867671206750018218149158191119401284579147210470369045919988336 \\
& 17707165364578323194662324199517761384329974 9,1\}\}. \\
5^{373}-1 = {} & \{\{2,2\},\{2239,1\},\{4350204259,1\},\{1368671486390599,1\}, \\
& \{974739719714982901997720702321205649353565728668293078842648318 \\
& 9237261806409217097289516440512139562408395382483277188507479689905014 \\
& 2165871590015294190621026382876094395171940099284353008092480477080458 \\
& 34624194394206146687596198519,1\}\}. \\
5^{375}-1 = {} & \{\{2,2\},\{11,1\},\{31,1\},\{71,1\},\{101,1\},\{151,1\},\{181,1\},\{251,1\},\{401,1\}, \\
& \{751,1\},\{1741,1\},\{3301,1\},\{1989151,1\},\{3597751,1\}, \\
& \{9384251,1\},\{28707251,1\},\{49892851,1\},\{183794551,1\}, \\
& \{317286001,1\},\{281057814001,1\},\{527559073501,1\},\{4032808198751,1\}, \\
& \{767186663625251,1\},\{38033863525621501,1\}, \\
& \{114325188700219508485150989127701764063766456048049316286280921 \\
& 974231310834688143728860978538113462241744523389270025 1,1\}\}. \\
5^{405}-1 = {} & \{\{2,2\},\{11,1\},\{19,1\},\{31,1\},\{71,1\},\{109,1\},\{181,1\},\{271,1\},\{541,1\}, \\
& \{811,1\},\{829,1\},\{1171,1\},\{1621,1\},\{1741,1\},\{4159,1\},\{4861,1\},\{6481,1\},
\end{aligned}
$$

$\{11071, 1\}, \{31051, 1\}, \{169831, 1\},$

$\{297315901, 1\}, \{921737881, 1\}, \{2332368714077641, 1\},$

$\{209305668209401393906696586829173386548951703282782780402553460$

$428192664808453855732006586516419275266485433899036862363379234776489 6$

$6550217562239016503354340470484047910021290325739572565374335 1, 1\}\}.$

$5^{419} - 1 = \{\{2, 2\}, \{839, 1\},$

$\{220094841901924571650987622467147400187094486771311371516144911$

$714015975930420006265667666086111341353000278410812354487905106550120 7$

$589429963512017504308159503343457840640327636864266817944697225937466 8$

$808204723627440950228738664568060266419699729631879254909886150256542 1$

$87002049015304329, 1\}\}.$

$5^{429} - 1 = \{\{2, 2\}, \{31, 1\}, \{79, 1\}, \{199, 1\}, \{859, 1\}, \{12207031, 1\}, \{17885869, 1\},$

$\{300128401, 1\}, \{305175781, 1\}, \{386478495679, 1\}, \{24615836446631, 1\},$

$\{608459012088799, 1\}, \{53130566763791958299, 1\},$

$\{700396364054946116190231244187233697567991757371291316621476117$

$679900684290662888533421344393704072844416852615673369743027047138535 4$

$366194103123949265868122585494202696963486281787967286745856145629 9, 1\}\}.$

$5^{455} - 1 = \{\{2, 2\}, \{11, 1\}, \{71, 1\}, \{131, 1\}, \{211, 1\}, \{631, 1\}, \{911, 1\}, \{4201, 1\}, \{19531, 1\},$

$\{4914911, 1\}, \{6481021, 1\}, \{85280581, 1\}, \{305175781, 1\},$

$\{1034150930241911, 1\}, \{20986207825565581, 1\},$

$\{146684795895102231260166784920352531108675064294400608294885034$

$090547033035637584893997748957171662561188159274171310210155175205040 2$

$392481389121656230195047636438859923133275759186668744193606229739684 0$

$59507314776329754126186679699826 1, 1\}\}.$

$5^{459} - 1 = \{\{2, 2\}, \{19, 1\}, \{31, 1\}, \{109, 1\}, \{271, 1\}, \{409, 1\}, \{829, 1\}, \{919, 1\}, \{4159, 1\},$

$\{4591, 1\}, \{31051, 1\}, \{90271, 1\}, \{317731, 1\}, \{26481241, 1\}, \{261594199, 1\},$

$\{466344409, 1\}, \{1203176701, 1\}, \{654652168021, 1\}, \{360143672909416579, 1\},$

$\{198791992856326005085331376433482969605877318722143940889431980$

$489463720509673026604871988483678150820044590744617705554146322309190 5$

$696844879221786697906263435690082126115004678725070762150832954448683 0$

$56984579063792129, 1\}\}.$

$5^{485} - 1 = \{\{2, 2\}, \{11, 1\}, \{71, 1\}, \{389, 1\}, \{971, 1\}, \{3881, 1\}, \{12611, 1\}, \{264811, 1\},$

$\{2387171, 1\}, \{5829701, 1\}, \{11401381, 1\}, \{4588270721, 1\},$

$\{899097422777008401851167409568439226688354258115294906618122747$

$782532492603622278702596836859210229192770344709344970245026277131870 6$

$624907141025954928434174867971014347304282351089880365913467133026437 7$

$613396645235087771549631301917507547715052361509520579630250551364924 4$

$61716018167949, 1\}\}.$

$5^{487} - 1 = \{\{2, 2\}, \{1949, 1\}, \{949651, 1\},$

$\{338030782923115295201942716301325798939609625814210230551939611$

$945015331201188803738938938594740946846584766858388469319558601961938 6$

$081716500876687481937695155899841474028396176088281346688772579227875 6$

$$\begin{aligned}
&3509269167736157038164038007165920417813999991264751741881801113872837\\
&0889507942642873762460448033520076465197895766637434176669, 1\}\}.\\
5^{495}-1 = &\{\{2,2\},\{11,2\},\{19,1\},\{31,1\},\{71,1\},\{181,1\},\{199,1\},\{331,1\},\{829,1\},\\
&\{991,1\},\{1171,1\},\{1741,1\},\{2311,1\},\{19801,1\},\{103511,1\},\{143551,1\},\\
&\{169831,1\},\{511831,1\},\{1731511,1\},\{12207031,1\},\{65628751,1\},\\
&\{190295821,1\},\{297315901,1\},\{578259991,1\},\{386478495679,1\},\\
&\{14300545887541,1\},\{22542470482159,1\},\{153560376376050799,1\},\\
&\{4713111284357051853953338515917077505133275031954145601486367 89\\
&3181842375952341924523640846996910941087490582379418488012396472231982\\
&8590848192281082549715482418421188963871352831829562 8721,1\}\}.\\
5^{509}-1 = &\{\{2,2\},\{1019,1\},\{219889,1\},\{3600024661,1\},\{1258198860123384481,1\},\\
&\{146973746295196128684452452818361140606771984634491835822894983\\
&8437884345459056777706312251825375554709899720188741443637459551194465\\
&3088447227394532932093811045757366103850647757673358506700903814558120\\
&7495055129900688439935425670675031729380609285642570582070360912250188\\
&855301473483498790749253787080053513313637308 51,1\}\}.\\
5^{515}-1 = &\{\{2,2\},\{11,1\},\{71,1\},\{1031,1\},\{3709,1\},\{28429,1\},\{41201,1\},\{96821,1\},\\
&\{236985509596761461,1\},\{326493403069610501,1\},\\
&\{889386641705408789383971855246449676478294659052010668700163546\\
&1202167877843986323894152311145359968823939470841480436415600261268441\\
&2231356140285550468181177445891205264842788328993576535458329913616307\\
&5737362864547921627805643054962647695001425717452682239655975646107070\\
&555628655055582091476084 2281,1\}\}.\\
5^{525}-1 = &\{\{2,2\},\{11,1\},\{31,1\},\{71,1\},\{101,1\},\{151,1\},\{181,1\},\{211,1\},\{251,1\},\\
&\{379,1\},\{401,1\},\{631,1\},\{1051,1\},\{1741,1\},\{3301,1\},\{4201,1\},\{12601,1\},\\
&\{19531,1\},\{28001,1\},\{50051,1\},\{200201,1\},\{519499,1\},\{1736701,1\},\\
&\{1989151,1\},\{8894201,1\},\{9384251,1\},\{22661801,1\},\{49892851,1\},\\
&\{85280581,1\},\{183794551,1\},\{1657309151,1\},\{37815352051,1\},\\
&\{119461537021,1\},\{21226783250214361,1\},\{38105263380318401,1\},\\
&\{238058276429599046633889495325950582786109822001395692046300078\\
&4429772902260954842067760053141367248182478411305061359649046387270446\\
&29087849693296660874466713715197797393692682501,1\}\}.\\
5^{567}-1 = &\{\{2,2\},\{19,1\},\{31,1\},\{109,1\},\{271,1\},\{379,1\},\{829,1\},\{2269,1\},\{4159,1\},\\
&\{4861,1\},\{19531,1\},\{31051,1\},\{280729,1\},\{504631,1\},\{519499,1\},\\
&\{2161279,1\},\{576448489,1\},\{19523494474219,1\},\\
&\{23792163643711,1\},\{305921358183421,1\},\\
&\{261192068102536376507282642127041664344171607867105650924528627\\
&4596750554018456723970355767575097657709712607823360174583088569608923\\
&3922967208332295114985706667211544260659083539213150528502893820571290\\
&9452582033133573304440459720050238572577147519276397563917711499090275\\
&245563536244218079,1\}\}.\\
5^{575}-1 = &\{\{2,2\},\{11,1\},\{71,1\},\{101,1\},\{251,1\},\{401,1\},\{461,1\},\{691,1\},\{1151,1\},
\end{aligned}$$

$\{8971, 1\}, \{173651, 1\}, \{689081, 1\}, \{1069501, 1\}, \{2855911, 1\}, \{9384251, 1\},$
$\{29028071, 1\}, \{40231601, 1\}, \{201781301, 1\}, \{824480311, 1\}, \{17223586571, 1\},$
$\{26135496851, 1\}, \{332207361361, 1\}, \{100062970166640331, 1\},$
$\{77633668734979047868314635337832502450929518761402001347075359 1$
$9805002280206672322343575762994364780445829682469859194831432195034544$
$6172559108807780088047953275092657800931383604622064865008749807192276$
$90767348504904485613954495531632894654471813823376091209854 30951, 1\}\}.$

$5^{577} - 1 = \{\{2, 2\}, \{2309, 1\}, \{23081, 1\}, \{185750794563808939, 1\},$
$\{5105323617700870926776103697758097168323743427802595490549717 74$
$2124893690644995450992445164854173535740877942826040901048177544675143$
$7580506967315106981145306893158190984187824850132155627924424389614166$
$6600492281665066313062961310952533955731856065326527481880479798198289$
$0021309195271550512997431030529858391463628455972943672282802764051571$
$847013350108217414997784668230815 51, 1\}\}.$

$5^{615} - 1 = \{\{2, 2\}, \{11, 1\}, \{31, 1\}, \{71, 1\}, \{181, 1\}, \{739, 1\}, \{1231, 1\}, \{1741, 1\}, \{15991, 1\},$
$\{62731, 1\}, \{248461, 1\}, \{960139, 1\}, \{1785961, 1\}, \{2238236249, 1\},$
$\{2908888501, 1\}, \{9025583299, 1\}, \{18383372131, 1\}, \{1256950067521, 1\},$
$\{4564849701718778949019898949338948989425748340479073214340764 84$
$4336840124334879270913831344323062846599274786231684269862095407509136$
$8429721039883744204100156921134385130885137644882818628077143058175378$
$0799510654162406823735374363119666524638907529637669605204522000923314$
$6022018078700722596519296410247434197042391835714241950758300 11, 1\}\}.$

$5^{629} - 1 = \{\{2, 2\}, \{149, 1\}, \{409, 1\}, \{1259, 1\}, \{94351, 1\}, \{9895429, 1\}, \{466344409, 1\},$
$\{13971969971, 1\}, \{2135840614741, 1\}, \{8737481256739, 1\},$
$\{1288358654468945881425349997303602828760255337740921056932206 06$
$3583181903622535809458323281611123060456914577332640166721834435375558$
$2682696973764171794240522314293748479753077359269414736447573007517612$
$8848878862980382978292383611919560838398187108302650498036060196950888$
$9263158799944805231357810168384171639932610148809690802333388615734964$
$360402793563340053161645626478021, 1\}\}.$

$5^{639} - 1 = \{\{2, 2\}, \{19, 1\}, \{31, 1\}, \{569, 1\}, \{829, 1\}, \{1279, 1\}, \{12781, 1\},$
$\{156309592445437909, 1\},$
$\{1543724076401780359727189598498555100758512911291954245552699 26$
$2151180799702488101869191170016008675155886745494085883369535880181037$
$6574225878317083666478751008127311387396308176011881133512689090922868$
$6903644013894877500614888718197308144346730221874873185264528060083209$
$1968451930075236859610433292856547181624398200251821438426075272184672$
$6943438867060846545462387370375740538769557305573684041489509036225466$
$9, 1\}\}.$

$5^{659} - 1 = \{\{2, 2\}, \{1319, 1\}, \{146299, 1\}, \{4706579, 1\},$
$\{1150750998079355888820212647519400940561556586374633776638536 67$
$2206325203222639995590991098386948972406876806450043309798100862551290$

$1292522223419502309561907598083365569206790474080180532330333932089730$
$5234632825645985917158221618002412810184902524062641497304214529600271$
$0875872724080886540431482317383576698771720528781378676437790220381565$
$5720738331175622319741590097048106646130065955094232814718643685476637$
$413014100006068869226112620421369, 1\}\}.$

$5^{699} - 1 = \{\{2, 2\}, \{31, 1\}, \{1399, 1\}, \{6991, 1\}, \{258631, 1\}, \{1318781, 1\}, \{412500871, 1\},$
$\{222831302398955532521959190646940823113894144155108483002198117$
$7501325222743556885240494533503982712196587074139199532772400503560172$
$8394673583737092361870300281877823040182496957960817930837500717198089$
$1287301747160520803485484678661057684054072997657392338164758307317505$
$0185369187191980829718153530185588994084797701815317910747398890149993$
$8540963893977695560902086517433868908057745827399178550040033902568727$
$56233797854303188005365115119685364392259235269, 1\}\}.$

$5^{719} - 1 = \{\{2, 2\}, \{1439, 1\}, \{8629, 1\},$
$\{730048752772368934341123593939801606098928950940927711629242732$
$5701371806504719514940286879758892289909895020967193984118283825131897$
$5095217791119534671414904910299957951194155009452322926943008430294331$
$5738406779650578446298115081840384401164237856813807289330906718730121$
$1258167224216416885377153479655206689197604845471750147165436960489423$
$4260556846443417973787106568044080428710483867270480445839244758346117$
$4698387105966302027813502158311672480998820250194897966997959158393251$
$419121375401, 1\}\}.$

$5^{725} - 1 = \{\{2, 2\}, \{11, 1\}, \{59, 1\}, \{71, 1\}, \{101, 1\}, \{251, 1\}, \{401, 1\}, \{1451, 1\}, \{21751, 1\},$
$\{35671, 1\}, \{97151, 1\}, \{1461311, 1\}, \{9384251, 1\},$
$\{22125996444329, 1\}, \{508005163062054101, 1\},$
$\{179361077822467174056605430720587201104180499067236863659757258$
$3246139207142222047982008477792446003969837501791733658935837497792483$
$4949686678977493215711816837355472847487615780543460344094576227386180$
$9144537104855211973547740874053104678976331674031690255482831339642669$
$4651516642482203138617240398608712932655656196080363400941650616538363$
$2684607155281790885121592091352815909222972036726964724382069134619985$
$920232193635928117761, 1\}\}.$

$5^{735} - 1 = \{\{2, 2\}, \{11, 1\}, \{31, 1\}, \{71, 1\}, \{181, 1\}, \{211, 1\}, \{379, 1\}, \{491, 1\}, \{631, 1\},$
$\{1471, 1\}, \{1741, 1\}, \{4201, 1\}, \{8821, 1\}, \{10781, 1\}, \{19531, 1\}, \{72031, 1\},$
$\{519499, 1\}, \{1736701, 1\}, \{24556351, 1\}, \{56117251, 1\}, \{85280581, 1\},$
$\{5071357999, 1\}, \{119461537021, 1\}, \{2035895040229, 1\},$
$\{12388179201301, 1\}, \{21226783250214361, 1\}, \{16650328910366149531471, 1\},$
$\{154576006904264792879340085445529610458109505205649255745195165$
$8399419576649313649300949233360011187902444795237240695638475373909128$
$2561586945544386967224867234036578784031410017827558434736486240113130$
$5918682029223872264116724727648750987159953478866870339784015222436398$
$6381771660879301476579224437916357908652304765272369996456557080301743$

$9822221, 1\}\}.$

$5^{749} - 1 = \{\{2, 2\}, \{1499, 1\}, \{6421, 1\}, \{19531, 1\}, \{81163139, 1\}, \{2379056191, 1\},$
$\{5486016363791, 1\}, \{35411885288978941, 1\}, \{172163150940018661, 1\},$
$\{69539984368841236753062916331417197025620117194172221354032138 1$
$236707412250646005114600269234763079160717714663282503114093822080820 3$
$037732253333087717741724010607887022613326030696009917535992064696702 4$
$667573646589718302737526649215557935998009672688380689916436190792385 6$
$573255666856203843737708064881377267921534344464272992323439577113344 6$
$172848259876057471651497131017518409894169485149806374150012496949083 3$
$8925101682753686547975084595962989 1, 1\}\}.$

$5^{755} - 1 = \{\{2, 2\}, \{11, 1\}, \{71, 1\}, \{1511, 1\}, \{1175989, 1\}, \{2237519, 1\}, \{4631171, 1\},$
$\{389439261961, 1\}, \{12799319666401, 1\},$
$\{184030733237458959129866412673683379407758187398209636619130930$
$814567338271296206203734402110919628136411060294596935464025729572684 2$
$879551064178482733850767882559872605971309273413162752130995412380968 6$
$319741681986508801853288589786821977193556129137961367035099738672143 9$
$689212319572858272684528926886521958273383376833929342609627185104845 6$
$968644263267862954530073029137179491033626445683198737106361647006580 0$
$028178149303774463678882380764153762271348619674560300422813852 21, 1\}\}.$

$5^{779} - 1 = \{\{2, 2\}, \{191, 1\}, \{1559, 1\}, \{6271, 1\}, \{23371, 1\}, \{3981071, 1\}, \{2238236249, 1\},$
$\{189875010211, 1\}, \{5079304643216687969, 1\},$
$\{209653389496537630045088488719663858162844739661887476946592841$
$199571133685354558637205112402667344270207102104507780072323802239946 7$
$011091654059443395614913333660293168333409476867144425031427231000950 3$
$517520795641388764746119009687750814408881727202533800042247103001603 4$
$855443919668868605296856444315704738254724073559764486425776733132577 7$
$735043415036711568064848009486643023511241865641528327884583917210230 5$
$010783610854767026669428390628400005491299072208726006035913117639393 7$
$99, 1\}\}.$

$5^{785} - 1 = \{\{2, 2\}, \{11, 1\}, \{71, 1\}, \{1571, 1\}, \{32029, 1\}, \{62801, 1\},$
$\{5129819, 1\}, \{25330381, 1\}, \{2107089779, 1\},$
$\{181814891234056248796721584200795539928556447993843185846179474$
$948821479941273571717104000903548333720434811571044254021694223250444 5$
$070797457122465685049522318760379401893188387229631377076378720330117 5$
$689604413739691527773287064742563966424216199427004951718560313255483 3$
$290089195976423206283654880997577163544716785025685927029384561234909 5$
$154648961195562172378748680472101130686098309927975704169129295616532 5$
$158614735068480734600915317161312519692922257880978808921867489805896 3$
$678423491506188261581231919, 1\}\}.$

$5^{789} - 1 = \{\{2, 2\}, \{31, 1\}, \{1579, 1\}, \{127819, 1\}, \{309289, 1\}, \{1523516701711, 1\},$
$\{260448881365314963308512653116427615309715340567379649856610317$
$616554168154691080458542693079552875897425106824909523340977765014989 3$

$$
\begin{aligned}
&0577001227146142902326868045250100042253239547899214321626257217774505\\
&4713359072744630844551955425840845648209119551734235805551254337392960\\
&6784490187219454965113266542835995143071639348870413160991647706274452\\
&9064774339803341614648212637196013044426089258739711079599730520793795\\
&7308731697654480867743431049535833076461079748257780288084171159526454\\
&35097687870689614922710033346996338673069, 1\}\}.\\
5^{809}-1 = &\{\{2,2\},\{1619,1\},\{50159,1\},\{397243271,1\},\{6516689161,1\},\\
&\{34833269732760609314860456226056487590874436120997199153924793 8\\
&1073650113837882192773808367170996510645200155426520255832304875377481\\
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&6649438817659900591483058480624118315701834824277208320476077200739265\\
&0021758031574544664295768690309439020727517790383887153829943837861103\\
&5378671741270846859703976887867265202246767102627093885570893896881394\\
&8054030739436355498384076152145928393029869673863327514926153308418690\\
&56022797844853293535387850190719558917122659497513826131, 1\}\}.\\
5^{905}-1 = &\{\{2,2\},\{11,1\},\{71,1\},\{1811,1\},\{1576511,1\},\\
&\{700931551,1\},\{478171613586061,1\},\{10906993756490561,1\},\\
&\{11338692524078631680950150810415706451610878637867841807887222 8\\
&3483062969267936887682647720492356427291921625892962160394553562902800\\
&2352166378273782439776027551591276863597125873525940375623631222669590\\
&8317488314654419698518671406379907066458892518546825848071613924677390\\
&3062876730878664939940225532110918846008291014101034102826997047771592\\
&3268604153995893310679605616607624297094601170492449800711879085240809\\
&7106898647323521268128439263577987062511524000680499145886478418391800\\
&6863072688827556603943390883801698080722504977589556175158258719160307\\
&7979787718613621692095064011, 1\}\}.\\
5^{935}-1 = &\{\{2,2\},\{11,2\},\{71,1\},\{409,1\},\{1531,1\},\{1871,1\},\{103511,1\},\{192611,1\},\\
&\{511831,1\},\{4200769,1\},\{12207031,1\},\{65628751,1\},\{190295821,1\},\\
&\{466344409,1\},\{53289827581,1\},\{1497355422751,1\},\\
&\{35167836680344146789230969792241037062706508328038786026955480 6\\
&4581904675475181281091167986329066013504927126276950771308858021188085\\
&3443314230103626183962925366462840336279850862571459945455910778479463\\
&9540406066785051791512230980737780195550854979952715899281959393891202\\
&5085419793173371786651223197242155610694816370122284300826099709248450\\
&6761061060762641307527660737826289166643154560060901898863704606355596\\
&2645997908443152199988046833641459055143430974924675078912148567415433\\
&5858877996845413465208312032312489840104474253321366178396521439673350\\
&9002346549, 1\}\}.\\
5^{939}-1 = &\{\{2,2\},\{31,1\},\{1879,1\},\{229651231,1\},\{37421136592412809,1\},\\
&\{10747311938216634458076493125648248951733619697529702897441585 4\\
&8039237126956598691612184079420046616200019364883071718530423973956830\\
&3491847575395277005226186685775803518061921567587573021112749975531419
\end{aligned}
$$

5876853515105575422709466710370352212018261971658049222452520739443502
2705840159707524094963475279126454154234900940126398582995963343457990
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4456650154972609033956227370737008218970700549883482566225952700991146
7961, 1}}.

$5^{965} - 1 = \{\{2, 2\}, \{11, 1\}, \{71, 1\}, \{1931, 1\}, \{36671, 1\},$
$\{295291, 1\}, \{1370755481, 1\}, \{749154725101, 1\},$
{4780313810578058451943381211643060390519477991821782072253036304
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7982991675889323812453524194215241723300512771984291137281387991586485
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0955325379073222391521790839453781132401241722723893692879942110501841
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7572828733413119566197309328951129043621981283232598353808112844291831
66715947107131, 1}}.

$5^{975} - 1 = \{\{2, 2\}, \{11, 1\}, \{31, 1\}, \{71, 1\}, \{79, 1\}, \{101, 1\}, \{131, 1\}, \{151, 1\}, \{181, 1\}, \{251, 1\},$
$\{401, 1\}, \{1741, 1\}, \{1951, 1\}, \{3301, 1\}, \{14821, 1\}, \{17551, 1\}, \{107251, 1\},$
$\{1659451, 1\}, \{1989151, 1\}, \{9384251, 1\}, \{49015201, 1\}, \{49892851, 1\},$
$\{183794551, 1\}, \{305175781, 1\}, \{933692761, 1\}, \{608459012088799, 1\},$
$\{1034150930241911, 1\}, \{3879644432625001, 1\}, \{20986207825565581, 1\},$
$\{142816611356873351, 1\},$
{1238506732780689876330102841921698392731229597199852977276369152
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7076318409387913654975682997635264156215815530059761699975190368601159
5763519229195641703485715570189856084639570896488105298903083243306117
2263624645207026611266834814525143594194915890358668911119031823796487
7997430526580714121651634957345583693411786187541585122387562067422016
4994672029728251, 1}}.

$5^{989} - 1 = \{\{2, 2\}, \{1979, 1\}, \{8971, 1\}, \{1644512641, 1\},$
$\{332207361361, 1\}, \{8345878028531, 1\},$
{5902950546743100935780125847276831087544589040792627161996067528
8573920869388812633714385114515020248410873951986034322638825921027042
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0744104237077496028332681961908257331091936788021216029911397200113336
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1

$8491223420842162546933312223547268698997347054902847094218958601219464$
$9962813507055665638644724530194971806814350774262546050724327752475443$
$668258823327416118774099439, 1\}\}.$

$5^{999} - 1 = \{\{2, 2\}, \{19, 1\}, \{31, 1\}, \{109, 1\}, \{149, 1\}, \{271, 1\}, \{829, 1\}, \{1999, 1\},$
$\{3109, 1\}, \{4159, 1\}, \{15319, 1\}, \{31051, 1\}, \{344145511, 1\},$
$\{13971969971, 1\}, \{966739015279, 1\}, \{8737481256739, 1\},$
$\{434809233528041113653416827379371108979955050516301210232063963$
$1124204598589716888398392755428616230226668224059176988475689602245467$
$1871190019101495256757495956664354892722143753317869611857118066336524$
$6704987291976313467047782831696890312730698425294021762596469390292092$
$5730018139576476971806439440455225968488213104962677500279076595124533$
$2322056510908588493820603662819038711145323161116775219361971106954593$
$5321887064833740491738372559140737515652416818059530764394957286121899$
$6695182344245409183515320167308174718663303309173343809352893406600873$
$6252533296229266753719073823896349201046169342781948376016332230633821,$
$1\}\}.$

$5^{1005} - 1 = \{\{2, 2\}, \{11, 1\}, \{31, 1\}, \{71, 1\}, \{181, 1\}, \{269, 1\}, \{1609, 1\}, \{1741, 1\}, \{2011, 1\},$
$\{4021, 1\}, \{26399, 1\}, \{126631, 1\}, \{5031363661, 1\},$
$\{2454335007529, 1\}, \{598266261966511, 1\},$
$\{110562601576528691339722276681709205161002122577562897413801590$
$3041055751095244566605372323402876447302420334083732561434566356269179$
$8857237126024490213052800329811796443634154541569511784715631881977604$
$1161942758931412495898115378039723411731572625857461082438216286013516$
$1138746891490408409832163961307278664049986838088043723917819046092511$
$4716571005678499798725578270665587324459806607359042789421610876308512$
$8492880002588483272737868689150485494567886437779334052055143276306049$
$3615717742208098273901785365766190630797753068351500638591766782933001$
$1714138096700565840208620492162367177110621603675602015521048706023775$
$39542514431, 1\}\}.$

$5^{1019} - 1 = \{\{2, 2\}, \{2039, 1\},$
$\{218251481952643588336462257707935661031801469393068350225927162$
$0291519798738517112167026149104964218307079293507636891618589930516482$
$0599400255148458830003149885022006199140073365474717435837528905557283$
$4423001837587873982423408594081231873341882410982696580642324731329943$
$1209715424031173013981118145612143650421825321628108467357350313990718$
$9169249554840422616636115668792075667997941626245214414823824341373149$
$9902997887818871374544147769791993306723621935437124636032454352410852$
$1473315200403538610997034882591162190528798454302547740675396926091751$
$0095979251758243275165878595182900732784434755606542314695194387825864$
$8954661498575071962692980980026394841210050766915389767367855878468329$
$2997473884646129, 1\}\}.$

$5^{1055} - 1 = \{\{2, 2\}, \{11, 1\}, \{71, 1\}, \{2111, 1\},$

$\{20876816861, 1\}, \{1532829720131, 1\}, \{5594292675731, 1\},$
$\{2194077372701082013629956460299549029026928111817163943941415578897791667026346743252530018606588533820299322215327694018977690165000917344940569129029237840618249865206070453213888112464590165592147399463029222508477355891132190856558570900028667520590042821747097119378328012045398933137195453286874417911003468988695091599882237160112371446163217332117337368325058534811715158865707368064457267310804784599712727426451979415396372143069095397095208132280131277916217072945194140186766797696101755642673921221924904801458840898615431280659381466496737031143826920540678172580132578124925428313501687189473873597943054471695483320912352084672632576855803565323547866623668299693215828201453642621, 1\}\}.$

$5^{1049} - 1 = \{\{2, 2\}, \{2099, 1\}, \{472051, 1\},$
$\{4182858724478818361270866174622679046946511054120922233806228084802666852059379762178022357888866503717210048931901995448480748982614480993787628362361667879499195201636384352925250270048486460097819990076044259924573200143207158111812470363353600787039668818700206443681902261614443395841785572117142468056868207890714401635095531258872880984506218463428268562223545586181062738943268098933020903839675004471797995368458781133365371307066844938919332736822214164707298958776307133848114481182282042172604656617869577297194309832407937625776372316802220794958655984454817053945679511294525533320861114153435820585627701770681415756431823752811250000873585539332909202954364444070909525418855593065230438086583447703638635169, 1\}\}.$

$5^{1065} - 1 = \{\{2, 2\}, \{11, 1\}, \{31, 1\}, \{71, 2\}, \{181, 1\}, \{569, 1\}, \{1741, 1\}, \{2131, 1\}, \{10651, 1\},$
$\{17041, 1\}, \{308851, 1\}, \{7484111, 1\}, \{8499626476216091, 1\},$
$\{156309592445437909, 1\},$
$\{1727389691566751537675549199286743019081390608417531663061644214189690123369378716610640063781794383565296376179134213098377939343412293835810496648362782157030609050177654383525473775520372927388255248350872193774758746160981890014243738401848602400575277241324699692953707067572593162538139920611019617804562558081809511240642220870834243054883637859590342260479163416763810208347824304220857440114735578310071843664995286171493559217690933544474824171001356306923536177601318137854681291328528104759469997970277925002438670435929251074098481597257894677986990771718189157722832691231398169658142028703908105745683522887007942904717760067893088665762897745112166019678391, 1\}\}.$

$5^{1089} - 1 = \{\{2, 2\}, \{19, 1\}, \{31, 1\}, \{199, 1\}, \{829, 1\}, \{2179, 1\}, \{3631, 1\}, \{12101, 1\}, \{143551, 1\},$
$\{486179, 1\}, \{1731511, 1\}, \{12207031, 1\}, \{386478495679, 1\}, \{22542470482159, 1\},$
$\{315269911213473908876921694295763790765675764885015556112624070471710470934260550435062712032147384976304465956769603775647102527487912713134291097958756824509773005049095938128510962698715596241881753 67$

$$
\begin{aligned}
&7593455432584442910301511639035210942217893094070012718879922636442627\\
&1912516464084637106575739420955271092487710252716868888150435521610753\\
&3010226847414439969521320715641295569168214391604793063309020454642278\\
&2757470131476772693279111176795139536531904896751108650399074543385032\\
&3958674637146061230669792512226263620148887674185118941338827527635590\\
&1414531874744431757959291215382151529721120117611802635175360141647833\\
&1688882443590499211505497038162629043763429520830311107319424895362619,1\}\}.\\
5^{1175}-1 = {}&\{\{2,2\},\{11,1\},\{71,1\},\{101,1\},\{251,1\},\{401,1\},\{941,1\},\{2351,1\},\{4231,1\},\\
&\{9384251,1\},\{20885861,1\},\{68308861,1\},\{61909208515151,1\},\\
&\{349569600611951,1\},\{640359292705751,1\},\{96766610646500911,1\},\\
&\{365133874274123109588983278736440576978269837005094163089889204\\
&3536974003444866851684254629249103727380093443470313102631007721295057\\
&1758562029372433780176321852994174821320240867623898123433437513903238\\
&0007616303229027376454441490740662030827282421051234521495964353477590\\
&4823730922362420098726752327908456766998142791691211781341617562323349\\
&2300819617095446631436215968809837585477736952200251886511230568785824\\
&4825744318828320504209402422755090049410856749405895889740889293391568\\
&7152278157978047563065698781578994209666630810242856105993506058224695\\
&8313377804245463699925731017945073829005760138325373541524229928629466\\
&5907416116951015354899986800455439045688046860176735606387346821661710\\
&26620332678059993786349151,1\}\}.\\
5^{1185}-1 = {}&\{\{2,2\},\{11,1\},\{31,1\},\{71,1\},\{181,1\},\{1741,1\},\{2371,1\},\{12799,1\},\{73471,1\},\\
&\{190391,1\},\{657439,1\},\{19834531,1\},\{60098461,1\},\{68175421,1\},\\
&\{38701052851841,1\},\{205367807127911,1\},\{809159522736541,1\},\\
&\{16961361791934811,1\},\{104449388937445035371,1\},\\
&\{241329614382387570241936215294982443103561047918220991508803358\\
&4225183148008332996501517989971915105707914904179177309791936416269013\\
&3979479132009878261479394937124012279598271936502418888452261414707356\\
&4203639646818944721841430283204815337681758909697111906273188576549342\\
&0292124816222693731221049708230173966340539871035791735592888206633080\\
&4210183379532412145805659889820693536108315346304904076345485371127802\\
&2520165654913040420342240947870724327081957425961070483010524253389127\\
&1391226393719469721163109253894579717257468128488932919009218111107306\\
&6167831058185933286421291813961812334752605936563683471314543850014035\\
&1380513276369843819703726870494730684196332019144095185838065126791681\\
&,1\}\}.\\
5^{1125}-1 = {}&\{\{2,2\},\{11,1\},\{19,1\},\{31,1\},\{71,1\},\{101,1\},\{151,1\},\{181,1\},\{251,1\},\\
&\{401,1\},\{751,1\},\{829,1\},\{1171,1\},\{1741,1\},\{2251,1\},\{3301,1\},\{11251,1\},\\
&\{169831,1\},\{1274851,1\},\{1989151,1\},\{3597751,1\},\{9384251,1\},\\
&\{28707251,1\},\{31248001,1\},\{49892851,1\},\{183794551,1\},\{297315901,1\},\\
&\{317286001,1\},\{1086749551,1\},\{281057814001,1\},\{527559073501,1\},\\
&\{1390332823651,1\},\{4032808198751,1\},\{84244323125251,1\},
\end{aligned}
$$

$\{767186663625251, 1\}, \{38033863525621501, 1\},$
$\{1614016126717641549108325232291581251575463158385048272584571481474910287081516012227880722551166360794595919863514608094274368604155244361761933486300565311140645523684270676881738125879362730654655670537898062310809790524362045624419273632726258092633428789287864298599349617364679411203587647877664130775288140742395408171387881770194157353630030851833326168084706990403126258447303922600490929901017148096089932368558506396883329057882124779535314020058163400169400791698264811670758794249297492030701533132607914209012759108488326629782690583697698300758777951, 1\}\}.$

$5^{1169} - 1 = \{\{2, 2\}, \{2339, 1\}, \{18371, 1\}, \{19531, 1\}, \{53441, 1\}, \{7330265231761669, 1\},$
$\{948403860842441853681488418640586385531042275970744491555365726155731564465467196236351153660291640056841355123508560399664887880926060469898828606795150711533892809137025015494820125992577670891369019350968707884274806035890919288681307665724823124949518092399305489722315102703160789562599098182345008010880318851742902023539427291224407154223033114647016294609527279297324475715263172787692983735192549106995963222580663870706239898292057660925122511654059364839291367397387025261174443885673242709782891478643768429409479475062311719852271518613285430540551056342654332182987530002365701355394809839822652374806632148319861847945752461245423618991900358541786676673925198739232183178552690827473143370358653343425203262050960965755806075883336915601354433879699458867270913916595 1, 1\}\}.$

$5^{1199} - 1 = \{\{2, 2\}, \{1091, 1\}, \{2399, 1\}, \{1007161, 1\}, \{1528399, 1\}, \{12207031, 1\}, \{222354551, 1\},$
$\{26553910894128929278112164336395077506282917949762521520068658488875063855121107528956776781203377066868436685350451277582265843469238335822438836266124984906578163384928322694181882343322465723495365405596922257632708557635484309533757547481763536142284638059466323572468697302200032739399198983937643260593822068256437215459924290610938758735469321368871758434275198671659226405229574772481492022736195252271148257286354410019351609320443826132145937974514034006102809957503645695435216527873539178517449934981913663041917792382301312222564462557421433979171980414888407889130745202934611287415666286369269445252690491855737148391441510153489137369648028532220922234913825048708168418093294334699828799155639786306710834360253613198730356341766421148392433030745016962558418162902104231308204116817860 1, 1\}\}.$

$5^{1205} - 1 = \{\{2, 2\}, \{11, 1\}, \{71, 1\}, \{2411, 1\}, \{79531, 1\}, \{578401, 1\}, \{882061, 1\},$
$\{464913101, 1\}, \{26991752705081, 1\},$
$\{2804771479250501, 1\}, \{19820497490365391, 1\},$
$\{851271289277091156669646623080447333139093945827882767582922395178447591253657568431339616559550440332828362329221076969713797822255377452278372966774652144777547594237249416608457766859950257122668694802646 27$

$$
\begin{aligned}
&3245548834810202797149320284877001333258994765524812949932893223050386\\
&2632535386901322174053558547085059002795365672449587077724288574081679\\
&4722017522092235184321700447891776234702194785015207790869176887232212\\
&0657409966300237373762578819073013343453983603090232598258133053377299\\
&1653957537554098411037884913381045657751547077562809419082919683542403\\
&8597585222260816182698439306464078559104080846477441782190457383736665\\
&2506594043108645746551934077661165322903925237445029665900383459938108\\
&1579428215635954472797387016437021492370556795683099006929636555531, 1\}\}.
\end{aligned}
$$

$$
\begin{aligned}
5^{1229}-1 = \{&\{2, 2\}, \{2459, 1\}, \{887977841981, 1\},\\
&\{1238553240967812171481952521770368903389382580430536882599274 09\\
&5136595666122657758579647259799758432570705386962476653361364732492506\\
&1355349104197848169924339664696905904553125204426439922432658528257282\\
&0395501904779952930209158161239168349964479049879204374198469192538709\\
&8757067398043587418185044619198809720115407930948191511514847807041131\\
&1629058643663814677795194268020918925671629860298071427335163237727730\\
&9178551219861327383068651009022255856613042058680632377312902996575452\\
&1960830899720230364318755945756207880228623296573012023714300928287445\\
&9107733609461558952828386388500139488972468433864531865429068546791981\\
&1947811561541393485111652629412862415140137486586470405105806587734733\\
&1297270149177350301467306887640692095060168187321219461001926604604628\\
&4089953962957102564114936913954058743274890516179436844050188994873386\\
&93573732139, 1\}\}.
\end{aligned}
$$

Appendix E Factors of $5^t + 1$

(1) $5^4 + 1 = \{\{2, 1\}, \{313, 1\}\}$.
(2) $5^6 + 1 = \{\{2, 1\}, \{13, 1\}, \{601, 1\}\}$.
(3) $5^{22} + 1 = \{\{2, 1\}, \{13, 1\}, \{89, 1\}, \{1030330938209, 1\}\}$.
(4) $5^{30} + 1 = \{\{2, 1\}, \{13, 1\}, \{41, 1\}, \{601, 1\}, \{2281, 1\}, \{9161, 1\}, \{69566521, 1\}\}$.
(5) $5^{32} + 1 = \{\{2, 1\}, \{641, 1\}, \{75068993, 1\}, \{241931001601, 1\}\}$.
(6) $5^{34} + 1 = \{\{2, 1\}, \{13, 1\}, \{1973, 1\}, \{20129, 1\}, \{45289, 1\}, \{12447002677, 1\}\}$.
(7) $5^{56} + 1 = \{\{2, 1\}, \{17, 1\}, \{113, 1\}, \{337, 1\}, \{11489, 1\},$
$\{2520917617, 1\}, \{3700770032797783 6913, 1\}\}$.
(8) $5^{62} + 1 = \{\{2, 1\}, \{13, 1\}, \{8124604717965111529, 1\}, \{10265135352352006785 1969, 1\}\}$.
(9) $5^{64} + 1 = \{\{2, 1\}, \{769, 1\}, \{3666499598977, 1\}, \{96132956782643741951225664001, 1\}\}$.
(10) $5^{66} + 1 = \{\{2, 1\}, \{13, 1\}, \{89, 1\}, \{601, 1\}, \{1453, 1\},$
$\{1030330938209, 1\}, \{650938718341740492478 4917, 1\}\}$.
(11) $5^{70} + 1 = \{\{2, 1\}, \{13, 1\}, \{41, 1\}, \{281, 1\}, \{9161, 1\}, \{234750601, 1\},$
$\{13148832195720299503896018648521, 1\}\}$.
(12) $5^{72} + 1 = \{\{2, 1\}, \{17, 1\}, \{1297, 1\}, \{11489, 1\}, \{152587500001, 1\},$
$\{273917785566730984560599580763 3, 1\}\}$.
(13) $5^{76} + 1 = \{\{2, 1\}, \{313, 1\}, \{457, 1\}, \{2136398150786037575 3, 1\},$
$\{21654472202540732126905077281, 1\}\}$.
(14) $5^{116} + 1 = \{\{2, 1\}, \{233, 1\}, \{313, 1\}, \{929, 1\}, \{33409, 1\}, \{397823386118610689, 1\},$
$\{668375848777845065895095866172488247660842757935193, 1\}\}$.
(15) $5^{138} + 1 = \{\{2, 1\}, \{13, 1\}, \{277, 1\}, \{601, 1\},$
$\{70942489, 1\}, \{22600337281, 1\}, \{66988220431117, 1\},$
$\{617324092426329883047504421365553829679018126806624959512 1, 1\}\}$.
(16) $5^{142} + 1 = \{\{2, 1\}, \{13, 1\}, \{853, 1\}, \{72497533, 1\}, \{8367932201, 1\},$
$\{200771218157, 1\}, \{554474665573, 1\}, \{1095832589179957, 1\},$
$\{10928221322658064610359890149344296102 37, 1\}\}$
(17) $5^{146} + 1 = \{\{2, 1\}, \{13, 1\}, \{293, 1\}, \{27133517, 1\}, \{84128413, 1\},$
$\{644660229355377335113807957224194811156238338635592348687079059$
$988333998767849825 17, 1\}\}$
(18) $5^{158} + 1 = \{\{2, 1\}, \{13, 1\}, \{317, 1\}, \{41081, 1\}, \{72997, 1\}, \{10390269663517, 1\},$
$\{106574974400179673389676477182530542427426505210152936524875177$
$941126368906224080123 7, 1\}\}$
(19) $5^{166} + 1 = \{\{2, 1\}, \{13, 1\}, \{997, 1\}, \{4649, 1\},$
$\{887141033302995056994713467520546276988118708626848063412547649$
$12252472658212583335342767503972760087406991 7, 1\}\}$
(20) $5^{170} + 1 = \{\{2, 1\}, \{13, 1\}, \{41, 1\}, \{1361, 1\}, \{1973, 1\}, \{9161, 1\}, \{14281, 1\},$
$\{20129, 1\}, \{45289, 1\}, \{14239201, 1\}, \{87393601, 1\}, \{12447002677, 1\},$
$\{126360649744930309735312408866105725994974215298711571613719605$
$70761, 1\}\}$
(21) $5^{176} + 1 = \{\{2, 1\}, \{353, 1\}, \{2593, 1\}, \{1827937, 1\}, \{2704769, 1\}, \{29423041, 1\},$
$\{392043794951301890898248014184097556075760058828614220171074252$
$726665644009078418441401613238668 9, 1\}\}$
(22) $5^{186} + 1 = \{\{2, 1\}, \{13, 1\}, \{373, 1\}, \{601, 1\}, \{1117, 1\}, \{1489, 1\}, \{10269119521, 1\},$
$\{8124604717965111529, 1\},$
$\{126061025159642854132157189752003696842811740308347145923678374$
$496478764980160805638161 61, 1\}\}$
(23) $5^{198} + 1 = \{\{2, 1\}, \{13, 1\}, \{37, 1\}, \{89, 1\}, \{397, 1\}, \{601, 1\}, \{1453, 1\},$
$\{6597973, 1\}, \{766142400277, 1\}, \{1030330938209, 1\},$

$\{161015770297358190717594484428694459546167619869592876477497342$
$880715274639219112507635764062933, 1\}\}$

(24) $5^{200} + 1 = \{\{2, 1\}, \{17, 1\}, \{1601, 1\}, \{11489, 1\}, \{25601, 1\},$
$\{103390853395201, 1\}, \{909456847814334401, 1\},$
$\{413358851438819728960350027504082170192096580314214059700983870$
$78334380286157790190547359523201, 1\}\}$

(25) $5^{202} + 1 = \{\{2, 1\}, \{13, 1\}, \{809, 1\}, \{3637, 1\}, \{5728721, 1\}, \{3375268261753121, 1\},$
$\{105174326056233443238401980067337481738107316832727246709094908$
$4443454114118856668976186837164720792055112948117, 1\}\}$

(26) $5^{216} + 1 = \{\{2, 1\}, \{17, 1\}, \{433, 1\}, \{1297, 1\}, \{11489, 1\}, \{177553, 1\},$
$\{39209617, 1\}, \{170735041, 1\}, \{152587500001, 1\},$
$\{238654529871987179722647739647506102676996568195151715383257622$
$082030820162304344903519245682369139479265761, 1\}\}$

(27) $5^{220} + 1 = \{\{2, 1\}, \{241, 1\}, \{313, 1\}, \{881, 1\}, \{21121, 1\}, \{148721, 1\}, \{632133361, 1\},$
$\{224873451236210728258358609190892744857970891291803720297188245$
$67306261711831534960777195126498748181020732174835607000422936081, 1\}\}$

(28) $5^{236} + 1 = \{\{2, 1\}, \{313, 1\}, \{1889, 1\}, \{66553, 1\}, \{38710282321, 1\},$
$\{297249242512358547491888615800368810090644753770361512598340893$
$5193081812901050589463714219229769780015528104157404091135948335$
$28895845309317193, 1\}\}$

(29) $5^{252} + 1 = \{\{2, 1\}, \{73, 1\}, \{313, 1\}, \{1009, 1\}, \{6553, 1\}, \{90217, 1\}, \{390001, 1\},$
$\{543097, 1\}, \{11824849, 1\}, \{1503418321, 1\}, \{2759159593, 1\},$
$\{59509429687890001, 1\},$
$\{819866248074619717120455482812290379487511077549784621303916037$
$4643036015305408138082028269249617715441177, 1\}\}.$

(30) $5^{260} + 1 = \{\{2, 1\}, \{241, 1\}, \{313, 1\}, \{2081, 1\},$
$\{51169, 1\}, \{632133361, 1\}, \{537181587281, 1\},$
$\{989466084526315544694016787821734221726297373828571596209131430$
$4065297658445343396703549131884726806439116360332352366665113663$
$745598178798333998289, 1\}\}.$

(31) $5^{262} + 1 = \{\{2, 1\}, \{13, 1\}, \{1049, 1\}, \{17293, 1\}, \{48821081, 1\},$
$\{586022872695362487415125212315099593756717010246775554400975007$
$0514328277914016286147107772387818856426543825163157869044664592$
$0005215251236583389002007904038591671053, 1\}\}.$

(32) $5^{266} + 1 = \{\{2, 1\}, \{13, 1\}, \{1597, 1\}, \{170773, 1\}, \{234750601, 1\},$
$\{4885168129, 1\}, \{126286794969133, 1\}, \{2864226125209369, 1\},$
$\{286729389765313125721925609608357082524297322817234204933292092$
$804164870508572541729524098355670110769591837945046868835094054837, 1\}\}.$

(33) $5^{276} + 1 = \{\{2, 1\}, \{313, 1\}, \{1657, 1\}, \{390001, 1\}, \{3368857, 1\}, \{150220315444217, 1\},$
$\{402297550539912605086596509583661220827334811807036762762868717$
$3078386032862664941787152719793655681013054394917306894307411392$
$3800046207479053341773213890632297, 1\}\}.$

(34) $5^{278} + 1 = \{\{2, 1\}, \{13, 1\}, \{557, 1\},$
$\{1421780856088393569434871474512274262838939257349709384666871351$
$22319737041156689018188194968991282673415275258596318868770157807$
$61896727104627241166150753931528110248588493803562075537618493, 1\}\}.$

(35) $5^{288} + 1 = \{\{2, 1\}, \{193, 1\}, \{577, 1\}, \{641, 1\}, \{597889, 1\},$
$\{75068993, 1\}, \{241931001601, 1\}, \{5207826497153857, 1\},$
$\{249063976463696053897541171205805194685595629840418536549159091$
$4367246690460291622641850079223807032451616325315875604043659018$
$33516758165042015104970817, 1\}\}.$

(36) $5^{292}+1=\{\{2,1\},\{313,1\},\{1753,1\},\{38907833,1\},$
$\{294339110898388933030666438674087732674377365128415431429470554$
$5331240383125935010573158144781268892750864406933733188794719258030249$
$2291089490612485533127588619024028737968920842242864491249,1\}\}.$

(37) $5^{296}+1=\{\{2,1\},\{17,1\},\{593,1\},\{11489,1\},\{177601,1\},$
$\{190923785574671131653743190449789430957207593248212541787481096$
$5332328967741176419199814392853404493381153581876550320297624403508261$
$3901266190175925748583706061771056186858601418196305366510257,1\}\}.$

(38) $5^{300}+1=\{\{2,1\},\{241,1\},\{313,1\},\{1201,1\},\{4001,1\},$
$\{132001,1\},\{390001,1\},\{776401,1\},\{632133361,1\},$
$\{268015942008645655782972335083605933065215410676092420009789340$
$8368820122825582473684672940279559807505636252923705499432233680678453$
$192167595783668319964687181557626562640l,1\}\}.$

(39) $5^{306}+1=\{\{2,1\},\{13,1\},\{37,1\},\{601,1\},\{613,1\},\{1973,1\},\{6733,1\},$
$\{20129,1\},\{45289,1\},\{4613053,1\},\{6597973,1\},$
$\{19724557,1\},\{34288117,1\},\{455983453,1\},\{2194449013,1\},\{12447002677,1\},$
$\{697062058837232690452475544278669319035031137004256857008937036378 33$
$965752208050695555732879539517834970108159899936795717297681428 53,1\}\}.$

(40) $5^{308}+1=\{\{2,1\},\{313,1\},\{617,1\},$
$\{496480091466470356616586471672297140921286022641270883635570153$
$9193596217204939536429817927496447289156215099970043755066010993644391$
$7435737418133661475547675715466522403922044612552361363606186324603385$
$2056953,1\}\}.$

(41) $5^{312}+1=\{\{2,1\},\{17,1\},\{1249,1\},\{1873,1\},\{11489,1\},\{247161617,1\},\{825256433,1\},$
$\{2220784177,1\},\{152587500001,1\},\{31308249137777,1\},$
$\{606074415387062678412559548607895202366463629864617011376740987$
$1229460244359687975532732870984667465498881690824336109844863077147895$
$5444613274070471745777,1\}\}.$

(42) $5^{317}+1=\{\{2,1\},\{3,1\},\{4084229,1\},$
$\{152837540196438489848172875512405615842958310737691522401156757$
$0071897952399828592532114824295851067914508479856351515268177612770134$
$3978523607986505515170434340632108442646838495540184527007012174263493$
$875473947749,1\}\}.$

(43) $5^{322}+1=\{\{2,1\},\{13,1\},\{1289,1\},\{424397,1\},\{234750601,1\},\{4195885464702037,1\},$
$\{835434658119628075220035657635966169042520482802263721821738480$
$3147739159410039153952222497500197147170505485125617423002057750608437$
$384198660933416877418828432974822797930611900580820726928 1,1\}\}.$

(44) $5^{326}+1=\{\{2,1\},\{13,1\},\{653,1\},\{2609,1\},\{189895001,1\},$
$\{869655550487180266344072498631650735681817697649453217338387875$
$0147004986203699135669385279111703381040686778239182004111154628467501$
$7625378824572082152572328861600485384920520491675698974411442827859200$
$804531413,1\}\}.$

(45) $5^{330}+1=\{\{2,1\},\{13,1\},\{41,1\},\{89,1\},\{601,1\},\{1321,1\},\{1453,1\},\{2281,1\},\{9161,1\},$
$\{245521,1\},\{1458601,1\},\{69566521,1\},\{4411608961,1\},\{1030330938209,1\},$
$\{2802204291103029121,1\},$
$\{630005246595051274112917064925622507047391294321702273077957398$
$5265187306363726979961697471339384054138187777745310141217039742942676$
$96079995983808277,1\}\}.$

(46) $5^{336}+1=\{\{2,1\},\{97,1\},\{673,1\},\{2593,1\},\{2689,1\},\{271489,1\},\{1149569,1\},$
$\{10922689,1\},\{29423041,1\},\{8906835789071809,1\},\{465314420407473281,1\},$
$\{188771740273710528624970810669294269237809741061817982897827641$

845384030613473089990525319874015931602230087063071795177204419583821 0
407280939453074591610093270220 9, 1}}.

(47) $5^{338} + 1 = \{\{2,1\},\{13,3\},\{53,1\},\{677,1\},$
$\{83181652304609,1\},\{349709778373997142089,1\},$
$\{38940502894440374345391870218084651722430190778726426498855466 5$
$54139280394178195200031678511009854170518591621639178263392582811562 24$
$51958048676735065451187746160784149821965301659978280635416 09,1\}\}.$

(48) $5^{352} + 1 = \{\{2,1\},\{641,1\},\{1409,1\},\{75068993,1\},\{369059329,1\},\{241931001601,1\},$
$\{900314420999410329315082241356645899367693864865800214773511137$
$391908264686703545645947779737367074113163197020924797677564009540450 1$
$136382610011951036983802922820006583812542352946080495822229107645192 2$
$315543041,1\}\}.$

(49) $5^{356} + 1 = \{\{2,1\},\{313,1\},\{2137,1\},$
$\{5092636615491426092127064415042207924275435223079359827443207 49$
$651601265844962214359210309005919666062942836788338197722009694602891 5$
$360791042916296956442320556091678947938308575045628583427029684854941 6$
$0218383095858740643423798517094256332073,1\}\}.$

(50) $5^{370} + 1 = \{\{2,1\},\{13,1\},\{41,1\},\{1481,1\},\{9161,1\},\{9769,1\},$
$\{40849,1\},\{5121318001,1\},$
$\{140680264279175963763498571557655236639189495448342387634086927$
$146771893880964259171836982236795342322083489351660160281529161992252 3$
$144789283325162470788292669194145660225632785121348427940581691161012 3$
$3210589802597659227215115841,1\}\}.$

(51) $5^{378} + 1 = \{\{2,1\},\{13,1\},\{37,1\},\{601,1\},\{757,1\},\{2521,1\},\{6597973,1\},\{234750601,1\},$
$\{4661402165281,1\},\{24587411156281,1\},\{147053007410401,1\},$
$\{563926930546876101445055662754938357880160974326483141239232493$
$309813151301405221982954676456659875589076424728946468941633905965390 4$
$194463173104699635859727663986214898839851948577115518145473685 3,1\}\}$

(52) $5^{386} + 1 = \{\{2,1\},\{13,1\},\{773,1\},\{157489,1\},\{3357768242667569,1\},$
$\{596991144619439286726859673687548389452047322110385930651836986$
$117524461588578690666585444879993386473893830155217016815749364952191 1$
$878881438676209920424726392869386452239599914676172638939555292834630 0$
$1204641374357350004446647331681352308853 57,1\}\}.$

(53) $5^{398} + 1 = \{\{2,1\},\{13,1\},\{797,1\},$
$\{747532461123312088180297390847669500544391312044935984072534107005967$
$830923209335944924666398419958866529783323531409620571534870956925068 1$
$444490011877908487602424602394615968996104697178215913697151563838345 6$
$083393133291430923054662991238884174386422247106338655289369319 33,1\}\}.$

(54) $5^{428} + 1 = \{\{2,1\},\{313,1\},\{857,1\},$
$\{881695186297,1\},\{1151785652724457273,1\},$
$\{264799137712800759892621927038377156545797963073050688447010502$
$604675817835055381426677129143136320087655654753913907666520531492017 8$
$999673567952095875053762400836756159699155695891460207260739754455701 3$
$975031239669196629282056414186240058206184129984920694136375 3,1\}\}$

(55) $5^{438} + 1 = \{\{2,1\},\{13,1\},\{293,1\},\{601,1\},\{877,1\},\{27133517,1\},\{84128413,1\},$
$\{7032466681,1\},\{1278677279155561,1\},\{4725897742147117,1\},$
$\{361697183870466527423525410190221612230419162055197629673670624$
$283441094590324937619710165263588590041105405367059157688795705252511 4$
$749062279156479250802369141396838057274368640720874503315292227254769 5$
$0735596143281854915400873273342511489 3,1\}\}.$

(56) $5^{450} + 1 = \{\{2,1\},\{13,1\},\{37,1\},\{41,1\},\{601,1\},\{1801,1\},\{2281,1\},\{9161,1\},\{14401,1\},$

{239201, 1}, {2516401, 1}, {5115601, 1}, {6597973, 1},
{20478961, 1}, {69566521, 1}, {424256201, 1},
{6794091374761, 1}, {25535754811081, 1}, {89620825374601, 1},
{1402227745415146762735756570721819953603429739301458701560302738
59223235109144771698721706943648449528414403209160929047859150867175923
09851155850214835539604541264775253449726649894654067021910975343180
1, 1}}.

(57) $5^{468}+1=$ {{2, 1}, {73, 1}, {313, 1}, {937, 1}, {51169, 1}, {390001, 1}, {543097, 1},
{1503418321, 1}, {537181587281, 1},
{3500855896329861441248551558276610251926056910746938460931106673
7329481569398340059330564505139721661339861548973840105434839297647987
7424615353398681890605270684986636144047489677900810246481112851865668
2355101313054665455182654117905531400118770194588563406204217445270833
232978057, 1}}.

(58) $5^{476}+1=$ {{2, 1}, {137, 1}, {313, 1}, {953, 1}, {623017, 1},
{938894988049, 1}, {1361753880209, 1}, {4959636586609, 1},
{59509429687890001, 1}, {121008044805780049, 1},
{2204290537853241257173958441497714289669018335307685047106387713
820457972242710179195505965797427440815747358294323352075351892184694
6334374002228589445832506727772294706984093361616132028556574933066005
97363861682264750959632621315325669857107670 33, 1}}.

(59) $5^{488}+1=$ {{2, 1}, {17, 1}, {977, 1}, {11489, 1},
{46455649, 1}, {13438797761, 1}, {188732040042673, 1},
{2782670740289564756529804800300191986155370548717832473685842741
346800899470085677056403681167873563332544060370682523398109759834149
0619820531900046828060902531325887480916478505197627467089238285442674
4359313360173417675580154580406271632389336212132098408775902766340942
4757552702616003374260544929, 1}}.

(60) $5^{506}+1=$ {{2, 1}, {13, 1}, {89, 1}, {1013, 1}, {2453089, 1}, {1030330938209, 1},
{8056739991333054407164477352800326654997987069708492715694203041
339488499303316022895147036094079333059901935968065565052360895220614
4638308837972583929612929460800791989642493473012635033847275665370792
5161662256589794256430846915868891511700529877446678168226035972274075
0841258161904478719531080718669688883516119099562822629 3, 1}}.

(61) $5^{516}+1=$ {{2, 1}, {313, 1}, {1033, 1}, {4129, 1}, {18233, 1}, {390001, 1}, {3084493201, 1},
{30185745097, 1}, {19511773794697, 1}, {45245904640297, 1}, {52396512454609, 1},
{5700523236198188305536590466437453875345475779078876507768874837
779586769813557816189995160817243502393658871307615023434696074830226
4133693780145484675690724396733046572964740416228460874570348401745205
2822479940026600956558468104516471127390141121204768905092660633385310
19686553, 1}}.

(62) $5^{522}+1=$ {{2, 1}, {13, 1}, {37, 1}, {601, 1}, {2089, 1},
{31321, 1}, {382801, 1}, {6597973, 1}, {506670253, 1},
{1504560496774389176871879970043812494858675667041096509938691144
676208555440398807594026590320501149837298067403736968456411083671475
7815162095922905013176049274011233964988977612521818011263591521388021
2024383537459305520283860535685242419619546302351140457412722294430269
787789174729375187959059363008758790749334444089844750269 3, 1}}.

(63) $5^{540}+1=$ {{2, 1}, {73, 1}, {241, 1}, {313, 1}, {2161, 1}, {8641, 1}, {73009, 1}, {390001, 1},
{440641, 1}, {543097, 1}, {1853281, 1}, {3314953, 1}, {632133361, 1},
{1503418321, 1}, {36280398313, 1}, {356646293281, 1},

$\{50735307557193841, 1\}, \{23320317172851318360001, 1\},$
$\{22182127424030579765013979990748295093858090184864842669022154249305632124963005613347878992385308567202200402345934772398555582064683481245902397416917464143405011569617574279148121153181777391138521886585301352768515986291216852665885281460850309611, 1\}\}.$

(64) $5^{546} + 1 = \{\{2, 1\}, \{13, 2\}, \{53, 1\}, \{157, 1\}, \{601, 1\}, \{1093, 1\}, \{2521, 1\}, \{8684521, 1\},$
$\{234750601, 1\}, \{3640732369, 1\}, \{4056854881, 1\}, \{24587411156281, 1\},$
$\{83181652304609, 1\}, \{667929705480493, 1\}, \{1636546425044077, 1\},$
$\{141437955091157453, 1\}, \{9202941751066205689, 1\}, \{199852066328056058281, 1\},$
$\{5322689540402803582045743093365926132069405127681076429564973043035116436582420564370745254646578301207725133470563342680116166782387466537239427225683247575242980074368147611761728130661966883880387931840095548215 3557, 1\}\}.$

(65) $5^{548} + 1 = \{\{2, 1\}, \{313, 1\}, \{1097, 1\}, \{591841, 1\},$
$\{26704002966403068660502593326248096112494118260060199991855759070303598450651034357067256569051364279614236988469889424436083181132180822287432711801164268286906240308909276231842805013779405165361402496114040344798887508631228411415654370707670826790048284858669655863217404211116986435226202723140038335644112033682430290490634461335054300148030279496390579341396639151 3, 1\}\}.$

(66) $5^{576} + 1 = \{\{2, 1\}, \{769, 1\}, \{1153, 1\}, \{3457, 1\}, \{740737, 1\}, \{1078601223169, 1\},$
$\{3666499598977, 1\}, \{4134866393089, 1\}, \{9258693158017, 1\},$
$\{58809801585833499492339195340234960557446146673875651069618544480827063086471659685323795086796075999308385479904969696335885522419265722460029197237453024243273517352855801986953301127349286255976897608969457487753768244934490544168041182179934192600227738067755376718317734840446871160925711481010590838970973491159596497591007141985 29, 1\}\}$

(67) $5^{596} + 1 = \{\{2, 1\}, \{313, 1\}, \{1193, 1\}, \{31908060963841, 1\},$
$\{16181075870903034274811533610889070816625428276301532384287116425986407337416202859219782300138252466271924815371582003127260214076167597072712614372766086415532082112486444449772293061072705395712591003530039315844920178757099873198476303297299409540438465726612174551022426584283698935971781612622052553966032891587698470844272574224408752995154247168265542715903546685579790186985810141058096377, 1\}\}$

(68) $5^{606} + 1 = \{\{2, 1\}, \{13, 1\}, \{601, 1\}, \{809, 1\}, \{1213, 1\}, \{3637, 1\},$
$\{145441, 1\}, \{5728721, 1\}, \{3375268261753121, 1\},$
$\{24008739792163957803888116032091842868065414126065589621813921886927521963628868678823914381269811509960047841617566060691632865067402068893325545185868295434808435695601142918943001001186575062583850839890082612970138783862094689580217207096021098911092452160960956660345657593827089139499083085249143299315543111657073590778325269865529238807144405074449513063969237995634565731449, 1\}\}$

(69) $5^{608} + 1 = \{\{2, 1\}, \{641, 1\}, \{1217, 1\}, \{69313, 1\}, \{75068993, 1\}, \{241931001601, 1\},$
$\{47931080903188778938093771366130360569308093779791505896074721392833800699711639485325511757993489172481133941696125062170663416448823285161017932596174057129880909071983844213558732122123159695460270101660497813640574009603048289388388625784499957432777576034390971519667420179264556475483029485745498620567534422946217784691359104486768916913339947482256220020370956889749372161945724318484481, 1\}\}$

(70) $5^{618} + 1 = \{\{2, 1\}, \{13, 1\}, \{601, 1\}, \{1237, 1\}, \{788569, 1\},$
$\{139210681, 1\}, \{968889277, 1\}, \{5223725024289757, 1\},$

$\{85600922442402346897465401872594792171236058181538742008972373 4$
$504249078068404151717652624804329641446000173334117989733835330783938 9$
$300804107340644116860625628916282553240364519473438868851294462171647 1$
$848774432784053546831216636114735319731679626505913232419759342539683 5$
$747733166885523704980892627814939602423424056588529857756304779112879 9$
$2453984242443816704194821343918429330229213, 1\}\}$

(71) $5^{638} + 1 = \{\{2, 1\}, \{13, 1\}, \{89, 1\}, \{1277, 1\}, \{8933, 1\}, \{35729, 1\}, \{1030330938209, 1\},$
$\{90222234437280577948370123027671851060451684149594505703794015 8$
$360859492849066892823455996875312188660777927093126122565414212465120 7$
$154111618462815390089491876872146593134197955628131553871561734537512 8$
$706730256666722127909910558004301963638463097640975104312058875244801 9$
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(72) $5^{686} + 1 = \{\{2, 1\}, \{13, 1\}, \{197, 1\}, \{1373, 1\}, \{41161, 1\}, \{234750601, 1\},$
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$626680738586994191632651708788155606435554122182412979269473422606505 0$
$40681510315308587722428896273795082688236961355064684699830082 49, 1\}\}$

(73) $5^{716} + 1 = \{\{2, 1\}, \{313, 1\}, \{1433, 1\},$
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$159041347097, 1\}\}$

(74) $5^{746} + 1 = \{\{2, 1\}, \{13, 1\}, \{1493, 1\}, \{3701716157, 1\},$
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$098627223590667141066149788364159833947246807666194332089363461009974 2$
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$4637445120260142623473201, 1\}\}$

(75) $5^{776} + 1 = \{\{2, 1\}, \{17, 1\}, \{1553, 1\}, \{4657, 1\}, \{11489, 1\},$
$\{89060376057679073482467546225669366914088820781232354889364318 5$
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$022772108645191698763502157260163642081742124945783269735940591672217 9$
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(76) $5^{806} + 1 = \{\{2, 1\}, \{13, 2\}, \{53, 1\}, \{1613, 1\}, \{6449, 1\}, \{83181652304609, 1\},$
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(77) $5^{818} + 1 = \{\{2, 1\}, \{13, 1\}, \{1637, 1\}, \{782009, 1\}, \{2161157, 1\},$
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(78) $5^{846} + 1 = \{\{2, 1\}, \{13, 1\}, \{37, 1\}, \{601, 1\}, \{1693, 1\}, \{5077, 1\}, \{12409, 1\}, \{412849, 1\},$
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(79) $5^{848} + 1 = \{\{2, 1\}, \{1697, 1\}, \{2593, 1\}, \{29423041, 1\}, \{53373121, 1\}, \{152929478928472033, 1\},$
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1, 1}}

(80) $5^{866} + 1 = \{\{2, 1\}, \{13, 1\}, \{1733, 1\}, \{5197, 1\}, \{17321, 1\}, \{7748969, 1\}, \{136590198133, 1\},$
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(81) $5^{888} + 1 = \{\{2, 1\}, \{17, 1\}, \{593, 1\}, \{1777, 1\}, \{11489, 1\},$
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(82) $5^{938} + 1 =$ {{2, 1}, {13, 1}, {1877, 1}, {20369, 1}, {75041, 1}, {234750601, 1},
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(83) $5^{956} + 1 =$ {{2, 1}, {313, 1}, {1913, 1},
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(84) $5^{966} + 1 =$ {{2, 1}, {13, 1}, {277, 1}, {601, 1}, {1289, 1}, {1933, 1}, {2521, 1}, {424397, 1},
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(85) $5^{996} + 1 =$ {{2, 1}, {313, 1}, {1993, 1}, {390001, 1}, {2058554713, 1}, {2183732799044857, 1},
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(86) $5^{998} + 1$ = {{2, 1}, {13, 1}, {1997, 1}, {1413169, 1},
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(87) $5^{1008} + 1$ = {{2, 1}, {97, 1}, {673, 1}, {2017, 1}, {2593, 1}, {2689, 1}, {271489, 1}, {1149569, 1},
{10922689, 1}, {29423041, 1}, {125837710159393, 1}, {8906835789071809, 1},
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(88) $5^{1026} + 1$ = {{2, 1}, {13, 1}, {37, 1}, {601, 1}, {2053, 1}, {20521, 1}, {22573, 1}, {6597973, 1},
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(89) $5^{1056} + 1$ = {{2, 1}, {193, 1}, {641, 1}, {1409, 1}, {2113, 1}, {6337, 1}, {27457, 1}, {325249, 1},
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(90) $5^{1076} + 1$ = {{2, 1}, {313, 1}, {2153, 1}, {93375281, 1},
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(91) $5^{1106} + 1$ = {{2, 1}, {13, 1}, {317, 1}, {2213, 1}, {41081, 1},
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(92) $5^{1118} + 1$ = {{2, 1}, {13, 2}, {53, 1}, {173, 1}, {2237, 1}, {2028053, 1}, {16146157, 1},
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(93) $5^{1136} + 1$ = {{2, 1}, {2273, 1}, {2593, 1}, {68161, 1}, {29423041, 1}, {2393066459969, 1},
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(94) $5^{1146} + 1$ = {{2, 1}, {13, 1}, {601, 1}, {2293, 1}, {1053557, 1}, {5412888049, 1},

$\{206218283317, 1\}, \{146292198125317, 1\}, \{234499916148809, 1\},$
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$7135167498355640062096016187990941328593828248301416030354210708014119$
$7932577645539914621961049032119679100100216646094174028656754007385450$
$8520136579116496454156533246130511550235868498081280261973335744084119$
$4651101549440461764538939110764056425030176769747752655570140966334751$
$6457814676769955833754204385450516883651074164016174647112155916042521$
$73295336363441905009227568224546172826647 7249, 1\}\}$

(95) $5^{1148} + 1 = \{\{2, 1\}, \{313, 1\}, \{2297, 1\}, \{2953, 1\}, \{737017, 1\}, \{5643569, 1\}, \{23579921, 1\},$
$\{811755457273, 1\}, \{59509429687890001, 1\},$
$\{130011421527587174042053118998687356397474274214523561150022322$
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$2386744272436693263000143855009361293164055564592413029099942265957976$
$5726829360373555308091984198021217519944660263043699891661313637127889$
$8891922881734031426108854809575762715719912102910143796290005601982990$
$2383693743843109497604614434068926776723734525710047675543675302923525$
$0114901938052375355768327421300122886925230502487318976159465991157946$
$8739738290426307968553944613124896886069455435341902778446709957293619$
$6457170971036487699246504639680465620502617629241162954137107144161869$
$6791361897136937647241907468121283945483857793296692229945521790878206$
$1089177766960374001518527229979285034812092820125329, 1\}\}$

(96) $5^{1166} + 1 = \{\{2, 1\}, \{13, 1\}, \{89, 1\}, \{2333, 1\}, \{1030330938209, 1\},$
$\{179378431261898945649166596314569100954763906609627684937958083$
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$0300970208834040783907662647584389424276384305997437787063633570811253$
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$1878339340289650383078479552077987031255074351926065478046176380203628$
$4591450110940948188118354350172892931472877246532440859963953591259728$
$3778351345699015037934886521934932641198864600790627729879153790058181$
$4623551669731501257884692896503229932920058043157819785695285786031507$
$2405973693910361312803019794785582515800192664520276712928987431279432$
$0460455071963327318188674808789824788620657447708841748369393921859102$
$10774811885477432105288810092289 97, 1\}\}$

(97) $5^{1178} + 1 = \{\{2, 1\}, \{13, 1\}, \{2357, 1\}, \{695437013, 1\}, \{4885168129, 1\}, \{2864226125209369, 1\},$
$\{408495765793286142074897354283745125163347203645197827804753366$
$2491869851859606041895838187951276121979102693488519767073186093655198$
$5124775969683493262932723830547590578432429397794624958538514691278674$
$3352692359622202526094807609725876086842921584987439216094414460936897$
$1779865912173219629767827242462066309412974115365596362398997380680553$
$3014380644152274808211550568721974042194329747711093373461456438603353$
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$6086666173589306853949454630701069296341493539926084208394077981684090$
$8396426053055208113670725728532360940816221261981196002405992844188882$
$0736992225788195449748261108405843431213461972230676071493461525123364$
$9153532397347880150910323443743175820608558744590534247606067772655366$
$049709087399366758096 1, 1\}\}$

(98) $5^{1188} + 1 = \{\{2, 1\}, \{73, 1\}, \{313, 1\}, \{2377, 1\}, \{73009, 1\}, \{390001, 1\}, \{543097, 1\},$
$\{1853281, 1\}, \{3314953, 1\}, \{331660297, 1\}, \{1503418321, 1\}, \{36280398313, 1\},$
$\{4352560631862697, 1\}, \{112835047682166217, 1\}, \{346866834441245977, 1\},$
$\{748003465152963352127485818438857767619524836938715873533649800$
$462368234495616148549523667801078438050580623576557051165656653140897$1
$4388815178521948708444803618569264630465809530179145066084558716830330$
$5430487092219258013388357422213686150149204231840704959341818632663680$
$4410020894027249204055431956142779984490784776164381363798138674718067$
$2434697367090233241073903249457819990987274618176065861359418672139150$
$1450852591230474399384864204526980378343372851825432628789429287289182$
$4915202488494314508384966098067849721177724249881280434451252949214674$
$5800565269580364027831108392354848409291279348778352685168631719039758$
$9954953429351794872084557944185677759706258821911609306647877561762955$
$1364997898831276891433, 1\}\}$

(99) $5^{1196} + 1 = \{\{2, 1\}, \{313, 1\}, \{2393, 1\}, \{7177, 1\}, \{51169, 1\}, \{4099889, 1\}, \{537181587281, 1\},$
$\{41383425174937, 1\}, \{150220315444217, 1\},$
$\{7288953048306073, 1\}, \{129044008326199409, 1\},$
$\{131161677173623738996438129908476011411205790413776421152759759$
$6005384677017730345802075703947584050957211196901495364640833236041780$
$1486655700195759513781581667265618742457465604719066281539360491419817$
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$5845410795092096260503690708084329602339944659269243416159648349798472$
$5314952106691211093020629226356320916383704094836809355159652018748154$
$4990780074239208326759827930283776296671473308981042711155056818483052$
$8111172892110213160482486552382469600148430201074841583545840868403031$
$9197252605564200492664374994864591072268434315125 7, 1\}\}$

(100) $5^{1208} + 1 = \{\{2, 1\}, \{17, 1\}, \{2417, 1\}, \{11489, 1\},$
$\{10640829750743377, 1\}, \{17718805276155041, 1\},$
$\{127443447430668730205889091330105122762626146088780506350565282$
$1279583685854753527124944822233684786070634581392854681048228875240624$
$2833188428721272925423813119292349235740611542593005118833155510117756$
$6508170298434855238797662013160349824019671640183293802799529691390905$
$0200432545770920012072271294288099032864212660192080488587004039381030$
$0531586917938996837991228379722890673936539954949747843611057096211344$
$4361743797637541126521779776997691996474869873776557321252002463342543$
$8734437441569977808090733494723102779216907646079284866193387134676640$
$9848223428458504873245185314420249272154681880974612052052530574404978$
$7922102543947321697923766727646761241711215807769509929950595193671767$
$1009029962440149247356160542923836007354293205252548715409483384916132$
$0189296338598497543398238380068941644652 9, 1\}\}$

(101) $5^{1218} + 1 = \{\{2, 1\}, \{13, 1\}, \{601, 1\}, \{2437, 1\}, \{2521, 1\}, \{382801, 1\}, \{234750601, 1\},$
$\{506670253, 1\}, \{918848200649, 1\}, \{24587411156281, 1\},$
$\{224350648831980416190188176664936920104575864809271676137598661$
$7068358769942242099667244181826045649320051253652991892050179079364492$
$4498463743875038012531661272291373986459146317716114731152525280702152$
$9465114926999530014243644712852715724632387207877709376665857768447036$
$9481585535481991214450874440286151381233522385188366914067326497869712$
$8348882383785807871703018530894423638354346523300295111294373209033938$
$1062625620570295796652865005348107146479127429274171986287993837242751$

8931195012824322428049807204505488001121481018949869965203676200385449
3871421294464553971850853589147447208090649983118852411136739825576390
7108375931204560533977308879002792104460155640842007607482090118932467
4388666980247022778037525615458620625008984556838325856746371224695309
597850531659274256010358779609, 1}}

(102) $5^{1236}+1=\{\{2,1\},\{313,1\},\{2473,1\},\{390001,1\},\{875089,1\},\{143724625337,1\},$
$\{111295544672183565855896129724234764282302277864323988990592680$
0657827722730965481161510105345239632977226342619750318148042057703234
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9472944536597522290448220910704756156897983601203335038248669509136649
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7670548268799957884861493061218269570336094970135710921729005516540197
8494957448993536100415158154613284706927824041267797860388181332042845
8865458259129786417300654602458524549494165031657145027811698507383022
209, 1}}

(103) $5^{1238}+1=\{\{2,1\},\{13,1\},\{2477,1\},\{66853,1\},\{12096323689601,1\},$
$\{405687580707536691887496200203391158297150270626210432597491772$
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6596918248299150290982442357118144687523470898198138895759736921559919
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5744527991967546217529920447111161018373473457266410129948052226615384
1969838521, 1}}

Appendix F Factors of $7^t - 1$

(1) $7^5 - 1 = \{\{2,1\},\{3,1\},\{2801,1\}\}$.
(2) $7^7 - 1 = \{\{2,1\},\{329,1\},\{4733,1\}\}$
(3) $7^9 - 1 = \{\{2,1\},\{3,3\},\{19,1\},\{37,1\},1063,1\}$.
(4) $7^{11} - 1 = \{\{2,1\},\{3,1\},\{1123,1\},\{293459,1\}\}$.
(5) $7^{15} - 1 = \{\{2,1\},\{3,2\}\,\{19,1\},\{31,1\},\{2801,1\},\{159871,1\}\}$.
(6) $7^{19} - 1 = \{\{2,1\},\{3,1\},\{419,1\},\{4534166740403,1\}\}$.
(7) $7^{21} - 1 = \{\{2,1\},\{3,2\},\{19,1\},\{29,1\},\{4733,1\},\{11898664849,1\}\}$.
(8) $7^{23} - 1 = \{\{2,1\},\{3,1\},\{47,1\},\{3083,1\},\{31479823396757,1\}\}$.
(9) $7^{25} - 1 = \{\{2,1\},\{3,1\},\{2551,1\},\{2801,1\},\{31280679788951,1\}\}$.
(10) $7^{27} - 1 = \{\{2,1\},\{3,4\},\{19,1\},\{37,1\},\{109,1\},\{811,1\},\{1063,1\},\{2377,1\},\{2583253,1\}\}$.
(11) $7^{29} - 1 = \{\{2,1\},\{3,1\},\{59,1\},\{127540261,1\},\{713\,169\,229\,849\,99,1\}\}$.
(12) $7^{31} - 1 = \{\{2,1\},\{3,1\},\{311,1\},\{21143,1\},\{3999088279399464409,1\}\}$.
(13) $7^{35} - 1 = \{\{2,1\},\{3,1\},\{29,1\},\{2801,1\},\{4733,1\},\{2127431041,1\},\{77192844961,1\}\}$.
(14) $7^{37} - 1 = \{\{2,1\},\{3,1\},\{223,1\},\{2887,1\},\{4805345109492315767981401,1\}\}$.
(15) $7^{39} - 1 = \{\{2,1\},\{3,2\},\{19,1\},\{486643,1\},\{7524739,1\},\{44975113,1\},\{16148168401,1\}\}$.
(16) $7^{41} - 1 = \{\{2,1\},\{3,1\},\{83,1\},\{20515909,1\},\{4362\,139\,336\,229\,068\,656\,094\,783,1\}\}$
(17) $7^{43} - 1 = \{\{2,1\},\{3,1\},\{166003607842448777,1\},\{2192537062271178641,1\}\}$.
(18) $7^{47} - 1 = \{\{2,1\},\{3,1\},\{13722816749522711,1\},\{6368151199641855045948 7,1\}\}$.
(19) $7^{51} - 1 = \{\{2,1\},\{3,2\},\{19,1\},\{103,1\},\{14009,1\},\{365773,1\},\{2316281689,1\},$
$\{2767631689,1\},\{10879733611,1\}\}$.
(20) $7^{53} - 1 = \{\{2,1\},\{3,1\},\{8269,1\},\{319591,1\},\{3890427601703518805637205183984 1219,1\}\}$.
(21) $7^{57} - 1 = \{\{2,1\},\{3,2\},\{19,1\},\{103,1\},\{14009,1\},\{365773,1\},\{2316281689,1\},$
$\{2767631689,1\},\{10879733611,1\}\}$
(22) $7^{59} - 1 = \{\{2,1\},\{3,1\},\{459257,1\},\{134927809,1\},\{550413361,1\},$
$\{35463932368454561298857764 9,1\}\}$.
(23) $7^{61} - 1 = \{\{2,1\},\{3,1\},\{367,1\},\{4759,1\},\{177237331,1\},$
$\{19146624498137276606805303260645919 07,1\}\}$.
(24) $7^{63} - 1 = \{\{2,1\},\{3,3\},\{19,1\},\{29,1\},\{37,1\},\{1063,1\},\{4733,1\},\{11898664849,1\},$
$\{26439999176607287878083969888 49,1\}\}$
(25) $7^{65} - 1 = \{\{2,1\},\{3,1\},\{131,1\},\{2801,1\},\{157951,1\},\{787021,1\},\{16148168401,1\},$
$\{4446437759531,1\},\{434502978835771,1\}\}$.
(26) $7^{69} - 1 = \{\{2,1\},\{3,2\},\{19,1\},\{47,1\},\{139,1\},\{3083,1\},\{402011881627,1\},$
$\{31479823396757,1\},\{2351696623950693563122 33,1\}\}$.
(27) $7^{73} - 1 = \{\{2,1\},\{3,1\},\{439,1\},\{3675989,1\},\{359390389,1\},\{1958423494433591,1\},$
$\{72226052281055362027576069 69,1\}\}$.
(28) $7^{81} - 1 = \{\{2,1\},\{3,5\},\{19,1\},\{37,1\},\{109,1\},\{811,1\},\{1063,1\},\{1621,1\},\{2377,1\},$
$\{3727,1\},\{2583253,1\},\{3368791,1\},\{707223088124016741749935333670 23,1\}\}$.
(29) $7^{83} - 1 = \{\{2,1\},\{3,1\},\{167,1\},\{66733,1\},\{76066181,1\},\{7685542369,1\},$
$\{62911130477521,1\},\{303567967057423,1\},\{18624275418445601,1\}\}$.
(30) $7^{85} - 1 = \{\{2,1\},\{3,1\},\{1531,1\},\{2801,1\},\{4931,1\},\{14009,1\},\{2767631689,1\},$
$\{138497973518827432485604572537024087153816681041,1\}\}$.
(31) $7^{87} - 1 = \{\{2,1\},\{3,2\},\{19,1\},\{59,1\},\{127540261,1\},\{2576743207,1\},$
$\{71316922984999,1\},\{196915704073465747,1\},\{358475907408445923469,1\}\}$.
(32) $7^{97} - 1 = \{\{2,1\},\{3,1\},\{389,1\},\{971,1\},\{416092415678495382141399910782579564691180758 4$
$55473572674863175790954776467 9,1\}\}$.
(33) $7^{99} - 1 = \{\{2,1\},\{3,3\},\{19,1\},\{37,1\},\{199,1\},\{1063,1\},\{1123,1\},\{3631,1\},\{173647,1\},$
$\{293459,1\},\{1532917,1\},\{12323587,1\},$
$\{14658652062814508164798530246267135510037033,1\}\}$.

(34) $7^{101} - 1 = \{\{2, 1\}, \{3, 1\}, \{607, 1\}, \{809, 1\}, \{6263, 1\}, \{1226962153072980952980067096888460031879516017670288348423584305043494594529, 1\}\}.$

(35) $7^{103} - 1 = \{\{2, 1\}, \{3, 1\}, \{17923, 1\}, \{1031658988265814866401843241274130415410385814525364533951092728772480584217175625 9, 1\}\}$

(36) $7^{113} - 1 = \{\{2, 1\}, \{3, 1\}, \{227, 1\}, \{2300919443481866466139853110309106017717326808587253608454630927182513271850920288156734283 63, 1\}\}.$

(37) $7^{115} - 1 = \{\{2, 1\}, \{3, 1\}, \{47, 1\}, \{1151, 1\}, \{2801, 1\}, \{3083, 1\}, \{188831, 1\}, \{1446701, 1\},$
$\{723461377501, 1\}, \{31479823396757, 1\},$
$\{8805698737308545237018344297984561577187373037 21, 1\}\}.$

(38) $7^{117} - 1 = \{\{2, 1\}, \{3, 3\}, \{19, 1\}, \{37, 1\}, \{1063, 1\}, \{1873, 1\}, \{322921, 1\}, \{486643, 1\},$
$\{2280097, 1\}, \{7524739, 1\}, \{44975113, 1\}, \{7687225261, 1\}, \{16148168401, 1\},$
$\{66135384230579134218722868484371534 1, 1\}\}.$

(39) $7^{119} - 1 = \{\{2, 1\}, \{3, 1\}, \{29, 1\}, \{4733, 1\}, \{12377, 1\}, \{14009, 1\}, \{34273, 1\},$
$\{2767631689, 1\}, \{305213588009240737, 1\},$
$\{89185490016677731671397038434179879045248666344565775 13, 1\}\}.$

(40) $7^{121} - 1 = \{\{2, 1\}, \{3, 1\}, \{1123, 1\}, \{1453, 1\}, \{65099, 1\}, \{293459, 1\},$
$\{965927365520136304449338827049103841305646805820126141875147843$
$2947875933991840016519, 1\}\}.$

(41) $7^{125} - 1 = \{\{2, 1\}, \{3, 1\}, \{251, 1\}, \{2551, 1\}, \{2801, 1\}, \{31280679788951, 1\},$
$\{128863605961145736707054168690536807555069447341593411117481172$
$6534504445255875725 1, 1\}\}.$

(42) $7^{127} - 1 = \{\{2, 1\}, \{3, 1\}, \{260461507, 1\}, \{13600544081983527726186578957448801595417$
$95023198314994427050839093112149090861720796705783594452 51, 1\}\}$

(43) $7^{129} - 1 = \{\{2, 1\}, \{3, 2\}, \{19, 1\}, \{10837, 1\}, \{63350976270733, 1\}, \{166003607842448777, 1\},$
$\{2672028853891888622110112128619876076347234164588616093 3$
$7071699981802929, 1\}\}.$

(44) $7^{131} - 1 = \{\{2, 1\}, \{3, 1\},$
$\{850534611647968019495395416395428057706663923306826733025308197$
$74105141531698707146930307290253537320447270457, 1\}\}$

(45) $7^{135} - 1 = \{\{2, 1\}, \{3, 4\}, \{19, 1\}, \{31, 1\}, \{37, 1\},$
$\{109, 1\}, \{271, 1\}, \{811, 1\}, \{1063, 1\}, \{2377, 1\}, \{2801, 1\},$
$\{159871, 1\}, \{185221, 1\}, \{2583253, 1\}, \{1527007411, 1\}, \{125096112091, 1\},$
$\{17556187692180231 1, 1\}, \{797937984757981841530188510084780901, 1\}\}.$

(46) $7^{137} - 1 = \{\{2, 1\}, \{3, 1\}, \{2741, 1\}, \{213721, 1\}, \{251533, 1\},$
$\{679092646604348848436230554836412626083799829151411191949277186$
$091088701917306401043179398523488875 77, 1\}\}.$

(47) $7^{141} - 1 = \{\{2, 1\}, \{3, 2\}, \{19, 1\}, \{283, 1\}, \{5960917, 1\},$
$\{249860742197241948192966954622328194519562899140309550615431262$
$02119564900064206662997135802885849751098206 3, 1\}\}.$

(48) $7^{143} - 1 = \{\{2, 1\}, \{3, 1\}, \{1123, 1\}, \{293459, 1\}, \{16148168401, 1\}, \{528808508322667, 1\},$
$\{418330546708532740480353541335711575735343912784641220943864002$
$340111144746922701991203, 1\}\}$

(49) $7^{153} - 1 = \{\{2, 1\}, \{3, 3\}, \{19, 1\}, \{37, 1\}, \{103, 1\}, \{307, 1\}, \{613, 1\}, \{1063, 1\}, \{4591, 1\},$
$\{14009, 1\}, \{44371, 1\}, \{365773, 1\}, \{2316281689, 1\}, \{2767631689, 1\},$
$\{10879733611, 1\}, \{350379330748449481709242776279131$
$669801940520613781858120522935782 51, 1\}\}.$

(50) $7^{157} - 1 = \{\{2, 1\}, \{3, 1\}, \{4397, 1\}, \{5653, 1\}, \{1039945344901, 1\},$
$\{308883753093321746997477641577980838910192868291755541765153082$
$117756750854911642404958745343035544914691732796 61, 1\}\}.$

(51) $7^{163} - 1 = \{\{2, 1\}, \{3, 1\}, \{653, 1\}, \{9781, 1\}, \{1260643, 1\},$
$\{116665034851129307902624357139195905667066806190114833939064618$

$494383002208259499163938362190977928250973584115543625739214443, 1\}\}.$

(52) $7^{175} - 1 = \{\{2,1\},\{3,1\},\{29,1\},\{701,1\},\{2551,1\},\{2801,1\},\{4733,1\},$
$\{2127431041,1\},\{77192844961,1\},\{31280679788951,1\},$
$\{368146726267000523206403038500165782925504879615058104729767146$
$296343146902066797513736881223461301,1\}\}$

(53) $7^{177} - 1 = \{\{2,1\},\{3,2\},\{19,1\},\{709,1\},\{89209,1\},\{459257,1\},\{134927809,1\},$
$\{550413361,1\},$
$\{518114623020975074018002739309396360820240761273419101595672842$
$3827589209624332398005305454923868786473959284779936 21,1\}\}$

(54) $7^{181} - 1 = \{\{2,1\},\{3,1\},\{1811,1\},\{13873726021,1\},\{618805947190964867,1\},$
$\{983847868716989204697137458368555065405633130650561289091916359$
$762908341074317072962503079475709433362652704944363102 7813,1\}\}.$

(55) $7^{185} - 1 = \{\{2,1\},\{3,1\},\{223,1\},$
$\{1481,1\},\{2801,1\},\{2887,1\},\{6661,1\},\{219041,1\},$
$\{942543691081369277773543497051844330874255595087425378964554439$
$4880868301031299089951 94$
$715072275531329676117964583356936940204323290 21,1\}\}.$

(56) $7^{189} - 1 = \{\{2,1\},\{3,4\},\{19,1\},\{29,1\},\{37,1\},\{109,1\},\{757,1\},\{811,1\},$
$\{1063,1\},\{2377,1\},\{4733,1\},\{2583253,1\},\{11898664849,1\},$
$\{651259195745827996896316544656062759285664227683642978154892763$
$465422334779852359508546076428650667715245119852206128 93,1\}\}$

(57) $7^{191} - 1 = \{\{2,1\},\{3,1\},\{383,1\},$
$\{112817278337559699810610328050113979428873512903449877053938675$
$951171112302901076711069776774714211878744002310011667823869269 0$
$03853437734035902480275113415479,1\}\}.$

(58) $7^{193} - 1 = \{\{2,1\},\{3,1\},\{14669,1\},$
$\{144334437423204232486585590464004980021928503419390813461808380$
$3650983887510085559095130207056419397592466215387544461142923560839401$
$0947949088430345102280102 9,1\}\}.$

(59) $7^{203} - 1 = \{\{2,1\},\{3,1\},\{29,2\},\{59,1\},\{4733,1\},$
$\{60089,1\},\{127540261,1\},\{71316922984999,1\},$
$\{465941598913454377106883133376466664725890139166855163821838467$
$8242557421304316103945154298716030603806966568704529767484855377352658$
$621,1\}\}$

(60) $7^{205} - 1 = \{\{2,1\},\{3,1\},\{83,1\},\{2801,1\},\{20515909,1\},$
$\{614419495366821529684317346059223964584194912846855305721440483$
$1768597271836450638822525073986039974017465321311451759587334029901332$
$0824699350907379619325907 83,1\}\}.$

(61) $7^{213} - 1 = \{\{2,1\},\{3,2\},\{19,1\},\{968299,1\},\{582053568181,1\},\{990643452963163,1\},$
$\{530843622319233468052258575559783397851526138568242511969574674$
$0445453578246760415887042361718235798297611223331504058841458542849352$
$963113117069,1\}\}.$

(62) $7^{219} - 1 = \{\{2,1\},\{3,2\},\{19,1\},\{439,1\},\{877,1\},$
$\{3675989,1\},\{359390389,1\},\{1958423494433591,1\},$
$\{350052570384438952909856290057637776638078892935130278004864659$
$6216462598019656046892083534715954735182304033668202205533353387492203$
$27885461620797,1\}\}.$

(63) $7^{233} - 1 = \{\{2,1\},\{3,1\},\{467,1\},\{79227874003,1\},$
$\{364331892843505027296810945837058370824234171587813242775720155$
$3776250997296568609753935062992480523710205974489800023295868212705247$
$217079455716816699544284311706067566834103569630 01,1\}\}.$

(64) $7^{237} - 1 = \{\{2, 1\}, \{3, 2\}, \{19, 1\}, \{2371, 1\},$
$\{2394847278769506843716429116665571903268843249671980431701269953766615830511447901288233785756775256772323788203171421130216403600807269564441973954070283656170233668543235557688144466228292440 83, 1\}\}.$

(65) $7^{239} - 1 = \{\{2, 1\}, \{3, 1\}, \{479, 1\}, \{871986721, 1\}, \{643384173113837, 1\},$
$\{590152912740383546405391589472730230353037907605911579446756624$
$345043061719165807378301979303376351137763615 1597787$
$848749314554190951736701031949191094038770259580403873964979, 1\}\}.$

(66) $7^{241} - 1 = \{\{2, 1\}, \{3, 1\}, \{1447, 1\}, \{28439, 1\}, \{33486078253526557, 1\},$
$\{563935264197604425767391369648190015332073194595000645196540604$
$0846243915457765763636379382438528586672787890669697456610083951476498$
$8288600174813891810007545918197353618197589621, 1\}\}.$

(67) $7^{243} - 1 = \{\{2, 1\}, \{3, 6\}, \{19, 1\}, \{37, 1\}, \{109, 1\}, \{811, 1\}, \{1063, 1\}, \{1459, 1\}, \{1621, 1\},$
$\{2377, 1\}, \{3727, 1\}, \{169129, 1\}, \{1382671, 1\}, \{2583253, 1\}, \{3368791, 1\},$
$\{556320970347209814897034030937552420240138410854444002078410534$
$889174201543076743361402681111820934427879395534500731216181$
$83288105144866681040155526 58777, 1\}\}.$

(68) $7^{247} - 1 = \{\{2, 1\}, \{3, 1\}, \{419, 1\}, \{6917, 1\},$
$\{16148168401, 1\}, \{22931541257, 1\}, \{4534166740403, 1\}, \{14975869588530371, 1\},$
$\{125454655753267202797365863162986582917855663926667180423229018$
$4801205923867264403869350563252415166745855607627536006024738516449922$
$74401607378216198999, 1\}\}.$

(69) $7^{251} - 1 = \{\{2, 1\}, \{3, 1\}, \{503, 1\}, \{27611, 1\}, \{8895962998159, 1\},$
$\{177669837175335119929176935701194838324252821819693597497081108$
$5363985997487004550001243529297221170285499484822210130977061003539415$
$509065640407017788987327060505024073474161052936711154653 31, 1\}\}.$

(70) $7^{257} - 1 = \{\{2, 1\}, \{3, 1\}, \{1543, 1\},$
$\{167370574641632502572384103567991159647198307969545382257664341$
$4741081747467696044279112555289002232175368458288635585784410406565303$
$9447077114058181760234621579650963117104467101671132607220834791233206$
$22966778807, 1\}\}.$

(71) $7^{261} - 1 = \{\{2, 1\}, \{3, 3\}, \{19, 1\}, \{37, 1\}, \{59, 1\}, \{523, 1\}, \{1063, 1\}, \{5743, 1\},$
$\{127540261, 1\}, \{2576743207, 1\}, \{83226515419, 1\}, \{71316922984999, 1\},$
$\{710074092185677, 1\}, \{196915704073465747, 1\},$
$\{190746601014030887864608468381789807871176990791091071309234221$
$7137658232874961755272581018171704448451395664347322116454543261 4167, 1\}\}.$

(72) $7^{265} - 1 = \{\{2, 1\}, \{3, 1\}, \{1061, 1\}, \{2801, 1\}, \{8269, 1\},$
$\{319591, 1\}, \{3041671, 1\}, \{19069931, 1\},$
$\{326807465819807698664810434867151304791310583012772124434804633$
$0001810484247056344701514282001017644502270562551132505922939451140708$
$160432936657199363943400765646104229565445008024308759126 4579, 1\}\}.$

(73) $7^{269} - 1 = \{\{2, 1\}, \{3, 1\}, \{2153, 1\},$
$\{166026527134191738669017246117715558254761946534264717237735920$
$2904266654414497656802586328365814688845871766391485540737993243972662$
$9980882705818126069237954655533417171726708563512476887150568030563589$
$828993766083011007017, 1\}\}.$

(74) $7^{273} - 1 = \{\{2, 1\}, \{3, 2\}, \{19, 1\}, \{29, 1\}, \{1093, 1\}, \{4733, 1\}, \{486643, 1\}, \{7524739, 1\},$
$\{44975113, 1\}, \{7304123737, 1\}, \{11898664849, 1\}, \{16148168401, 1\},$
$\{434230520613371097330409506950589908590603411661538993453297817$
$1022240582981318653732106062735677777169176587762334543654115724827134$
$117740601581408782382141319375104306 1, 1\}\}.$

(75) $7^{281} - 1$ = {{2, 1}, {3, 1}, {563, 1}, {8431, 1}, {341614931501, 1}, {4420135738596254569, 1},
{69030029387946731872580279118107858433772685922727849412503890432255046903602377704692239324706194331412995515595997236444409764503480503519019425751472990866406060745178201267888210392265449385908193, 1}}.

(76) $7^{283} - 1$ = {{2, 1}, {3, 1}, {644316289, 1}, {182739497717749501, 1},
{20590293436181167829810240785077618271771591833728155301043612981267608244546986002680455979846394918248618896624938577015478852825743463384569393075657811525494018980546799598254701012863146448540067959 3806467213, 1}}.

(77) $7^{293} - 1$ = {{2, 1}, {3, 1}, {587, 1}, {15823, 1},
{73730562034668545261089017916584277231840430344173779570681825017818127957564035941180846490176918109781877513051574094262809098450170572726312667936273335791334222244969867032297622930275468789727720634 6738438488327308295011451230700046301, 1}}.

(78) $7^{303} - 1$ = {{2, 1}, {3, 2}, {19, 1}, {607, 1}, {809, 1}, {1213, 1}, {3637, 1}, {6263, 1},
{25012453367194976268964922922147487467662824008683432671032193489772612113142492908721136222974009832380540912218484132139528609278346483285416579124741617517032352627319438437181113141607115516500127860 648988903754924870753750365755574 09, 1}}

(79) $7^{309} - 1$ = {{2, 1}, {3, 2}, {19, 1}, {619, 1}, {17923, 1}, {220400431, 1},
{16328807345998109291101591101272558367289620280715354944294030471909463232631196464616298010054467893413585089109331266724760421964891836653958609125022740991792042267923506388743680814495698839059209762 5982779593569796676831306825867950580 0919, 1}}.

(80) $7^{311} - 1$ = {{2, 1}, {3, 1}, {3733, 1}, {106363, 1}, {310379, 1},
{90489522462589381298809121317833644842788583075019925457982020759268305938564382187271981237123620810349912959428940653643009833176396455633249723589075290773968553785932225666553217320800343546877962394 2825581341701126229124729675135954607607 10077, 1}}.

(81) $7^{321} - 1$ = {{2, 1}, {3, 2}, {19, 1}, {643, 1},
{859121000285791, 1}, {23884590227767303, 1}, {477280063808143381, 1},
{87758328280214000193264155607792896067084189610274674333547666168074409855680036081803425180519728417741331145267717091994238609412263876741731967024657616960716232656547073388025894443432191529410296366 32000334190927, 1}}.

(82) $7^{323} - 1$ = {{2, 1}, {3, 1}, {419, 1}, {647, 1}, {14009, 1}, {2767631689, 1}, {4534166740403, 1},
{32388024103200381561142405419341718484940758456114021819971732062765169175084719918626729080447712806709732377037038596386764461087257769643549274642275374272053418451975679564675745290760899763552107347 3028980749565536923025266552219472018 3, 1}}.

(83) $7^{343} - 1$ = {{2, 1}, {3, 1}, {29, 1}, {1373, 1}, {3529, 1}, {4733, 1}, {8233, 1}, {49393, 1},
{734021, 1}, {1074473, 1}, {13473433, 1}, {83517610741606021, 1},
{51314007790235585134274964866125795643304512641846493858029480643677747032599019930601218569895313033727422769899772877770885297325078699422515610967309872822767053019367395599291739593443661018899591776 121648725541902044825589426973, 1}}.

(84) $7^{345} - 1$ = {{2, 1}, {3, 2}, {19, 1}, {31, 1}, {47, 1}, {139, 1}, {691, 1}, {1151, 1}, {1381, 1},
{2801, 1}, {3083, 1}, {159871, 1}, {188831, 1}, {1446701, 1}, {402011881627, 1},
{723461377501, 1}, {31479823396757, 1}, {430907693063791, 1},
{376720035234076771, 1},
{849138250161001102357523697384563489983132248356861435290718983

$9155674455325336922868252448825736138714840135092635324042980061278082$
$6500783925034546954928397201454738596445364852203, 1\}\}$

(85) $7^{355} - 1 = \{\{2, 1\}, \{3, 1\}, \{2131, 1\}, \{2801, 1\}, \{25561, 1\}, \{990643452963163, 1\},$
$\{112787554991591748609446118316967746548630991690591447185175668$
$5490632594097525660306356133279367860420205822648680558621581778080725$
$6103372050556683948807495980871744846528142484058220756519112855262668$
$0636930149450198377036387747248338424169506260806622437554679861531172$
$9, 1\}\}.$

(86) $7^{357} - 1 = \{\{2, 1\}, \{3, 2\}, \{19, 1\}, \{29, 1\}, \{103, 1\}, \{1429, 1\}, \{4733, 1\}, \{12377, 1\}, \{14009, 1\},$
$\{34273, 1\}, \{365773, 1\}, \{1175001241, 1\}, \{2316281689, 1\}, \{2767631689, 1\},$
$\{10879733611, 1\}, \{11898664849, 1\}, \{23992463461, 1\}, \{305213588009240737, 1\},$
$\{467359137012519524745085559680145177955576162943849370422675752$
$7545380691244773959907809874702898502514707910713081851858120570837398$
$8543695335677495322203258430771665005714942189741151269869229 7, 1\}\}$

(87) $7^{359} - 1 = \{\{2, 1\}, \{3, 1\}, \{719, 1\}, \{2051327, 1\}, \{3093863, 1\}, \{22452201769, 1\},$
$\{399504166411930750130060066333350120458321100187652796764254466$
$1340392371765053820295350615233064691592099826565322042292209634978840$
$7558234059615473189861841278901449748850068530739850957509245062002161$
$8193131622344312301908097910841648846690567658842644750697896472086452$
$9287, 1\}\}.$

(88) $7^{363} - 1 = \{\{2, 1\}, \{3, 2\}, \{19, 1\}, \{727, 1\}, \{1123, 1\}, \{1453, 1\}, \{3631, 1\}, \{65099, 1\},$
$\{293459, 1\}, \{1532917, 1\}, \{3191497, 1\}, \{12323587, 1\}, \{11795261559086963, 1\},$
$\{294629918693044686536577670072041952949482099054592094102293885$
$0472637307592911440297452280017202573604595981604156780625944532985003$
$3012645476751789588307555286670643504684649488438359564418489045024676$
$8668599410503263759386843085818528440739523, 1\}\}$

(89) $7^{367} - 1 = \{\{2, 1\}, \{3, 1\}, \{5713457, 1\}, \{1232836358179, 1\},$
$\{334984751006532638999044491536734803873372303151368396835965652$
$7753729565790949210627097330568856757176117295431570999797780110009347$
$2087167188336164696709263301535370447850236385953473729513224804853991$
$9773275058057737689763862070923808764007580831046256992824296786410399$
$236230986324568019, 1\}\}.$

(90) $7^{373} - 1 = \{\{2, 1\}, \{3, 1\}, \{2239, 1\},$
$\{123983411278666201465275824886878236062619639609393056374034439$
$0672549926340487645362799478663369621241108503516173197080189856051337$
$0823807426267871952628198206351615193956723736950307792274066753963392$
$7306306905552274991888700375323573382208582496376688706418342193399013$
$9483638383773485882492317504768766 39759, 1\}\}$

(91) $7^{393} - 1 = \{\{2, 1\}, \{3, 2\}, \{19, 1\}, \{787, 1\},$
$\{493774760688250301025743342014503276925509144831235100252208497$
$8489036738064057339456047841654751845219451365647980146008919404707456$
$7583442241031066697971446985404262365898946325422380623984100640858305$
$1651654768336929258593693732157555883648978574995751233108862859799673$
$1459848132278290710590170525053184325321940830462611 39, 1\}\}$

(92) $7^{397} - 1 = \{\{2, 1\}, \{3, 1\}, \{2383, 1\}, \{11117, 1\}, \{232643, 1\}, \{7843916473, 1\},$
$\{110010980972989993361557141323866023203663554796277769167636671$
$3157440212345503588231355559135637602354822704276850020342308140773926$
$1650874075262659232463446752142762476195039884132707150094613594505979$
$0253223646345808633437129955820952494401651878802257710137199785861040$
$4003449859297709651129371099832646597169, 1\}\}.$

(93) $7^{419} - 1 = \{\{2, 1\}, \{3, 1\}, \{839, 1\},$

$\{2478366640674397061990825993503191908502742221235223689539846230586223391782632962073387669776856234022072911851876546479087908633411324457796761355912191830172357496454029858808129391148196691357715846257015210413750085725321386427825653886624646958671549112903622009681963529680256131393154036199315003078835077145645666828968565244527128015 59864063, 1\}\}$.

(94) $7^{429} - 1 = \{\{2, 1\}, \{3, 2\}, \{19, 1\}, \{859, 1\}, \{1123, 1\}, \{3631, 1\}, \{293459, 1\}, \{486643, 1\}, \{1532917, 1\}, \{7524739, 1\}, \{12323587, 1\}, \{44975113, 1\}, \{16148168401, 1\}, \{528808508322667, 1\}, \{377341692572662931188133035082603892338658591856975927188200180085777235758856167046920116277287645840220249984333719895043603456965429046155590230354160601542351344804969462964152864686049371009354614835050144792275811013076454937345992749437597454954587283969132028028201 68169533928217, 1\}\}$

(95) $7^{433} - 1 = \{\{2, 1\}, \{3, 1\}, \{1733, 1\}, \{885919, 1\}, \{341532829437623, 1\}, \{268952162809985221897141157821615038103735770342502660185627855086424563646378070582530885681296035934409343279047433969544629647076181575559460215868038294388764610231229607693100682256162867762852499509575935768721668064925236917874261311773153509683938727340398606803365801259364253633851530717295982026386127653883009134923078993631730181, 1\}\}$.

(96) $7^{443} - 1 = \{\{2, 1\}, \{3, 1\}, \{887, 1\}, \{111637, 1\}, \{409333, 1\}, \{982815371658598023237972434879107363433114542369440869126771930778272219104399433193176660516265088884036897325988411274381789583588480693505409332111936252848734821698738291978028576164226799782342668306907462058631500227256517113617670241298941100529659299989657691574308752401947938612263616707627113193090031546663727031395403485259813295024090577680585991, 1\}\}$.

(97) $7^{491} - 1 = \{\{2, 1\}, \{3, 1\}, \{983, 1\}, \{696239, 1\}, \{213636046315531171819814770033633796110206120743960531746437437648598529111843266383866921990216754714432832352630058479864028189107105468586703903102733880062635164943560004197138055888626560041251686601170886046472643280510837137774426320912787965970942528041786780248192251140173275893199153788076705565327814412162700679458941644275493058240503153175039467011468431930699807606798471153494245653210896 1, 1\}\}$.

(98) $7^{499} - 1 = \{\{2, 1\}, \{3, 1\}, \{1997, 1\}, \{85829, 1\}, \{2841035543, 1\}, \{106026323287223, 1\}, \{163255653904702308743046167822624592993565768865361681563884440039442610634435784242886133139139502398228697786879334223669033855061581885169693940841827278494019512076668571936536756026834550302035937839912250286010618436362150695661923360330368584084850339923012224844656065217494648152413871973437677060412067914161919240931453560097701220568474769671043789091101816706361630173538326601, 1\}\}$.

(99) $7^{517} - 1 = \{\{2, 1\}, \{3, 1\}, \{1123, 1\}, \{2069, 1\}, \{293459, 1\}, \{13722816749522711, 1\}, \{33547155065898563, 1\}, \{437269108109919626538333361896365344143689087264636569330106946340538265954326234569498283743557454874731309685792349607462068499581114503013580326337757528064536883133363580781139805295918933332280958314268857093681019810116033683507937652851286079298017578110784036877031678524989713369267999450976140178481328324949220709527605047922172463672070855876620176411477043962965499027288348717 29, 1\}\}$

(100) $7^{519} - 1 = \{\{2, 1\}, \{3, 2\}, \{19, 1\}, \{1039, 1\}, \{9343, 1\}, \{8248987, 1\}, \{14735135904138340407318033698053755297858736655302117505306597 1$

$8214736337714733537726756138808016543579495190007747843403695929606896$
$0774189957714003631045024091527325056220428223624480795767219485645621$
$8037153493729623641021251315970706271146422877420191720937461294246832$
$0886124811313317253978827473301034162436595179222042823485912096805043$
$4873558752127441572879774160507917610231284250492734282844808663375205$
$5002564499, 1\}\}.$

(101) $7^{545} - 1 = \{\{2, 1\}, \{3, 1\}, \{1091, 1\}, \{2801, 1\},$
$\{2156171040211, 1\}, \{2470529440391, 1\}, \{14551288401737861, 1\},$
$\{2665434454782751262102020116407947359354687458981339759743170 08$
$0045842947967845111636142557649417651322461860294863738470446727797664$
$5057686996284696504879526519616358292975682016770610492897833301773976$
$2861759005943053334978808496228988999701696926256990952228470166830695$
$0096592799634754868524778401231534511894979434215650697325036360127208$
$2232103124774546921074755968719748519837643020938568608041107549781651, 1\}\}.$

(102) $7^{553} - 1 = \{\{2, 1\}, \{3, 1\}, \{29, 1\}, \{2213, 1\}, \{4733, 1\}, \{6637, 1\}, \{33181, 1\}, \{402592849, 1\},$
$\{135151732264934206729504864995218031842940332378546653979092857$
$6164790629975110628544084036212010571734803988451399376303213732623957$
$3162012576439455892558353520725116375943892452877507335281300660208176$
$6316128980697687126258619116721800882412314747059743686896808626662490$
$4789248744045194447708183427026359987334469616395197685930585388000507$
$5159282346757950068349449729278843436220698244583132224007074525737593$
$66946007551802054980126690237, 1\}\}.$

(103) $7^{567} - 1 = \{\{2, 1\}, \{3, 5\}, \{19, 1\}, \{29, 1\}, \{37, 1\}, \{109, 1\}, \{757, 1\}, \{811, 1\},$
$\{1063, 1\}, \{1621, 1\}, \{2269, 1\}, \{2377, 1\}, \{3727, 1\}, \{4733, 1\},$
$\{2583253, 1\}, \{3368791, 1\}, \{11898664849, 1\},$
$\{131595417902225378190845108794977413431720870526695171336243254$
$6290042674637870166928620009245766230661636552071056923168076336676421$
$4850958742935412625536692680366084168140243694207129500937524793734996$
$3429521824797862254871202982999628774729528044523381954113062699725115$
$8415501660754069621136920655197513337446866156883309819359370807673365$
$4508259187409337036700880448762779689142186004620457151014811086771236$
$508200319, 1\}\}$

(104) $7^{583} - 1 = \{\{2, 1\}, \{3, 1\}, \{1123, 1\}, \{2333, 1\}, \{8269, 1\}, \{82787, 1\}, \{166739, 1\}, \{293459, 1\},$
$\{319591, 1\}, \{3896341085695040687, 1\},$
$\{750689448124948247816237094042250176504177915583992805848791674$
$1612233825150993960453409641227100348272808677440916262268104839050219$
$7295070456711529518084239208737425065939287987031414234336598675997348$
$5278269654930986770154567380871085723529409885868307011760087111669804$
$5072005528253710903997980150760916764842028545541009177237586686495105$
$0668290813588805001071976374912138261991688108477579556544115354626293$
$50284597561249000264896682073, 1\}\}$

(105) $7^{559} - 1 = \{\{2, 1\}, \{3, 1\}, \{2237, 1\}, \{5591, 1\}, \{16148168401, 1\},$
$\{7823113057733767, 1\}, \{166003607842448777, 1\}, \{2192537062271178641, 1\},$
$\{744612452058572976318327781749257232398433706414089279801654488$
$6309794285163450665215661369721602403823862622726148437248048472559626$
$2399942952877495218360398320252120994019479986310109402236300328877205$
$0651923238022669072258630626002794080633914281658542775790101476340667$
$3429968260416711955999272172286866191270484710423419316042184734892916$
$2643514098390438226422458046124321652524310283709273693675 09, 1\}\}.$

(106) $7^{611} - 1 = \{\{2, 1\}, \{3, 1\}, \{1223, 1\}, \{298169, 1\}, \{16148168401, 1\}, \{80614984399, 1\},$
$\{3364901012227, 1\}, \{8715379679683, 1\}, \{13722816749522711, 1\},$

{197524667816924853493381822387435860231232615556825468656946727
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4421042942354413972624303816216956397948798438098005122356386851312236
4400949541010677315073370928710414882448909059183274234335380996174705
0472169276820436157785323340431987063380541878501945934805757589908963
88237035302721817488855194974439, 1}}.

(107) $7^{615} - 1 =$ {{2, 1}, {3, 2}, {19, 1}, {31, 1}, {83, 1}, {1231, 1}, {2801, 1}, {18451, 1},
{159871, 1}, {267403, 1}, {20515909, 1}, {8736087301, 1},
{126734235588747502855138501515705918224796131162528527045435276
7209258217453375321560378933987563415436844534721243844813090955132137
5506470103825555106316944986921061752623483369372539789008898639449521
6903480337396563107648299927210637220034081053653773728391794269455647
6507315553507454374509342125257550112953518924541833792261673818240712
3220242814199775141434509920655814905829798609439101804139222378553457
266547940674207823930842955458419351336531724454430319748853581, 1}}

(108) $7^{629} - 1 =$ {{2, 1}, {3, 1}, {223, 1}, {1259, 1}, {2887, 1},
{14009, 1}, {308827679, 1}, {2767631689, 1}, {150757054187851, 1},
{419980621112513304192306330823995821280066683350396866248395839
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4579242326929289195079790408061026833847139284389397891361888781405818
0271266456305964433949461582149314203272992713354799013368325453652385
3462609362089286065293930689330028114692169256401572549883367335054920
6631924707383271018772872845513924809132508021671581011032225760886874
7897314196451332685483164343619324822923122176992287446283093066597104
991, 1}}.

(109) $7^{639} - 1 =$ {{2, 1}, {3, 3}, {19, 1}, {37, 1}, {1063, 1}, {1279, 1}, {7669, 1},
{968299, 1}, {582053568181, 1}, {990643452963163, 1},
{471266017668915946914465845185545449452039130814322125370206044
6907210013599683944055720253846530749120885278407880380867790232122168
2852286251981029815580066785442764250028215084778943407580603335742849
8277641309727702841005709560635904209557877385919705332743113249343278
6350739752528247370569740859236566210557583325968979256684301402806441
8471667418016592287300506292512856977292996210764652231315579947248850
1507977801649170495860300938806974662958373667651150138087133567071417
4012833431, 1}}

(110) $7^{645} - 1 =$ {{2, 1}, {3, 2}, {19, 1}, {31, 1}, {1291, 1}, {2801, 1}, {10837, 1}, {31391, 1},
{159871, 1}, {822161, 1}, {63350976270733, 1}, {166003607842448777, 1},
{679668326738868015843129875427170473182525255856689586930814430
3221372530503323613205654575005180649641185541683714140659211707416634
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3399613847235546141291912567815183247729818924123007215374299324945085
1888127541937261305667689459997734998584007680545262959149090790356577
1083578160174790014872793179013467203007941395076226404762187818789247
4296098303447460991267296713529417735207605553929221153874712812465746
9, 1}}

(112) $7^{653} - 1 =$ {{2, 1}, {3, 1}, {1307, 1}, {3919, 1}, {270475213, 1},
{849748793036383123981364808737337709281070886985941254135396635
5493248438026964379224469439546371191838136120257678016407180190336738
0192567843849858771948655056966985238663305808740254044488352923807212
9324197195632675913072988542158875179519926906498052583376946076673916

6224603711752135645342375310513590145838724730287813206612401408628306
2738962841489220932383245036689341715968467211032214134686688698041510
2462532986216436229923597279854042201376563486410117590931089109387150
31808265096051855013316354250751345993382022559826369, 1}}.

(113) $7^{659} - 1 =$ {{2, 1}, {3, 1}, {1319, 1}, {5273, 1}, {282053, 1},
{706031095169933204062094887134964662176902494363276931963753011
2554063996570435332285886722534242768040584051769314130234194130998954
3229088125474966919396518611007849111206630179359296123168940075069266
1024445119088876205798256486780016099586228848682352170731637675712768
9896882706187113637494011250062252908682830426575405521870330419734009
7188943236725647565634642103393170904730061106786544955502228305886287
2344734862658288180371190212538009606764201211883824439789278661568494
7038541661557693475834084488295505356376634654474351221699787, 1}}.

(114) $7^{699} - 1 =$ {{2, 1}, {3, 2}, {19, 1}, {467, 1}, {1399, 1}, {391441, 1}, {79227874003, 1},
{763527975915837345372727844865944590404083572316310215043076724
6974383210980226672747362091878350268126980480929278845045610612662980
3062700517815735712650910714217972060823187757737934157044122148610765
7755330924889617393361722473054112348270488298087058585154704193719925
6709581229578801293994385282688330331264423778015269770600851854855868
4387478786577135278626192393710920956567764783141202405523360562707585
4601111030007304334580477543475921638481228332626578229159214520958668
7822368613623406439997722181261183748861545482417381166541553218231533
9755149665439, 1}}.

(115) $7^{713} - 1 =$ {{2, 1}, {3, 1}, {47, 1}, {311, 1}, {1427, 1}, {3083, 1}, {21143, 1}, {5451599, 1},
{31479823396757, 1}, {3999088279399464409, 1},
{640931687414161184785903385322136074480249136327973631586621238
1530323996784634794578225418083414826754732539487035076101194743462143
7063545890167430061132606588691492672398934129897274637803255705941207
9576453233904889632760944443012395144933735790816539329637572387049931
2435141803627972556708506932164483087712165587600949642858944761796572
2429279448674579103253129696987794992635708906167600181685958871330201
3723991772138517747893218220780504715987084382777915051255776712003094
28821696782850213923220959938914282184569200870014459952617551013, 1}}.

(116) $7^{741} - 1 =$ {{2, 1}, {3, 2}, {19, 2}, {419, 1}, {1483, 1}, {6917, 1}, {7411, 1}, {14821, 1},
{19609, 1}, {486643, 1}, {7524739, 1}, {44975113, 1},
{879399649, 1}, {16148168401, 1}, {22931541257, 1}, {4534166740403, 1},
{6957533874046531, 1}, {14975869588530371, 1},
{108298732378534933789088359287735866001474951245023775391029716
0707687570398750140687694199769911248719060168580527791338491089487607
0535073592215052709699475110166812588202834303937461599218355936520439
5666018755396243674558787326350576568103566192362346411253090701319854
2856022747379537943904840802123689282409383953433437835266672646085357
9095278120794754792053950570759650907570403819377954231995456418917733
4549998869114400426008492178559246955680111563273494957356288353393715
752118088392955261253563, 1}}

(117) $7^{743} - 1 =$ {{2, 1}, {3, 1}, {1487, 1}, {205069, 1},
{11713923500359, 1}, {97986253240756675481, 1},
{385135864714658907353934583309851755516134366646349055542017998
9695308564344402786454912806645885004619646676576819396515459064501250
8181872914035263846059114113080024278640510969085964321692867944673943
1877893486157222521845489434108946774532805680059739721785291790667501

$1534879827446614634240909436868204038086528412353759789691583826625683$
$7280611467094616570674307898873685128127623913110282718105745598654896$
$1705703611616717708558809915828472844769087463065469427444065882443680$
$5108086238931674921604477989398299201719821889132503842562365664933807$
$165343914599824452988084725245261, 1\}\}.$

(118) $7^{755} - 1 = \{\{2,1\},\{3,1\},\{1511,1\},\{2801,1\},\{36241,1\},$
$\{332201,1\},\{6788519081,1\},\{115356256117,1\},\{36076003832567,1\},$
$\{129615117850063249409028029604792628510020043125918208852361576$
$4293495450004503424396362121525124177697324077771954005149044954757141$
$8333427655954104221522409596898917492471953814326661638431808177703590$
$5914769393520872575626932612676899891766092459090545065342841113030884$
$1183978216703705889213423608382513303182036982630487096700913401303737$
$3314714856139900047928959963426678068078653523268850399425300800839677$
$1895577304108600939013635610398457377895216067638348617966008321444618$
$5957252581569544041784271835115685550247755811076187011349495530702384$
$069869334811567539512429654029973, 1\}\}.$

(119) $7^{779} - 1 = \{\{2,1\},\{3,1\},\{83,1\},\{419,1\},\{1559,1\},\{140221,1\},$
$\{20515909,1\},\{4534166740403,1\},\{1253761670999103569,1\},$
$\{403150674199670165537040121423837975649488300269950963540862964$
$5483767682579967794585560627381062787670139482228143508032809375940200$
$2983866215180926914190411582440168312465175565155818661087707616829790$
$2141397606496350451847463182793332480563744896582374776181870696586656$
$3563113700680696403746151572640058101598974761071321469687240153138961$
$3595611858619117139243981798941948952610243756880945983443628231839623$
$2011261480355717401457577828975758907337740254206661481525663729517330$
$9988704835673824924304907977358746843779398800636997411663369841005850$
$820478350857205702436692408220425313806450729752554813, 1\}\}.$

(120) $7^{783} - 1 = \{\{2,1\},\{3,4\},\{19,1\},\{37,1\},\{59,1\},\{109,1\},\{523,1\},\{811,1\},\{1063,1\},$
$\{1567,1\},\{2377,1\},\{5743,1\},\{86131,1\},\{375841,1\},\{2583253,1\},$
$\{127540261,1\},\{768631951,1\},\{2576743207,1\},\{83226515419,1\},$
$\{71316922984999,1\},\{710074092185677,1\},\{196915704073465747,1\},$
$\{415831983474196104805174275626152034262137695646498991155957538$
$6464440146769846494389879409780484238969206034663175493596444106453340$
$7470950875666655119523677026857759057569826088015218028127177797525104$
$0667604341225814236664608309481663924470513181306502918491362814401171$
$4641987433742180225414239868721057112194668334558261983667701449055739$
$4267041135139566284973021829435249647973496626749600539739415034730913$
$4063614790267508818642994799186046625942527366276358633553678397949037$
$569925949333392962378876972420598905199329347691581, 1\}\}$

(121) $7^{785} - 1 = \{\{2,1\},\{3,1\},\{1571,1\},\{2801,1\},\{4397,1\},\{5653,1\},$
$\{1039945344901,1\},\{1263410268121,1\},$
$\{292637850687560867301661576750236411379685228882495094213408261$
$5881737031408016565703416127004122189676962543400691126395018467944328$
$7874840709854775004389708658604062139195431248778368822728044212941851$
$0504169616754773540652315525889498340630624145801855768818902369514611$
$6823717703327033868697925341612317578890264984828599217438815990629200$
$4099773145815323736348040348060403095451257614774066141406295831733293$
$3013201330887902307537917058080066727290016418603927128391549857552166$
$9648515969515052634944308437925297121346624875931155517970499843294553$
$0642920565689109967604545986589953183311343594240058544218367390466400$
$71, 1\}\}$

(122) $7^{813} - 1 = \{\{2,1\},\{3,2\},\{19,1\},\{1627,1\},\{430891,1\},\{6780421,1\},\{164514887,1\},$
$\{4339738747229042444616699373884083094982706993362212215204668294$
$4552555100699358235671397833426625555856713268411447914165031115670866$
$0399285079467987205474821775538089480787172342643442455364958283367303$
$0177611083093661053116153677377082448059574387682966243001895426991304$
$1576476494676045025661445918452428916700455608548275160517777712666818$
$1751052531146897340878218736073482175601114766178712551935177108913799$
$5776167596186033089929705466273830871932413603976874200063259836863524$
$2697892517428121165656380705855275155058614584532715921503853719821223$
$3075528749973645014001537213391964087512648912953850135798375560382605$
$26643404410454709282286342588367182587,1\}\}.$

(123) $7^{849} - 1 = \{\{2,1\},\{3,2\},\{19,1\},\{1699,1\},$
$\{25471,1\},\{644316289,1\},\{182739497717749501,1\},$
$\{17661962305772146949466590519771257317085822852723419593131759 6$
$1715248385180458725476382563595453077528090430415169066751989727332491$
$4437476333640928874112714139653712614137218165215142502557106396935684$
$8884180817376460752593443514212042763791134461183706105344577899910384$
$6244899110407521827145367037470014894496145986282341980510870674517449$
$1205784623592735742964035825661062354643716467400896679017514848674282$
$0679941050005893493477008829608598027056442900478169773807135348578501$
$0233101802347360681234893584538322862944701923987979297168063027335980$
$2218502595071383435563093065328403568313908048715070734528021335041976$
$5784308923228336951377799498298688104142388687330845828195 3,1\}\}$

(124) $7^{891} - 1 = \{\{2,1\},\{3,5\},\{19,1\},\{37,1\},\{109,1\},\{199,1\},\{811,1\},\{1063,1\},\{1123,1\},$
$\{1621,1\},\{1783,1\},\{2377,1\},\{3631,1\},\{3727,1\},\{5347,1\},\{173647,1\},$
$\{293459,1\},\{1532917,1\},\{2583253,1\},\{2904661,1\},\{3368791,1\},\{12323587,1\},$
$\{27170749,1\},\{57156633859051,1\},$
$\{712383793896664420694508055707541026895532017325610737876304833$
$5257028028592336278603208240152505532546859443034108019381968285987534$
$7625737942824984475921928934624941799812104269496229396801974189139950$
$4638914671075058594961033339307926614929686768297886248628777627646595$
$4021973293919904183799308758827670033164751267287366567611896881770308$
$6431336906265018730947849113560863635496830558582063864199847849415461$
$7278886015697605419369522578801409409716945403152614811261161039962106$
$5320222524717429137386500274956588616391113957339529533482766158816466$
$1970630920478069425960126590366373039480689864563423359922778802860859$
$758201048771414590988256 81,1\}\}$

(125) $7^{911} - 1 = \{\{2,1\},\{3,1\},\{1823,1\},\{32797,1\},\{382621,1\},\{10664977216349,1\},$
$\{523375850555417066359627992485313473766730966562254880741120704$
$3714546473895334713926465701979213242468843184918648407395875112521053$
$9189287576339148485175593578998505570448552591557742513594608294113407$
$1718738039876862956809193547896574170988493195571435544460922125742959$
$8839224040540648811404874329189416716305642854438943370504879800904778$
$6829111984363480925105101647937864763639685398479942677027232306381872$
$1340503853779404678262192250718044514905243953162016352647188620096769$
$1996141346759017899565120585923949098120646227376979816068207320647839$
$9451560009448367965089144626205422289161663125941562405239340798335024$
$0352002066751752817843397332329790726622749157568945127229692768490947$
$3136750468421807089668704448607629198338837922564 3,1\}\}.$

(126) $7^{923} - 1 = \{\{2,1\},\{3,1\},\{1847,1\},\{34055009,1\},$
$\{16148168401,1\},\{11112633233131,1\},\{990643452963163,1\},$

$\{15806443509476190768509551311424051179317051877050962283763248529227488539158245011177328437296667102321821847290179004772959946979977250579963977283282947195487484574872261813904876432221394508307611995372465889806313443767456327405279851696442430825901289650630177579378859726956266062352711954360571243029066232858568169126064797884355772159102830869189580056675332527651165124527170590916062037267244721330475462145294517111800338844163559372881144260417701085152690915978730673919581287637411143143422763325231585042802433519980298357551044796818903246910705171219759747916121308593184358681210535126798642133335658368203446333542687893860996975490422132518834384727783331386169016909841369682810129960329495336812933851318503, 1\}\}.$

(127) $7^{933} - 1 = \{\{2, 1\}, \{3, 2\}, \{19, 1\}, \{1867, 1\}, \{3733, 1\}, \{106363, 1\}, \{310379, 1\},$
$\{3806799569946705928532830639157549329796601117217032934554515137022960743464420609390180528124824805348932740897145412379343168964486221736623573583472371835023849338274658926332575334858460061730059954176901718157375856510730757888072627143184100072148831902055023803809735763216507188479361132656449053837244037073260485241031036386576336027745477703709845163516487020031530632973535518390632065853759021861744219055311620399569130477581007600073999515721633262732493229783457734921060175498135349860313449742472388119227504896370695064659103778541628897960295572314367984501127664003820717247513386893537534852441907016727918434907173142406772656065869545248281939891615696063394394609061773485807373608696141218434629475564025591616911104439001626702463406347670919, 1\}\}$

(128) $7^{939} - 1 = \{\{2, 1\}, \{3, 2\}, \{19, 1\}, \{1879, 1\}, \{50707, 1\}, \{9450044264774621, 1\},$
$\{114447135581366713082578980606578212845180629562906779241097648092565212097096727077886645254098128038553510403105381525282930557583477531422551314376471329275498532504675086323427043540139247256883144658408791411428928546689306660518897520988201731644651313309465734844788254914032234100694179703395963894181339299102550284163763403729264441966908915091882573947126653981789995128728860157345516054375903000937312614462404465839839855393039154672722757321224589279598837681361635889894186547662153230970387520613709714429804827707653842878091929543043666396211130340646574910260503937004819548031121454876815176938349869368339788716875352764741580652464963154392945916864171634937237257114802785458493147453325169482166351724526348971980738778741647799221929838158177, 1\}\}.$

(129) $7^{953} - 1 = \{\{2, 1\}, \{3, 1\}, \{1907, 1\}, \{32778579857, 1\},$
$\{63729433276655204713479597317985047404217245995612140417269517356217962568764465348779885019675994649608030803363141942520044383546280179934824302540125114315713824652869380915812251630130263920381736447479441440653521295283304323113007892495053357632002950058400754557107496465750264132905772814901710958721799682151421303912448878717972478112327529776804818830836222308279495295719763981625077510315577331814200679283848416646085963193691829093552876221015381054431530337014486729243254784843101300814661503515508678583666265159859047147922730667609914091940043148336356881894244122186341986187248367654606981985184395337585701725799668288317201960293737539215789897596890128451432730285442036834161267580245775319652930842763534606213074026674282105893828785184922432525705688364676376299, 1\}\}.$

(130) $7^{965} - 1 = \{\{2, 1\}, \{3, 1\}, \{1931, 1\}, \{2801, 1\}, \{14669, 1\}, \{627250310113980671, 1\},$

$\{11079510859469769139128539008863930529292207414391187956214679253837697344722372087102987911061051639165580106164064445917476834476784714448678638465429323733543809109059998863507655409937320330164026685169925807934200710635597226088463481430135749466670242174286778815573961090429553385725794461831252244753218060051626061048580418084701462752028088765399423173583615156563850339520411589193178120718947572863347193821980550307739498285671890625485283999850012594161868840194085030855358386842715762373104503004455661932140365764325760437302492777835552003428619312542623581416371283821300382424336461445745133760202489348173016025764829691453944146592650634460861575690914588646561849838607640778619338256470042965437978801662070479933590498053258031571788965542382258862974324494261291, 1\}\}.$

(131) $7^{975} - 1 = \{\{2, 1\}, \{3, 2\}, \{19, 1\}, \{31, 1\}, \{131, 1\}, \{1951, 1\}, \{2551, 1\}, \{2801, 1\}, \{29251, 1\}, \{157951, 1\}, \{159871, 1\}, \{486643, 1\}, \{787021, 1\}, \{7524739, 1\}, \{44975113, 1\}, \{567538141, 1\}, \{14046059431, 1\}, \{16148168401, 1\}, \{3960157950451, 1\}, \{4446437759531, 1\}, \{31280679788951, 1\}, \{434502978835771, 1\}, \{16363993407921369374016815178370851129006721712914838047513791206854667233863111951747566606652244939874844293648527127108425692322932461399282643956712594938508286074037178498798090246281631336576743101310079215822791239865423136158162304221947912405085627212418018160063023393146576588359619845253864184094404848240994662672997715508381464507571627184398051520453364871408494715407683097917032883783149679005427127751608865560459281289558177299830487251369367815263584908530772989701819763495533471146046153604327755481346161085094437784054636124505184108507970527263750425972435081081333381761897862851060956923505778817247566935002727579657948973247749201890972144876613964597509810, 1\}\}$

(132) $7^{989} - 1 = \{\{2, 1\}, \{3, 1\}, \{47, 1\}, \{1979, 1\}, \{3083, 1\}, \{670543, 1\}, \{1501303, 1\}, \{31479823396757, 1\}, \{11624274954629007063522572031339822236573144986242172970663700301318388346382927170452895615353992923259691345481252257185481997593044241831730134784002845360917435077570925321548504360697283392436976244675003045089170543513972527774502092413943631684644063831059130108573313559812956877258749895184948537932501664072704890607256438329544917295457877660726692960376765653509626783668359015106730747941714618178859048244519971517154113863744588743765550267437475175847070800572791029278417809972541466016162659840287512477289542135659360097610290519759783488446632223408497684022917882288033384978048553418678454039934722874187339551413519987969227040040326061122168722093542965652788170419756535339420747885602382639177735830905938997011238248201426964226162512287314116855299247331532946178034863795231, 1\}\}$

(133) $7^{993} - 1 = \{\{2, 1\}, \{3, 2\}, \{19, 1\}, \{1987, 1\}, \{7283, 1\}, \{25819, 1\}, \{66170604553, 1\}, \{17997613911938779523943339110134911772235229557987851557085794734619271221147501525734720499055165054289481275739472564452382346141652285041995301335877757032895249226692319165859739545164377599232935918383485803225655657386310679642627920474468348920881416876387641918815347014677874707988786514207875873120843891368222162013183617651695195846557221575803546851775510607413776814199403224993314361993705246343103171331477789625707061589359488426479612821283319957634579600685449016309517857052082108243052310743225080255273324111830428543505515673169777006559991603096763565541340661862377867046820408146845689846244756605434205836511689883613779928876339835467898539438477378694193149204271,$

02339392045323413998319438131009372089648737964059631202168317656972732093908364880117290309116960996936146743793819871819, 1\}.

(134) $7^{1031} - 1 = \{\{2, 1\}, \{3, 1\}, \{2063, 1\}, \{285960223, 1\}, \{871749679, 1\},$
\{640813058375167219449763593588812763167792559500329558015768558
3154271065797312389797589383732158068642085443426448207905982655165733
7073129304834387194211659300115388372369477620481832796836077250352517
210824650947301288609058900248393242005555865904049634334260247602541
6174461060508950825827403615817243499778432386753785896338544257957551
4075476522229495170553655253169462767447693354028717010689726202058332
2570465311380286472464244210079472945111312905223591121678045313360525
3027420106486526448956039479541449773502837359756882377517023014215110
7474274303808769699971018929725648361246956853505106230797542104928312
2137859453491989078895603147962418386605352961551527710184377891501724
0351512945439081080368470508356231663477826217228285817192473197132731
5493381699554002612073538060405393059457028337929522737026390081928998
33179449277333367, 1\}\}.

(135) $7^{1049} - 1 = \{\{2, 1\}, \{3, 1\}, \{2099, 1\}, \{48992497, 1\}, \{420164363, 1\}, \{284330642001457, 1\},$
\{436825347906424797401289373041438960887095681951458264406595082
9378119871598869227232939600341115409793699165095703858112429161713674
7023659125040143436617116791718564639915983118949234877707325789447055
5246508840774196675925899257568853893256815256458002192821250373363909
3703619766166127806528021267867637395000292463244002730666595356973122
0587972839825824530539760447040343821563055063925553752147809989939264
5347035545097834388399669103675511711017720567977042518548188006995217
1290319100913444165962731267687744107152596203803113937289136338767816
4188286244417091635649138083757100509015688942271685324206391375783404
2300625512774141178135853953501353936140843150667266554282812827211578
1312765037930747726143242489229379641044179300963334162062135023268108
0493992916600271920044808439645793493842916857998671573120662868858946
1062105651324877137, 1\}\}.

(136) $7^{1101} - 1 = \{\{2, 1\}, \{3, 2\}, \{19, 1\}, \{2203, 1\}, \{5713457, 1\}, \{9127291, 1\}, \{12088981, 1\},$
\{1232836358179, 1\}, \{555169875729277, 1\},
\{872849908290455430254595660525157858726258626175343655889104626
0621556438728200613258087030148119378717267690876278420982644910307584
0351971975008253249999709879284802086691673810649367769374685715694678
1642316153894950827730914493830619262722442627531778628717593401700067
3064777276022180886658401609599261717349609469660171824624734105979353
3627224068700327850837546357816522191932789188321809908250711517912967
4226510880139618299070832429290204913736789170254805950325833598264670
7438536341511265720370259535864556576327671429633588661925841883872329
8357277321639699949176934350713270146196027044843816060232896622387054
7403763629120470527947791286252644382250304077180292619873793572126352
6138829533781270640488782058899534112933771233119688270061566527071124
5207437580882115816264199971817007276825600598624548978731748472051849
80173442823952053900286823502259807616826331, 1\}\}

(137) $7^{1121} - 1 = \{\{2, 1\}, \{3, 1\}, \{419, 1\}, \{2243, 1\}, \{8969, 1\}, \{459257, 1\}, \{134927809, 1\},$
\{550413361, 1\}, \{4534166740403, 1\},
\{289481342287376532691829948925003693983795819012520488989672831
9094718147097855569042441423255350241361917004176284294355641771205918
1081986428070569022658888002207647015720366769127542307384823609386550
2171451726689993548951096093181401240734493246374491479647065804472268

$613814302342445940580005482505158952951115141007791095064298237563593909643017305957823226081427892717523693080280400050121737745066677231975358545618935512731588760773775894851625922012983093784637813732140814043229862623258747007660957298085615719122122137532221329142601657273813382023494393636538053268012331647494538389557637889983417793886308237782621588090155765400170352298300161903730950788634490992245624996707089210670206244464192941914443220060987532795277398756190348478788755223730638131888099317355625630445840176321559478322392963781623800099215027735594929585442858082137536314414266911815412316846442830721532 03, 1\}\}$

(138) $7^{1133} - 1 = \{\{2, 1\}, \{3, 1\}, \{1123, 1\}, \{2267, 1\}, \{17923, 1\}, \{293459, 1\}, \{344210899583, 1\},$
$\{113321953624108734148501587859108700112951593165094394602122174$
$2085179619773833454298336344980860257105288813529240951536823161713287$
$8603429559620203644586476956594729892598778203353680394916813882952869$
$1392064539738496260451861255316451892592584621949970034526705054502109$
$6914598373692681791224841579538953152705210257382188908583239236619651$
$5940429733504513181472830574120712855213354392383721463255662888425788$
$0546392570314161902741929614330238714153852095447231081315914923597068$
$6471566616073694933834473141025344224212561740683120626837466106584749$
$8394058798291354224039633997726551506265957774744267087318257070923024$
$7783287308549135790956428285574541169829616723688169479112314889908075$
$1135665473645386366135736152889016377681603501281426739356730475725154$
$3391644279656425503401715249076968927606432967976089325247048939206942$
$9503306769363915243765841053475047442940889091702624019695704077176638$
$4437732142964696868168142 31, 1\}\}$

(139) $7^{1143} - 1 = \{\{2, 1\}, \{3, 3\}, \{19, 1\}, \{37, 1\}, \{1063, 1\}, \{2287, 1\}, \{130303, 1\}, \{2208277, 1\},$
$\{10920223, 1\}, \{260461507, 1\}, \{9809667961, 1\},$
$\{11699360074183, 1\}, \{3451948895547127, 1\},$
$\{295833553111890954448612314093327906546577022386017975692244985$
$0928004739744048055261444047972549962631885500379716973033115286228724$
$8853056544418521802908869281010902238238746889432995180028912089446372$
$7528805380136624347334558793667584981389441900209443237645424545985354$
$3046186759643498083262960988764147527952796892706626337543646849079561$
$6425193696882898176500309357583940049466890280046025517211835100637816$
$6974752695164801106903971405633971030527754036587643595825193115170207$
$9590606982008883097581680623216753373745545637888375058449399006746009$
$5946759330948212706616071376698102857979522884139730908777892378894674$
$1087926233918077656443998959854088836815761508505528993621189605191555$
$0133806379359566599324682486105743827849058795499621811934050813819402$
$6159353044169298136013271921128218790519497759392736541913125580567596$
$53949602230517395185558917700822424113451066990014203739 21, 1\}\}$

(140) $7^{1175} - 1 = \{\{2, 1\}, \{3, 1\}, \{2351, 1\}, \{2551, 1\}, \{2801, 1\}, \{77551, 1\}, \{138711101, 1\},$
$\{31280679788951, 1\}, \{13722816749522711, 1\},$
$\{210063937677829240482609778912761612020923670468129558980782110$
$8676376216803652173847908777443876027871384631031676802192588116602210$
$4079976536754175236154473480555658182514436736190188005263064151102060$
$0769004001033031496387090821018302543746168747450529753244466885609384$
$5530946632372598747262413070184584093090731720202967225286876744320577$
$2967407039846239554139279153326809251991857317111887918650177869410049$
$0284378516888218171891248660750631175055595558508893803488199515986488$
$1081195251082914939593283201651824154980967582824922119647365896170406$
$0366235306818554185446002019922355159040825893793823053779883819169374$

$0593430405467036578596179490544181727990953251878701742516841549480063$
$7405233193364442766013168296626956124967956828362718761368220751570410$
$6212234471601312103237350998470909097998739547432269179443283344250951$
$6853624193713513904884117463547027070633231799407418357752365317072489$
$2412271700280514331716414396502279787, 1\}\}$

(141) $7^{1199} - 1 = \{\{2, 1\}, \{3, 1\}, \{1123, 1\}, \{2399, 1\}, \{59951, 1\}, \{293459, 1\},$
$\{1583492923, 1\}, \{2457808519, 1\}, \{38793043519253, 1\}, \{14551288401737861, 1\},$
$\{299795689062637983704249728058685524733298287883656536648078247$
$4770136752367329062687529364482737701175421615127173474927122160279360$
$6216509662515101562194531109908487120579117648877413388550085527837495$
$8512515889819833081166032050227404189975793583830429241661626851141009$
$4641842313641522735702683520336260527742078237314147511005284290784955$
$9302860526514428885202610014429565938939802010715698345116208376011855$
$8088674811324990314818356380230217419218144789164032905127193667043092$
$1575239278025273514371969213854599086945819456921179822733719616922466$
$8226212135764764466676103646237755489492035468765298436833748461783309$
$0471216990680614719483896502945835825566395109446192378873371921689272$
$1573262521320774628510437217950876113128178310780880647642329345575208$
$0731219737786555970212419779433770638372509398257202649393244109295030$
$1684792442943187139140241244625090960123510546038397123746742744650014$
$402943673578806851167892604968329424970860269, 1\}\}.$

(142) $7^{1205} - 1 = \{\{2, 1\}, \{3, 1\}, \{1447, 1\}, \{2411, 1\}, \{2801, 1\}, \{28439, 1\}, \{595271, 1\},$
$\{1471180803061, 1\}, \{33486078253526557, 1\},$
$\{98544033012195431, 1\}, \{580351872045743557, 1\},$
$\{788001439410563834670596313758909272514128954482820092062783513$
$0192529880516274450648722239947115081550028544445092274701705996936026$
$3952139550327990097559765599730102955830977428267731112966492919245803$
$0390455270975558863529020294474015266670930078177261061012446415774435$
$8196200052959993507638938322214790716198036929811964619099943989370719$
$7663458607378690513024503857733119721476063903744080959634768360131724$
$5265344292318804867768980954788583361338952031491535094834948909521391$
$5962762668999609455607060522009233758797546923220352583625024538449801$
$8121414178850613034259241271715843955820230100565073593239447362201863$
$8595429501038128407435350791200359216490950343746056026446659476319150$
$1427438941908854200686618956523875734031219269601793715470783880408067$
$1782771543233625114242749197784689485030870974396521363532704984034889$
$7942939842687831822880222901904937094739086686131645691670871050793801$
$7331189363226777538374225661943, 1\}\}$

(143) $7^{1233} - 1 = \{\{2, 1\}, \{3, 3\}, \{19, 1\}, \{37, 1\}, \{1063, 1\}, \{2467, 1\}, \{2741, 1\}, \{213721, 1\},$
$\{251533, 1\}, \{443881, 1\}, \{575401, 1\}, \{9712753, 1\}, \{44807221, 1\}, \{109781389, 1\},$
$\{329437051, 1\}, \{27500724319, 1\}, \{8350989090751, 1\},$
$\{748462250678523518049517456077821793235105640889442254843797349$
$2130808709118007170851869798035153873029468042251699417376580973596096$
$7138759506033392185534668445576218515502008906669094142836764844622442$
$8182192504002034282542832223931987197650259392783449319738712666900643$
$0971750186303349048943837423939277083401757122840463876099145798628936$
$1805637095131104971998640438917987369335084761801082309047102582376364$
$7382216363233087789210249592779978762626280296917990065384000347544703$
$0722796882541703851744122470606029371127569844588927367141961826283674$
$2673473512799524007946308558839130075516332180087328157328692522158583$
$2717341792716761318719533723120991928549921200336234675920916761615354$

$4632928749024689398931212271521547766134325611234411977913499694495044$
$5525788437443773548219999649461554558590181395160568353215685660027415$
$1601974498798869987073338087892180324801599531249597434514933046649381$
$2246383073753033461520751952640969472230824246 97, 1\}\}$

Appendix G Factors of $7^t + 1$

(1) $7^4 + 1 = \{\{2, 1\}, \{1201, 1\}\}$.
(2) $7^6 + 1 = \{\{2, 1\}, \{5, 2\}, \{13, 1\}, \{181, 1\}\}$.
(3) $7^8 + 1 = \{\{2, 1\}, \{17, 1\}, \{169553, 1\}\}$.
(4) $7^{10} + 1 = \{\{2, 1\}, \{5, 3\}, \{281, 1\}, \{4021, 1\}\}$.
(5) $7^{12} + 1 = \{\{2, 1\}, \{73, 1\}, \{193, 1\}, \{409, 1\}, \{1201, 1\}\}$.
(6) $7^{14} + 1 = \{2, 1\}, \{5, 2\}, \{13564461457, 1\}\}$.
(7) $7^{16} + 1 = \{\{2, 1\}, \{353, 1\}, \{47072139617, 1\}\}$.
(8) $7^{20} + 1 = \{\{2, 1\}, \{41, 1\}, \{1201, 1\}, \{810221830361, 1\}\}$.
(9) $7^{22} + 1 = \{\{2, 1\}, \{5, 2\}, \{661, 1\}, \{1409, 1\}, \{83960385389, 1\}\}$.
(10) $7^{26} + 1 = \{\{2, 1\}, \{5, 2\}, \{157, 1\}, \{1195857367853217109, 1\}\}$.
(11) $7^{28} + 1 = \{\{2, 1\}, \{337, 1\}, \{1201, 1\}, \{2129, 1\}, \{517553, 1\}, \{515717329, 1\}\}$.
(12) $7^{30} + 1 = \{\{2, 1\}, \{5, 3\}, \{13, 1\}, \{61, 1\}, \{181, 1\}, \{281, 1\}, \{4021, 1\}, \{555915824341, 1\}\}$.
(13) $7^{34} + 1 = \{\{2, 1\}, \{5, 2\}, \{137, 1\}, \{59361349, 1\}, \{1330880393737662309, 1\}\}$.
(14) $7^{40} + 1 = \{\{2, 1\}, \{5, 2\}, \{661, 1\}, \{1409, 1\}, \{83960385389, 1\}\}$.
(15) $7^{44} + 1 = \{\{2, 1\}, \{89, 1\}, \{1201, 1\}, \{8713, 1\}, \{8206973609150536446402438593, 1\}\}$.
(16) $7^{148} + 1 = \{\{2, 1\}, \{97, 1\}, \{353, 1\}, \{104837857, 1\},$
$\{47072139617, 1\}, \{108604397663266369, 1\}\}$.
(17) $7^{50} + 1 = \{\{2, 1\}, \{5, 4\}, \{101, 1\}, \{281, 1\}, \{4021, 1\}, \{13001, 1\}, \{25301, 1\},$
$\{3832796630023190929 1101, 1\}\}$.
(18) $7^{52} + 1 = \{\{2, 1\}, \{313, 1\}, \{1201, 1\}, \{85094881, 1\}, \{137745463534253746093500815 4217, 1\}\}$.
(19) $7^{56} + 1 = \{\{2, 1\}, \{17, 1\}, \{449, 1\}, \{673, 1\}, \{169553, 1\},$
$\{39648001, 1\}, \{21535258550401, 1\}, \{142256806230113, 1\}\}$.
(20) $7^{58} + 1 = \{\{2, 1\}, \{5, 2\}, \{233, 1\}, \{136853089, 1\},$
$\{55716067510309, 1\}, \{116714640028973541741413, 1\}\}$.
(21) $7^{78} + 1 = \{\{2, 1\}, \{5, 2\}, \{13, 2\}, \{157, 1\}, \{181, 1\},$
$\{344518212206157602484419548088667547339993308388 1531303593, 1\}\}$.
(22) $7^{86} + 1 = \{\{2, 1\}, \{5, 2\}, \{173, 1\}, \{1033, 1\},$
$\{53372188628311276616238880882996409313455606853704297384639789 5\ 277, 1\}\}$.
(23) $7^{100} + 1 = \{\{2, 1\}, \{41, 1\}, \{401, 1\}, \{1201, 1\}, \{810221830361, 1\},$
$\{10108781944425033996020948969923519911594210849831293746365778 3601, 1\}\}$.
(24) $7^{114} + 1 = \{\{2, 1\}, \{5, 2\}, \{13, 1\}, \{181, 1\}, \{229, 1\},$
$\{814233839701286841407600426610905218940681274414001855960390665$
$813894796898753204671776 61, 1\}\}$
(25) $7^{120} + 1 = \{\{2, 1\}, \{17, 1\}, \{241, 1\}, \{881, 1\}, \{169553, 1\}, \{542081, 1\},$
$\{33232924804801, 1\}, \{2312581841562813841, 1\},$
$\{506124771462012241725144686069293242582322872276 4561, 1\}\}$.
(26) $7^{126} + 1 = \{\{2, 1\}, \{5, 2\}, \{13, 1\}, \{181, 1\}, \{1009, 1\}, \{250993, 1\}, \{2304793, 1\},$
$\{13564461457, 1\}, \{13841169553, 1\}, \{31864919689, 1\},$
$\{195489390796456327201, 1\}, \{37806375386040524773988014755308 7649, 1\}\}$.
(27) $7^{128} + 1 = \{\{2, 1\}, \{257, 1\}, \{769, 1\}, \{197231873, 1\},$
$\{190845781233779568947114378127612565912345052718201774886297099$
$41792454027327157612573467077889, 1\}\}$.
(28) $7^{132} + 1 = \{\{2, 1\}, \{73, 1\}, \{89, 1\}, \{193, 1\}, \{409, 1\}, \{1201, 1\}, \{1321, 1\}, \{8713, 1\},$
$\{532489, 1\}, \{98138029441, 1\},$
$\{482121267192714407437280028916490787579914240064587528159545948$
$6484414160017, 1\}\}$.
(29) $7^{134} + 1 = \{\{2, 1\}, \{5, 2\}, \{269, 1\}, \{3217, 1\}, \{4591913, 1\},$
$\{39144122221793, 1\}, \{27982218262152281, 1\},$

$\{8043066638625731183783562402927490226742147207772021658106978528 44101, 1\}\}$.

(30) $7^{138} + 1 = \{\{2, 1\}, \{5, 2\}, \{13, 1\}, \{181, 1\}, \{829, 1\}, \{21529, 1\}, \{26497, 1\}, \{26681, 1\}, \{649981, 1\}, \{15513961, 1\}, \{5755716973, 1\}, \{48779529425506209716775381028420304083102551973096491103093750559 14548053, 1\}\}$.

(31) $7^{142} + 1 = \{\{2, 1\}, \{5, 2\}, \{569, 1\}, \{853, 1\}, \{29537, 1\}, \{810253, 1\}, \{17374141308469551469718648900907789008049467465982479621099340956319170026167267707873542786350098 66473, 1\}\}$.

(32) $7^{146} + 1 = \{\{2, 1\}, \{5, 2\}, \{293, 1\}, \{20149, 1\}, \{36793, 1\}, \{22307839956922814080823571734550155373921027529367509223868368808440205692487006276717269425895541140714651 2673, 1\}\}$.

(33) $7^{154} + 1 = \{\{2, 1\}, \{5, 2\}, \{617, 1\}, \{661, 1\}, \{1409, 1\}, \{5302529, 1\}, \{13564461457, 1\}, \{83960385389, 1\}, \{80495297458997560527130316928471283711289948509615954606724481783541711761172675099569755257, 1\}\}$.

(34) $7^{160} + 1 = \{\{2, 1\}, \{641, 1\}, \{7699649, 1\}, \{134818753, 1\}, \{163344191041, 1\}, \{531968664833, 1\}, \{14209781432972995915719176096710879594160687783872918717248567475215750584159203240471457715521, 1\}\}$.

(35) $7^{170} + 1 = \{\{2, 1\}, \{5, 3\}, \{137, 1\}, \{281, 1\}, \{1021, 1\}, \{4021, 1\}, \{59361349, 1\}, \{15425560249042501, 1\}, \{12829076159336934664065675209967637324402039778019986577288788990444811691865608161310987056313352306899829, 1\}\}$.

(36) $7^{174} + 1 = \{\{2, 1\}, \{5, 2\}, \{13, 1\}, \{181, 1\}, \{233, 1\}, \{349, 1\}, \{136853089, 1\}, \{55716067510309, 1\}, \{1527744125503041544999763194496616690945916633912866884406082679389348820995120597908439868345697350118919752750 9737, 1\}\}$.

(37) $7^{192} + 1 = \{\{2, 1\}, \{1153, 1\}, \{35969, 1\}, \{55681, 1\}, \{12644353, 1\}, \{1110623386241, 1\}, \{2474441815297, 1\}, \{11308065554892809903403244526077911039468313764027968898644975133006042944929223059791867913583416051929003284243949313, 1\}\}$.

(38) $7^{198} + 1 = \{\{2, 1\}, \{5, 2\}, \{13, 1\}, \{181, 1\}, \{397, 1\}, \{661, 1\}, \{1409, 1\}, \{29569, 1\}, \{257401, 1\}, \{2784937, 1\}, \{678240817, 1\}, \{13841169553, 1\}, \{83960385389, 1\}, \{116320403133001, 1\}, \{25256185377842270097857830518096843237063296965139665608090401267332692010409073133723030 91533, 1\}\}$.

(39) $7^{216} + 1 = \{\{2, 1\}, \{17, 1\}, \{433, 1\}, \{2161, 1\}, \{19009, 1\}, \{129169, 1\}, \{169553, 1\}, \{455761, 1\}, \{25726609, 1\}, \{51385969, 1\}, \{635311009, 1\}, \{338325042961, 1\}, \{33232924804801, 1\}, \{609928137331215931325743981240140163846633907421573770915524903023638310252054897264558136285624787013787 3, 1\}\}$.

(40) $7^{230} + 1 = \{\{2, 1\}, \{5, 3\}, \{281, 1\}, \{461, 1\}, \{4021, 1\}, \{26681, 1\}, \{649981, 1\}, \{15513961, 1\}, \{48203401, 1\}, \{1853224081, 1\}, \{9118947413041, 1\}, \{55681942361860463813, 1\}, \{148380801538465137230106248713887408397705874784316169458289482226910803482493259028339119907427804070043125754977821, 1\}\}$.

(41) $7^{244} + 1 = \{\{2, 1\}, \{977, 1\}, \{1201, 1\}, \{2441, 1\}, \{279181359188709713738914433888025302486858307167679087718299549063095128510464163556374981142594860176765189462446476211611646186785207606664938978223668998882782792307539865377532263808078066379 93, 1\}\}$.

(42)$7^{254} + 1 = \{\{2, 1\}, \{5, 2\}, \{509, 1\}, \{3049, 1\}, \{15241, 1\}, \{23128733, 1\},$

$\{16515496979290887896490570217141499558459589183739345785366873087744704695018672013258340367450667515014783912648886646916524985711814573765990832727326063778748140390930464141515573433292094098 49, 1\}\}$.

(43)$7^{260} + 1 = \{\{2, 1\}, \{41, 1\}, \{313, 1\}, \{521, 1\}, \{1201, 1\}, \{85094881, 1\}, \{810221830361, 1\},$
$\{480003171956223529669924055211056738057773418463805298730047823$
$583610970401563630468510614588931790782326108755592216107516852706503 3$
$810660211151300206528306837101541526983017467890835617377, 1\}\}$.

(44) $7^{270} + 1 = \{\{2, 1\}, \{5, 3\}, \{13, 1\}, \{61, 1\}, \{181, 1\}, \{281, 1\}, \{4021, 1\}, \{9901, 1\}, \{26893, 1\},$
$\{68347801, 1\}, \{13841169553, 1\}, \{154625887441, 1\},$
$\{167039577217, 1\}, \{555915824341, 1\}, \{2295769290481, 1\}, \{590297313273013, 1\},$
$\{7554632105540366589152331484661728227545476182930874046627278 32$
$237036197570492139675451647561794425727620690384575395504253918 1, 1\}\}$.

(45) $7^{282} + 1 = \{\{2, 1\}, \{5, 2\}, \{13, 1\}, \{181, 1\}, \{1129, 1\},$
$\{86857, 1\}, \{95881, 1\}, \{4574857237, 1\}, \{23146232881, 1\},$
$\{177405126822295090374135014406451784279375509548214737594895898$
$503469735005432170542560988450632599527727874400367050172559756865146 8$
$048637891942552652635748253315415117692002185955890699952801715571 57, 1\}\}$.

(46) $7^{288} + 1 = \{\{2, 1\}, \{577, 1\}, \{2113, 1\}, \{7699649, 1\}, \{9917569, 1\}, \{134818753, 1\},$
$\{596878081, 1\}, \{16356157057, 1\}, \{531968664833, 1\},$
$\{187521384109685338945221918793894283983642255950664916404230950$
$251823642924406300906827234116768835258480120650188785869986379073338 0$
$548076049665351144901848133954361130197690906425433 7, 1\}\}$.

(47) $7^{296} + 1 = \{\{2, 1\}, \{17, 1\}, \{593, 1\}, \{1777, 1\}, \{169553, 1\}, \{15731809, 1\}, \{274166594399233, 1\},$
$\{537898242355005363187080995372272918566624712854651450653774432$
$286046809310192815045180470242640166934219746727983801028686574703854 5$
$023318253819260069813183159094256019490442490242980825902361476626434 3$
$4918210070753, 1\}\}$.

(48) $7^{300} + 1 = \{\{2, 1\}, \{41, 1\}, \{73, 1\}, \{193, 1\}, \{401, 1\}, \{409, 1\}, \{601, 1\}, \{1201, 1\},$
$\{100801, 1\}, \{12913561, 1\}, \{810221830361, 1\}, \{8556026185965589764 1, 1\},$
$\{274186405564458718822730366656762022450762143859152488613621009$
$821437236790338569954549040216610463227994942413349321560225699367001 7$
$804664203292362084118703404079394610084356510519536806170201, 1\}\}$.

(49) $7^{324} + 1 = \{\{2, 1\}, \{73, 1\}, \{193, 1\}, \{409, 1\}, \{1201, 1\}, \{1297, 1\}, \{40177, 1\}, \{42409, 1\},$
$\{137089, 1\}, \{7821361, 1\}, \{32952799801, 1\}, \{317328896377, 1\},$
$\{189027978606574721147194185783957778308052370125560933766132969$
$746716890904902424632258616311153306154362947626502259156728927624491 8$
$666974996413479083086480306734919883548860170143317638633269866713210 6$
$050871780296857, 1\}\}$.

(50) $7^{338} + 1 = \{\{2, 1\}, \{5, 2\}, \{157, 1\}, \{677, 1\}, \{2029, 1\},$
$\{407752988127621883728249622256495347982646900218466489535182883$
$741676812341131359164126215529503437784928075852539954085408764080933 6$
$025098047862656909899474654616803696127022216684989781593887707243449 8$
$070506070660489990274459107798619222662942561456489501156692144743252 5$
$749, 1\}\}$.

(51) $7^{366} + 1 = \{\{2, 1\}, \{5, 2\}, \{13, 1\}, \{181, 1\}, \{733, 1\}, \{4497653, 1\}, \{133440673, 1\},$
$\{390762221068173008631919921610897124445518925029000609050379099$
$811125222640177995021520540630980000258998511053543983330030004016775 8$
$528464525340409008941833717610849461812799936780352757122343594160350 8$
$659732227636100633238147103159359034197647145953122736265801040366705 2$
$68266736460993, 1\}\}$.

(52) $7^{380} + 1 = \{\{2, 1\}, \{41, 1\}, \{761, 1\}, \{1201, 1\}, \{44118001, 1\}, \{700197881, 1\},$

$\{1081419961,1\},\{810221830361,1\},\{1295161807633,1\},$
$\{1299254811081649,1\},\{135214434686939513,1\},$
$\{29719214653004098369323659800254997470312808298066656106462893876884576782859167220664053934131086814187604678448875843621904963072235916371312472660093608762961299121460113543772724495648394354130061611132006960086716578403931024 81,1\}\}.$

(53)$7^{386}+1=\{\{2,1\},\{5,2\},\{773,1\},\{64849,1\},$
$\{378881050637,1\},\{286400155926593,1\},\{18154096793869693,1\},$
$\{32684352222168432005099608928732062215066842070112727303870012289156391009503316055215587584847643267076292280134341603557457910112664124655956826479422504201363921551366051616192722428756882747021378097133403123582008552642936615464569588443363089898701756718994872901847493,1\}\}.$

(54)$7^{398}+1=\{\{2,1\},\{5,2\},\{797,1\},$
$\{56052062803682845641781909532515411123332644015041414812455862930655215382351782962716235294071500412639231492345404379459962466479247503510046204313557670197137257954048530336296807243274052846674457241742388485237076805867456135711160866287146655301020045802952209940193782792456728304261958421492680816234265743216657413256205317,1\}\}.$

(55) $7^{428}+1=\{\{2,1\},\{857,1\},\{1201,1\},$
$\{24457231878660696714412156115890735638923139561002717875562050147773707554841400907790116522131454339279864401108211795995669872610050466643353078984633268486302517704713837561253234444886288015634933426961197077371924127888818219114495906150940411406283362207203329420434326192013323894897397450400686523460744256743584803424296314324964724512114681787593,1\}\}.$

(56)$7^{464}+1=\{\{2,1\},\{353,1\},\{929,1\},\{47072139617,1\},$
$\{43242043805164797719374137140672402758925991209035836575493204152554518130943180461761347665984429285902900971824212244405608033706093251354236228756081115960401604718398433166114295760494480535808961567368355498166440141271220466039238195429185934575572111319884304659175403725179649468141179336159064019991035447223582129133891267459392567554122522095082387058592606387807 69,1\}\}.$

(57) $7^{468}+1=\{\{2,1\},\{73,1\},\{193,1\},\{313,1\},\{409,1\},\{937,1\},\{1201,1\},\{8737,1\},$
$\{42409,1\},\{45553,1\},\{137089,1\},\{85094881,1\},\{32952799801,1\},$
$\{12169888604519670576002219899793644348534408589952866546806273612632634379642558785465121180744066305311973202146630035239334112036143193629649201198461799924510817091399701023266014269903285898559022313971585739172522637622423834487663597949992077492772195185724433660887472127237804830655009737178865367776619577501016140661870494369949263281,1\}\}.$

(58) $7^{470}+1=\{\{2,1\},\{5,3\},\{281,1\},\{941,1\},\{3761,1\},\{4021,1\},\{95881,1\},$
$\{4574857237,1\},\{148625151254595761,1\},$
$\{24099222846777369521951265828190263955621853958610738869339858360058026377556935670492537667214901655216466740130714157900912047165583297561760273066063742738085354311145303873787486406987904751484142175230966723471892854846612781396372325409337677076728637755827834257604429390971694152770841219396396511278144391824649940093005920788044270012919 7241,1\}\}.$

(59)$7^{498}+1=\{\{2,1\},\{5,2\},\{13,1\},\{181,1\},\{997,1\},\{1077673,1\},$
$\{5715442173411036567996490596903946947789333428037605900617743 52$

6075595972156196240857472998444961169839581838344803041539749821209293
3674844904422952862337955372165667849404912458039251079138702444564196
5432561201230632566257457026684864301295041156572662235973961484578904
9031569952403070878080320707570403298351496983780683939254080749570323
8780839368294569580164678608794005643734312625492103006844629493, 1}}.

(60) $7^{506}+1=\{\{2,1\},\{5,2\},\{661,1\},\{1013,1\},\{1409,1\},\{3037,1\},\{26681,1\},\{173053,1\},$
$\{649981,1\},\{15513961,1\},\{83960385389,1\},$
{74369101960618279510690568163788139379303366253256229612637655 5
1610973660854319923976766230611889336008246241030630446416382691450144
7053990439443742113783919435105338376907343716117293534455632124927756
6841412134228766697356883353409623549357269625171979269795489906360869
8241234316260510758429795677117790697681047603096968969957910260915461
505501504254369823609276249897633524 1, 1}}.

(61) $7^{524}+1=\{\{2,1\},\{1049,1\},\{1201,1\},\{63929,1\},\{8306449,1\},\{183191479867795697,1\},$
{27669791790108677380160413405366641410277861427343950936522396 1
2836002199569711683699520371118976814342035357891832897719576179414832
6803950887027924851652369800607549871857865395875005228447390694399070
5162645529078030218068323821452037526128681733290880995526449244134605
0165899529462157566088421740109800960501386899505915139227519231761090
8332998057590913556135095817129784911204568679363226021472093857 7, 1}}.

(62) $7^{534}+1=\{\{2,1\},\{5,2\},\{13,1\},\{181,1\},\{1069,1\},\{2137,1\},\{1154656453,1\},$
{617342848774588830389301879846307895110724491110603268986697 70
3966187229860896854353144530707253120268528533455441225623951688026919
1903551352735441980853051426229640441268423914516986301794331414520246
5615711546305426746207699379312487541120270090919501704082489405763833
2453521865694037699244648921847645655083995040426846341885001699296741
2957045094702281113131125660186996135076585565695621229094386267984835
883866043322888372 1, 1}}.

(63) $7^{548}+1=\{\{2,1\},\{1097,1\},\{1201,1\},\{1384249,1\},\{56779677401,1\},\{3885538791193,1\},$
{161468536478521209631397377810414069933660635009246965996491189
0910038631971775428038603752556615673874564609815708431079172986471838
6381596762319101105858014315639239616774317149468978463381942297386449
1091460580999640371704003399543367777192017966195705197052094795386959
1218028508962895536411029907119047594364077087363068520781660975077340
6823884319994704883796025506301444311196333282587375785955363021573487
341884219626769, 1}}.

(64) $7^{554}+1=\{\{2,1\},\{5,2\},\{1109,1\},$
{275684670287677801721447274913248486262516103861123516308456547
4782139968524965006530354658458567611576378105861992799147922745435972
0242986855823222373353715408774267984182094021632877832797582096171549 3
7288557893803527800388066955283410889750491815713541492814002988139111
0380454919335107873412451751502526945083180410171837609849138158682311
3834130189985657412418101646802077246686728398162582361317879105644583
96503892885820526074585821920412167538664072507785 3, 1}}.

(65) $7^{590}+1=\{\{2,1\},\{5,3\},\{281,1\},\{1181,1\},\{4021,1\},\{149153,1\},\{165437,1\},\{1024477,1\},$
$\{109508800175101,1\},\{551659699804141,1\},\{1084026572852341,1\},$
{7339834198661211209928539097774832066760465901679171200405282 49
7556950634485161659210044658724146139462197411174468025069076024750769
3932278578399696036890464615491499564353959216233003356023520246302241
0079033498261465922786762264994404769710202556208182061978590841348095
8686406572210101817502685735684044766224196493831420561582898904621545

65460534259754668435017258685413357238109000646113919221578038876736583
39382021776177, 1}}.

(66)$7^{596}+1=\{\{2,1\},\{1193,1\},\{1201,1\},\{100129,1\},\{3668884217,1\},$
{45302744866656106619294907465068883144692783832143220591344645210522859079484052762361485801563670811343697312336175515190325126635009455788681671187756771863515750141274506976497581312523218062105130447653663677281179907260757111857203678995920563371331108014321276278978115083090778755875608280093099729421111196607586389607991288699058790460395978832573247174641760177398856883280185183393279607885477324739944605939019420795994882129385980772189663725432475288587776381828118124 9,
1}}.

(67) $7^{608}+1=\{\{2,1\},\{1217,1\},\{908353,1\},\{7699649,1\},\{134818753,1\},\{531968664833,1\},$
{540661960800636410432261551798363257159976259191748263657563339553165365673878924665806446502216015652358166986349825524326304370822727760441255530807625672555757765899904209815723803046817607605197551424608248222330406066611968883209902252487012128531362243702190524967488897649363364997271745966592055151070840363473248370654807584677969726111013566838573238591498589720784412177674407414202842264915644411362001088478203937113787374590418380380708914848216275126028678974720 1, 1}}.

(68)$7^{618}+1=\{\{2,1\},\{5,2\},\{13,1\},\{181,1\},\{1237,1\},\{2473,1\},\{3709,1\},\{51913,1\},$
{115361, 1}, {924529, 1}, {38774969, 1}, {122087137, 1},
{532928691060592778270264275079103462818997384124815158212559429293557699165214976173550788657447825385987773059461116768206375561031090683951069627216059655399906778398070813606279380009556916642021402709561258218954108546179610317988126311734203928731199885992737412639899815688482478073378288716364498711199809974260518034697147008739354336577431644205290558090662298485021128701497684210958512428319188352446202501483115594586899110597972966796470850438274794269310738501 37, 1}}.

(69) $7^{624}+1=\{\{2,1\},\{97,1\},\{353,1\},\{1249,1\},\{4993,1\},\{3307201,1\},\{41237249,1\},$
{104837857, 1}, {47072139617, 1}, {1409781090433, 1},
{541367896596308901297974565812641949903548092953130254763475930876818333931109796396329560841324257560720229181632093393454793073640056772848732381105437702440625029465062113189979519681933043173123250636979721147079471755655112044744493411291304200426743275301755234645334581403907275134879799343635006048686648469889107334645549011811755633046986672071859901980985410861109627581133224240080580203379820412658339303643829195774735517205430416292897602969675169668119201, 1}}.

(70) $7^{638}+1=\{\{2,1\},\{5,2\},\{233,1\},\{661,1\},\{1277,1\},\{1409,1\},\{1984181,1\},$
{136853089, 1}, {83960385389, 1}, {55716067510309, 1},
{845336231282845839193231315417379311551764282289934245445374003594876427715724889140252901840571218750081326694640793208980294026154480793003032673472960635778199165471516599583427307122613276859907482613387310660172992106612071349533539791038357880340820368566794519520813307669772639866576323190648606323213481542741771868736020749652350169390254469231251035717802651838123090362476681062900499628536853619065682040445985970034766297864595125841073043721041049124805754373371689054
2149, 1}}.

(71) $7^{650}+1=\{\{2,1\},\{5,4\},\{101,1\},\{157,1\},\{281,1\},$
{1301, 1}, {4021, 1}, {13001, 1}, {25301, 1}, {134358901, 1},
{1599176476336052193015740438886156806830047257113343861868318582327574700182265918333738395283244292790015993235028081076741688614751

2604119351725778584068760639824227908938897350500099258222668153868633
4524314361133165194069770767600215223575755160260286992976988693389546
1193347502516675639658581052196573990371599632392863764502396314419657
3725361233066079506280676635089090907195246485743968222889287447444402
6607238051847333614830215680746271746544855195638556772330058235684245
5910169625330567548519139647670409, 1}}.

(72) $7^{680}+1=\{\{2,1\},\{17,2\},\{881,1\},\{1361,1\},\{169553,1\},\{542081,1\},\{2354161,1\},$
$\{173539292561,1\},\{5388077170097,1\},\{49853897949169,1\},$
{664001044859779160166480957521739378730361653192287148213517070
3101731457401821114876867047922053708439137967566363537398888006051166
1291067988032091394502403720190532574240797476384808048205578456561059
6290632610982925760267734050538063774615898336288556680729114815754381
3397703692087067005345214743887634859863364841156164876173468961133732
3802862640834148351873958567880764170121592197272018509084166657595871
3727913639528631866222316229030555403081152573879670169474714329270825
4973537383319241108034968081, 1}}.

(73) $7^{1068}+1=\{\{2,1\},\{73,1\},\{193,1\},\{409,1\},\{1201,1\},\{2683661857,1\},$
{988103554184969480814372452535171515127558675953793259908534715
6993889803509846103836387289744340241860992203459584611128386805287422
9548463921663466052112961551029166716843971614632646077460204268094400
9597783806520335927176836962425990492142333193100478687853536791088668
4227385410807561525134885758522089921457143852217518773389405130119595
6873415275998384552748908327684684972200595453417310967500761993071303
8012051858512068072011691907361024969501022641550992235850275122520900
0019787661044182759562583346932911405148692695467876927983003335256912
9302226707177699544810972352592278600473565623148698563999561528428778
2814823488666573644881618152853125505090917646376482173480570668693518
6570771274666610390151981540807856254966814210402953067646498827335321
8042502668574828271919607408558899747431772697766199275730427112812650
59741517172685945796785744633674435271649156631393, 1}}.