MEASURING THE MUSTARD SEED: AN EXERCISE IN INDIRECT MEASUREMENT AND MATHEMATICAL MODELLING

Authors: *Amit Dhakulkar, Nagarjuna Gadiraju, Samir Dhurde*

Brief Description: We designed a simple teaching/learning activity from real life using commonly available mustard seeds as an introduction to mathematical modelling, using a cluster of trans-disciplinary skills and concepts.

Abstract:

As a first exercise for middle or high school students in indirect measurement using physical and mathematical modelling, we present here a simple task where students are asked to find the average diameter of mustard seeds. The resulting observations lead to a simple linear mathematical model which has accessible physical basis from the real world. This simple task also provides a rich opportunity and a context to learn several topics in measurement, modelling, graphicacy and statistics. We present this as a template to be used for developing a series of activities for learning indirect measurements, physical and mathematical models.

Key Words: mathematical modelling, real world data, graphicacy, indirect measurement, graphs,

Introduction

After students learn about measuring dimensions in real life using direct methods, it is important that they learn how to measure very small and very large dimensions or dimensions that are not directly accessible, like large trees, the distance between the Earth and the Sun, or very small objects like microbes, pollen, or the thickness of a hair. Under these situations, scientists use *indirect measurement techniques.* Indirect measurement often requires the use of alternative physical models and the corresponding mathematical models. For example, in order to measure the height of a building, the alternative physical model may be an analogous triangle, and the mathematical model would be the equations describing the proportionality. In some cases, even the physical model may not be amenable to direct experience or direct measurement. In such cases mathematical models provide a way to measure the required dimensions that often depend on some assumptions and approximations about the physical world,

In order to prepare students to understand this core aspect of scientific literacy, we have designed learning contexts that help students understand physical and mathematical models. This paper describes one such preliminary exercise in physical and mathematical modelling, in which both types of model are directly amenable to experience and measurement. The task is designed to provide discussions on a variety of concepts as shown in the mind map in Figure 1.

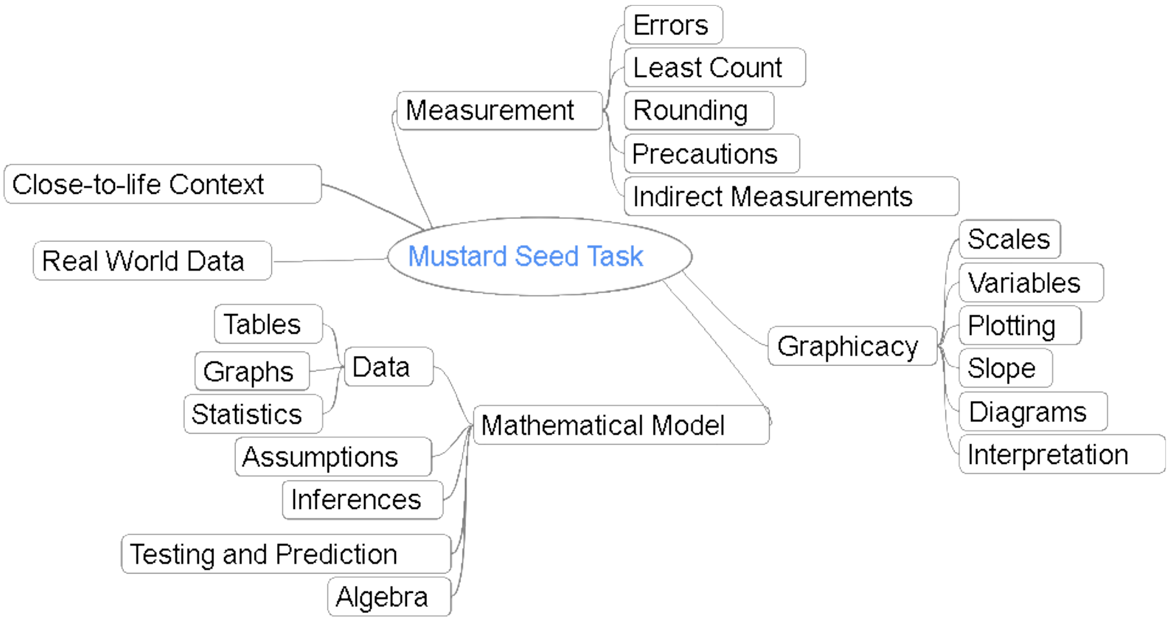


Figure 1: A cluster of interrelated topics and skills that can be learned through the mustard seed task.

Mustard seeds

The Indian kitchen is a versatile place where many spices and ingredients mingle to produce a variety of cuisines. Though each part of India has a unique style of cooking, there are many things that you will find in all kitchens. One of them is the mustard seed, of the family *Brassica*. In many of the cuisines the mustard seed is a must, to give a *tadka* or flavour to the food. In many Indian languages the mustard seed is metaphorically used to denote something of a very small size. There are three varieties which are commonly found, and all of them are so small that they are difficult to measure with a scale. Two varieties (*B. juncea and B. alba*) have seeds almost double size of the other one (*B. nigra*), as shown in Figure 2. Variation in the size and shape in the seeds of the same species can also be seen in this figure. These two characteristics of the seeds provide for opportunities for classroom discussion as we will see later.

Figure 2: Commonly found mustard seeds, and their sizes. From left to right Brassica juncea, Brassica alba and *Brassica nigra.*

The designed task was field tested in three consecutive summer camps for class VIII students (age 13-14 years) from a variety of urban Indian schools. The task was performed over two days. On the first day a few different possible approaches to measure the diameter of the mustard seed were discussed with the students. The students were to do actual measurements of seeds in their homes. Also, how and why a mathematical model (both algebraic and graphical) may fit the observations was discussed. Students were asked to write detailed reports on the task.

On the next day data was collected from all the students, and plotted on a projector. An interesting pattern emerged from this collaborative plot. The mathematical model was again discussed with the data from all the students which helped in understanding the physical meaning of terms in the mathematical model. In what follows we narrate the significant events when interesting learning took place to highlight how a simple task can be so rich in its learning outcomes.

Warm-up Discussions

All the students were familiar with mustard seeds. Two examples which are similar to the mustard seed task involving indirect measurement were discussed in the camp. One of the tasks was the indirect measurement of the width of a thread. This is usually done by winding the thread on an object (for example a pencil) and finding the width for a given number of turns. We would get the average width of the thread by dividing this length by the number of turns. The second task that was discussed was how to find the thickness of one page of a book. This task also involves measuring the thickness for a given number of papers and then the average thickness is found out by dividing the length by the number of pages.

After either of these two warm up discussions the students were asked to guess the approximate diameter of one mustard seed. For this purpose some mustard seeds (*B. juncea*) were shown to them. During the discussions that ensued the students came up with guesses from a few millimetres to a few centimetres.

How to measure the mustard seeds

In the next part of the discussion the students were encouraged to come up with ideas for measuring the diameter of a seed. Some of the students came up with some ingenious methods of measuring the diameter. One of the students suggested that a thread should be wound on the seed, and then the length of the thread can be measured easily with help of a ruler. Another student suggested an even more elaborate method: we can find out the volume of displacement of water due to one seed and then from the volume of the water displaced we can find out the volume of the mustard seed and from this volume we can find out the radius and hence the diameter. The students were already familiar with the properties of a circle (area and circumference) and a sphere (volume), which are to be used in the above two methods. One of the students said that a divider from a geometry box could be used to hold the seed, and then the distance between the points of the divider could be measured on the scale. Yet another student rolled a seed in a piece of paper, and measured the diameter of the roll. Despite the discussions in the warm up on similar tasks it was interesting to note that it never occurred to any student to use more than *one* seed in the measurement. This may indicate the transfer of mathematical knowledge from one context to another is often not easy.

The teacher asked the students to look at the mustard seeds and to tell whether all of them were of exactly the same size and shape. As we can see from Figure 2, there is a variation in size and shape even in the same species and this was noted by the students. They said that the seeds were not of the same sizes, some were larger and some were smaller. Some of the students responded to this trigger of variation in size by saying that an *average* of many values needs to be taken. Thus the rationale of doing multiple measurements on different seeds and taking averages of the readings was brought in. This discussion helped the students to realise almost on their own that variation calls for measuring the averages.

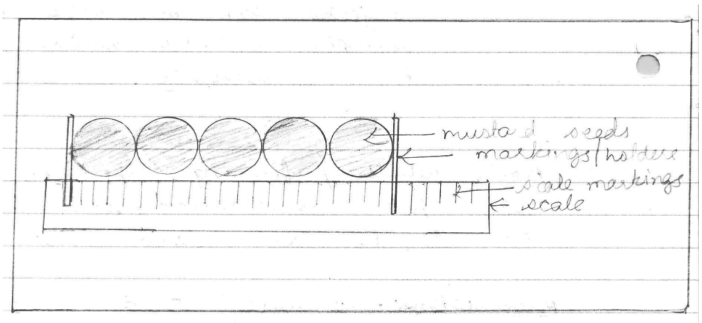
After this they finally decided to use a scale (smallest scale division is count 1 mm) to measure the average diameter. In the discussion that followed the procedural details of the task were worked out. The method involved aligning a number of mustard seeds along a scale and measuring the length covered by them. Then the average diameter for each one of the sets of seeds was found. The precautions to be taken were discussed. Figure 3 which shows a photograph and a drawing made by one of the students to explain the procedure. The students were guided to take measurements of sets of 5, 10, 15, 20, 25 and 30 mustard seeds, though some students went up to 40 seeds. After this, possible ways of making a mathematical model from these observations was discussed along with the assumptions that were required for such models. The discussion on modelling was further elaborated on the next day, with graphs and will be discussed later.

Peculiarities in the Reports

The students submitted written reports which were supposed to include the procedure, precautions taken, possible errors, observations, tables, pictures and conclusions. In their reports, the students were also to plot a line graph for sets of each number of seeds versus the total length that they measured on the scale.

The observations on the different sets of the seeds were recorded in a table. The table had data in the following format: Number of seeds - Length in mm/cm - Average diameter for 1 seed. Some students used calculators to get answers up to 4 decimals. This provided an opportunity to discuss significant numbers, the concept of least count, and rounding off in the class on the next day. The most commonly

(a)



(b)

Figure 3: (a) A photograph showing the placement of 5 seeds along a scale (b) A drawing by one of the students. Placeholders which hold the seeds together can be seen in the drawing.

reported error was of the alignment and placement of the seeds with the scale. This error was the most irritating for some students although it was fun for others. Although, the students did not realise it, the problem was aggravated at times by the presence of static electric charge on the seeds. One of the students actually glued the seeds on the paper to overcome this problem!

Plotting

Almost all of the students who drew the graph, could plot the data points correctly. Not all of them drew best-fit lines through the points that they had plotted. Some of the students drew the graph on a plain paper rather than using the graph paper they had been given. Only one student drew both a bar graph as well as a line graph. The students were not given specific instructions in choosing the scales, but they were asked to write the scales on the graphs that they drew. While most students chose a scale of 5 seeds per unit for the X-axis, various scales were used for the Y-axis. Many students chose the same scale as the actual readings, with 1 mm on the graph paper being equal to 1 mm of the actual measurements (Figure 4). One of the students plotted the values of the average diameter that was obtained from the measurements against the number of seeds in each set.

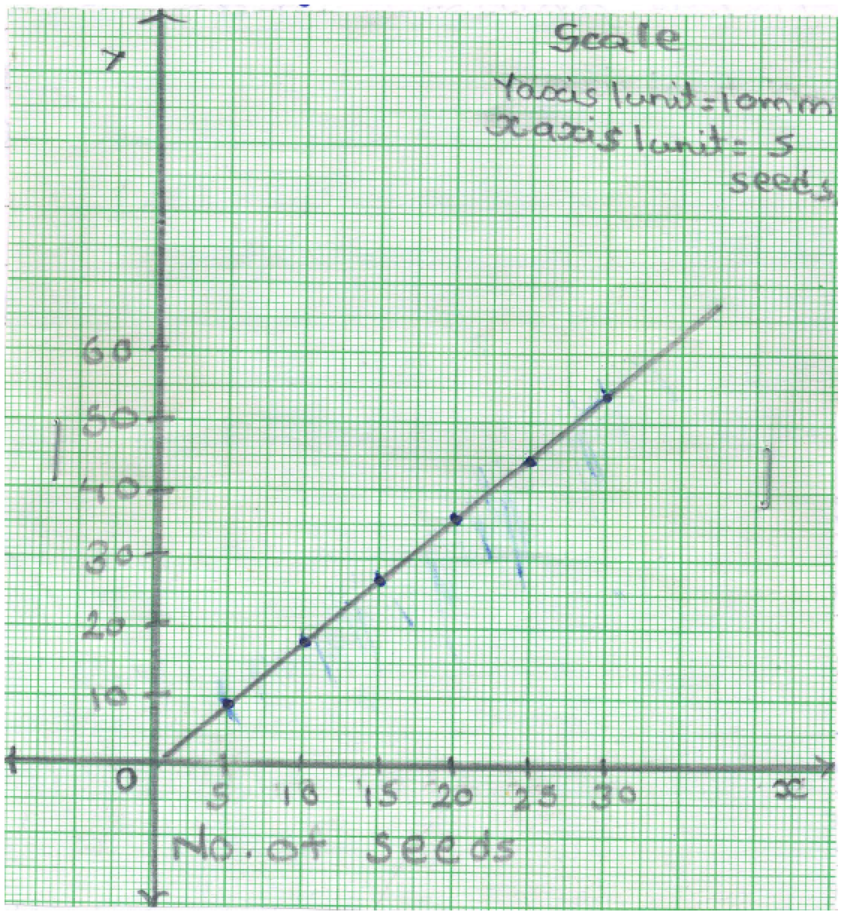


Figure 4: Graph one student made of number of seeds vs. the length. Here the Y-axis has scale 1mm = 1mm and X-axis has scale of 5 seeds = 1 cm.

Modelling

On the next day the students came with their written reports and they were asked what average diameter they had found. Their answers varied, sometimes more than double. Through some probing on why the answers varied, it emerged that there were in fact two different varieties of mustard seeds. This students did not realise initially, because each of them had only *one* of the varieties at their home, and so had the data only for one type. This provided us with an opportunity to discuss what the existence of two sizes would mean for the mathematical model.

During the discussion the students were asked if there are any mathematical relationships between the number of seeds and the lengths that they had measured for them. First of all it was noted that as the number of seeds increases, the total measured length also increases. So, it was agreed upon that the measured length of the sets of seeds L is directly proportional to the number of seeds n:

$$L \propto n \qquad (1)$$

After agreement that these two quantities are in direct proportion the discussion was taken further by introducing a proportionality constant d. Hence the mathematical relation between the two quantities L, and n can be written as

$$L = d \times n \qquad (2)$$

At this point the students were reminded of the straight line equation

$$Y = m \times X \qquad (3)$$

where m is the slope of the line. We then compared the two equations for similar terms. The total length L and the number of seeds n in equation (2) is analogous to the Y and the X values respectively in equation (3). The proportionality constant d in equation (2) can be seen analogous to the slope m in equation (3). All this leads to the fact that equation (2) is indeed an equation for a straight line.

After showing that the mathematical relationship that is expressed by equation (2) is a straight line, it explains why we can draw a reasonably straight line passing through all the points.

Collaborative Plotting

To underline the understanding of the slope of the line and its meaning in the mathematical model, a collaborative exercise was done on the second day. The length of each set of seeds (5 seeds, 10 seeds, 15 seeds, ... etc.) were collected and displayed on a spreadsheet in GeoGebra a dynamic mathematics software (GeoGebra 2013). After this the average lengths of each of these collected values were plotted against the corresponding number of seeds in each set (Figure 5). When the points were joined, two distinct lines emerged, corresponding to each type of seed. Why did we get two distinct lines? Discussion followed on what is the physical meaning of the slope of the line. In this case we have a concrete physical observation that the sizes (diameters) of the two types of seeds are different. On the other hand in the mathematical model, the slopes of the lines are different. Thus we can relate the abstract change in the slope of the mathematical model to a concrete observation regarding the size of the mustard seed. This point was discussed at length in the camp. It was helpful to use GeoGebra in order to visualise how the lines would have looked if the slope was different, meaning if the size of seeds was different. For example if the seeds had an average diameter of 0.5 mm or 3 mm, where would the lines be with respect to the lines drawn. This way the use of graphs for understanding the meaning of the slope in terms of associated lengths was made clear. The students were also asked to plot the collaborative average on their own graphs. Drawing this 'average plot' shows how much deviation the students readings have from the average values. This led to interesting discussions about different aspects of statistics like averages, standard deviations and need to take multiple measurements.

Predicting

Another use of graphs in the context of modelling is their use as calculating/predicting devices. The students were shown with example in GeoGebra, how to find the length for a given number of mustard seeds. For example to find the length for set of 50 mustard seeds in a row, we need to take a line which is parallel to the length axis (Y), and find the point of intersection of this line and the line made from observations. Similarly, we can also find the number of seeds, if we know the length for the set of seeds, by using a line parallel to the number of seeds axis (X). The students were asked some practice questions for these types of predictions. They solved these questions using both the algebraic and the graphical method, and compared the results with actual observations (for a given number of seeds). The results were in agreement between the methods and the actual observations, within the error margins.

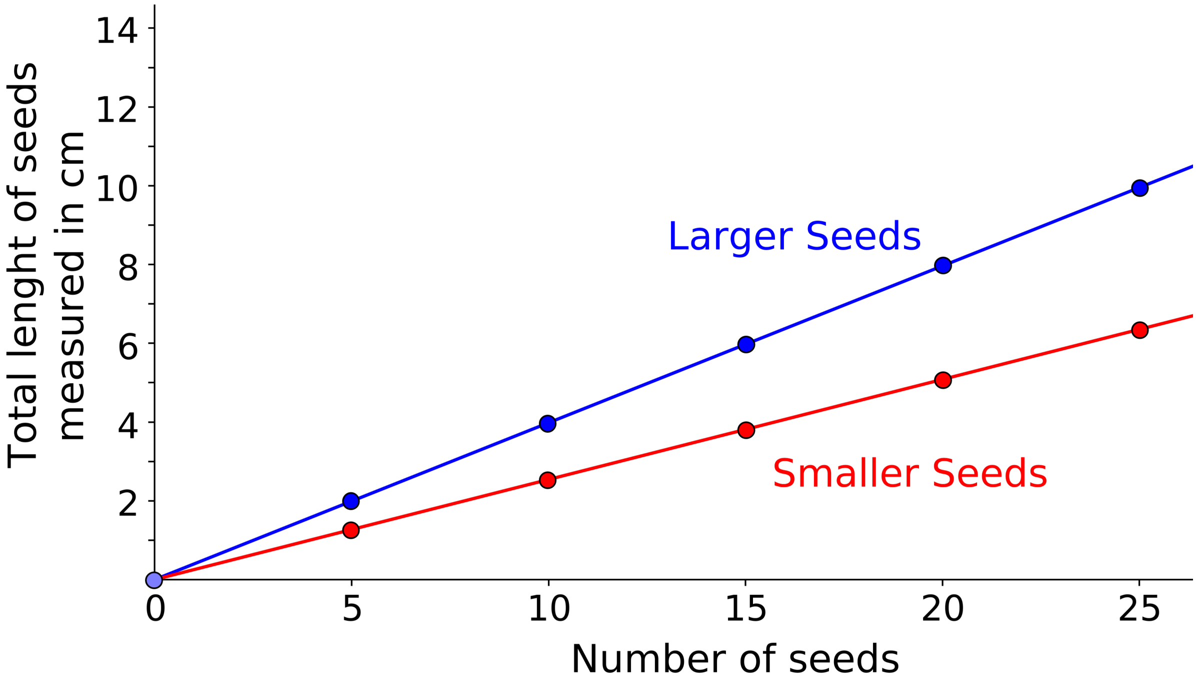


Figure 5 The graphs of lines for two different types of seeds drawn in GeoGebra from the average readings by the students. The difference in their slopes is linked to the difference in the size of the seeds.

Conclusion

The activity described in this paper is a part of a larger work on developing critical graphicacy skills. G*raphicacy* is defined as the ability to understand and present information in the form of sketches, photographs, diagrams, maps, plans, charts, graphs and other non-textual, two-dimensional formats (Aldrich & Sheppard, 2000). In the book *Critical Graphicacy* Roth et al. take a stance that "our aim as critical educators is not just the provision of opportunities for students to become graphically literate; rather, we want students to develop critical graphicacy, that is, we want them to become literate in constructing and deconstructing inscriptions, the deployment of which is always inherently political." (Roth, Pozzer-Ardenghi, and Han, 2005). In our study on Indian textbooks we found that the presence of graphs is limited and opportunities to use them are almost non-existent (Dhakulkar and Nagarjuna, 2011). In such a scenario it is important that such opportunities are provided to the students. In this case the emphasis was on using and understanding graphs in the context of mathematical modelling of real world data and measurements. This can be seen as a first step in the direction of making students graphically critical and literate. This task also provides a concrete context for the students to use multiple symbolic systems (tabular, graphical and algebraic in this case) and to understand the relationships between them.

While designing this activity we followed following principles: the learning context should be close-to-life; the constructed model should be abstract enough to find applications in multiple places; it should provide opportunities for linking several trans-disciplinary skills; it should be easy to do in the classroom; and it should not consume too much time.

Though the task and the model were simple, not all the students could come close to the expected result. Some students could not go on to make the mathematical model on the first day. Only after the discussions on the second day they could do so. This shows that bridging the gap between abstract mathematical knowledge and the real world is not trivial. By making students aware of the fact that the *same* mathematical model can describe different objects, one can perhaps hope to overcome this problem.

It is vital to bring to the classroom tasks, which are simple, but rich enough to raise discussions of several interrelated concepts in a close-to-life context. Other tasks such as measuring the thickness of paper or the diameter of a thread can be done in continuation to this task. This would emphasise the power of mathematical modelling to the students: that using the same general linear model, we can model systems which are not similar to each other. A few such experiments can act as a spring board to scientific modelling, and would help the students find the links between the models and the real world.

Acknowledgements

We acknowledge the inspiring Arvind Paranjpye and the interactions with him about the "mustard seed puzzle" that he poses to students. Authors are also thankful to Karen Haydock for helpful comments on the article. We also appreciate the help of summer interns at IUCAA, for their diligent help in data collection. The study would not be possible without the facilities provided by IUCAA.

Authors:

Amit Dhakulkar is a research scholar at Homi Bhabha Centre for Science Education, TIFR, Mumbai India.

Email: amit@hbcse.tifr.res.in

Nagarjuna Gadiraju is a faculty at Homi Bhabha Centre for Science Education, TIFR, Mumbai, India.

Email: nagarjun@gnowledge.org

Samir Dhurde is incharge of Public Outreach Program at Inter University Centre for Astronomy and Astrophysics, Pune, India

Email: samir@iucaa.ernet.in

References

Aldrich, F. K., & Sheppard, L. (2000). ‘graphicacy’: The Fourth ‘R’? *Primary Science Review*, *64*, 8–11.

Dhakulkar, A., & Nagarjuna, G. (2011). An analysis of graphs in school textbooks. In S. Chunawala & M. Kharatmal (Eds.), *Proceedings of epiSTEME 4 - International Conference to Review Research on Science, Technology and Mathematics Education.* India: Macmillan.

GeoGebra (2013) Dynamic mathematics and science for learning and teaching. Visited May 2013 URL: http://geogebra.org

Roth, WM., Pozzer-Ardenghi, L., & Han, J. Y. (2005). *Critical Graphicacy (Science and Technology Education Library)* (Vol. 26). Netherlands: Springer.

Word Count: 3313 (Excluding Figure captions on this page.)

Postal address for correspondence:

Amit Dhakulkar

Homi Bhabha Centre for Science for Science Education,

Tata Institute of Fundamental Research,

V. N. Purav Marg, Mankhurd

Mumbai 400088

Phone (O) +91-22-25072224

Email: amit@hbcse.tifr.res.in

Figure Captions:

Figure 1: A cluster of interrelated topics and skills that can be learned through the mustard seed task.

Figure 2: Commonly found mustard seeds, and their sizes. From left to right Brassica juncea, Brassica alba and Brassica nigra.

Figure 3: (a) A photograph showing the placement of 5 seeds along a scale (b) A drawing by one of the students. Placeholders which hold the seeds together can be seen in the drawing.

Figure 4: Graph one student made of number of seeds vs. the length. Here the Y-axis has scale 1mm = 1mm and X-axis has scale of 5 seeds = 1 cm.

Figure 5: The graphs of lines for two different types of seeds drawn in GeoGebra from the average readings by the students. The difference in their slopes is linked to the difference in the size of the seeds.