Relational E-Matching

$$= \subseteq R \bowtie S$$

senior presentation 

Yihong Zhang

ongoing collaboration with  

E-graphs are everywhere!

E-graphs are everywhere!

Spores

Herbie

egg

Glenside

TenSat

Szalinski

Z3

Metatheory.jl

Diospyros

CVC4

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Metatheory.jl

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. . .

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Equality saturation

- Stores many expressions in a single data structure.
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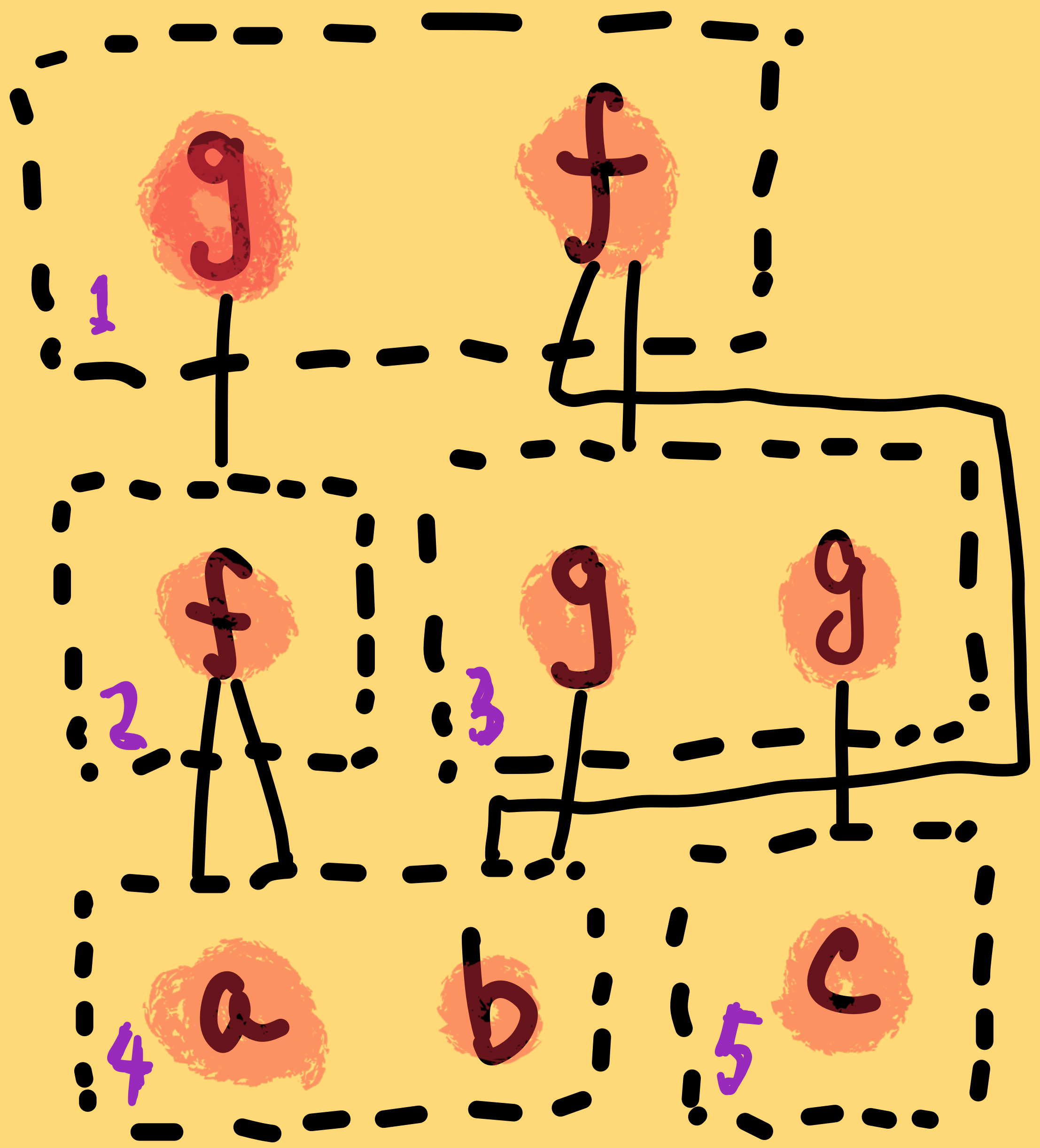
SMT Solver

- Used in the solver for the theory of equality with uninterpreted functions.
- The "glue solver"

E-graphs

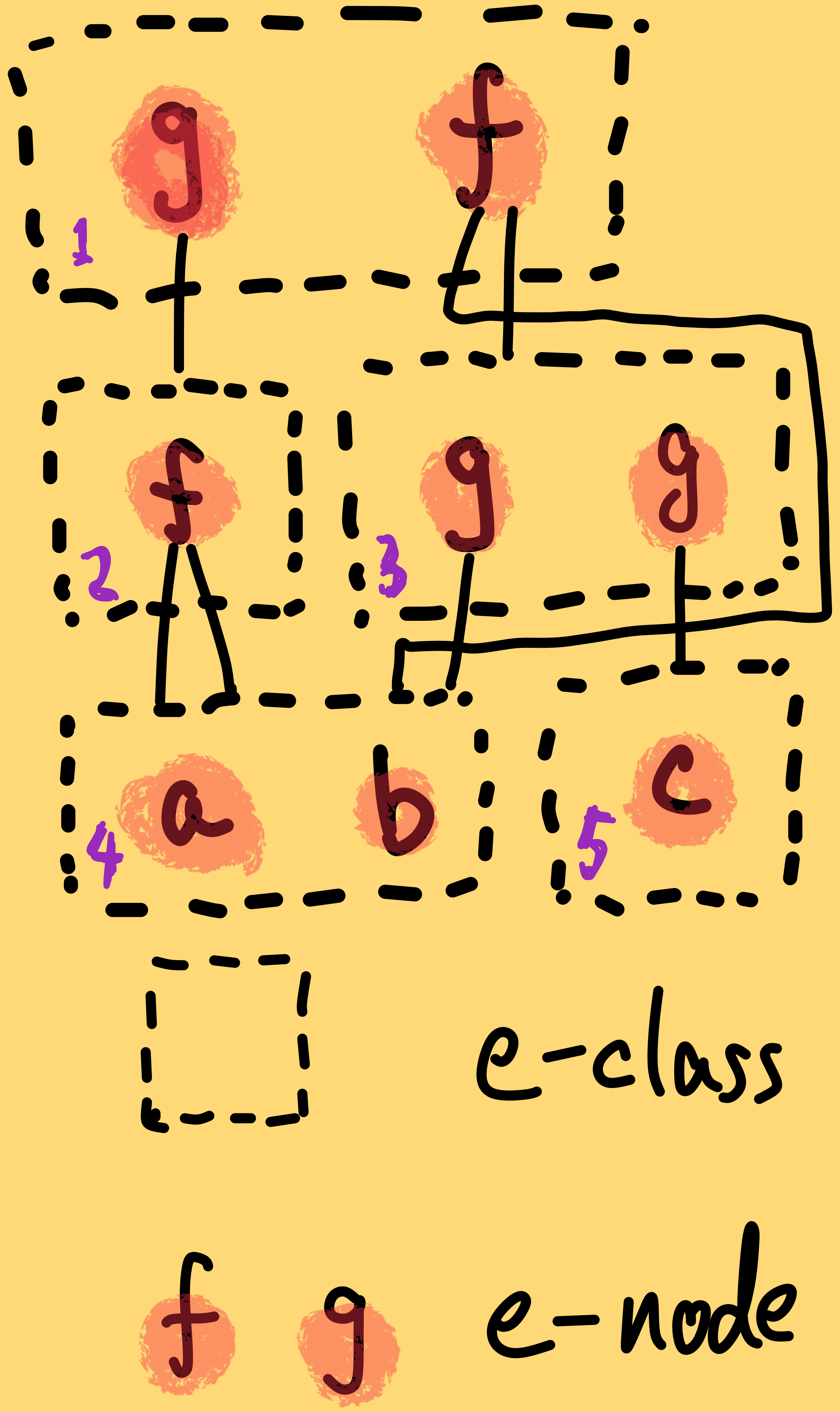
E-graphs



e-class

e-node

E-graphs



e-class 1 represents

$g(f(a,a))$ $f(a,g(a))$

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$f(a,g(c))$

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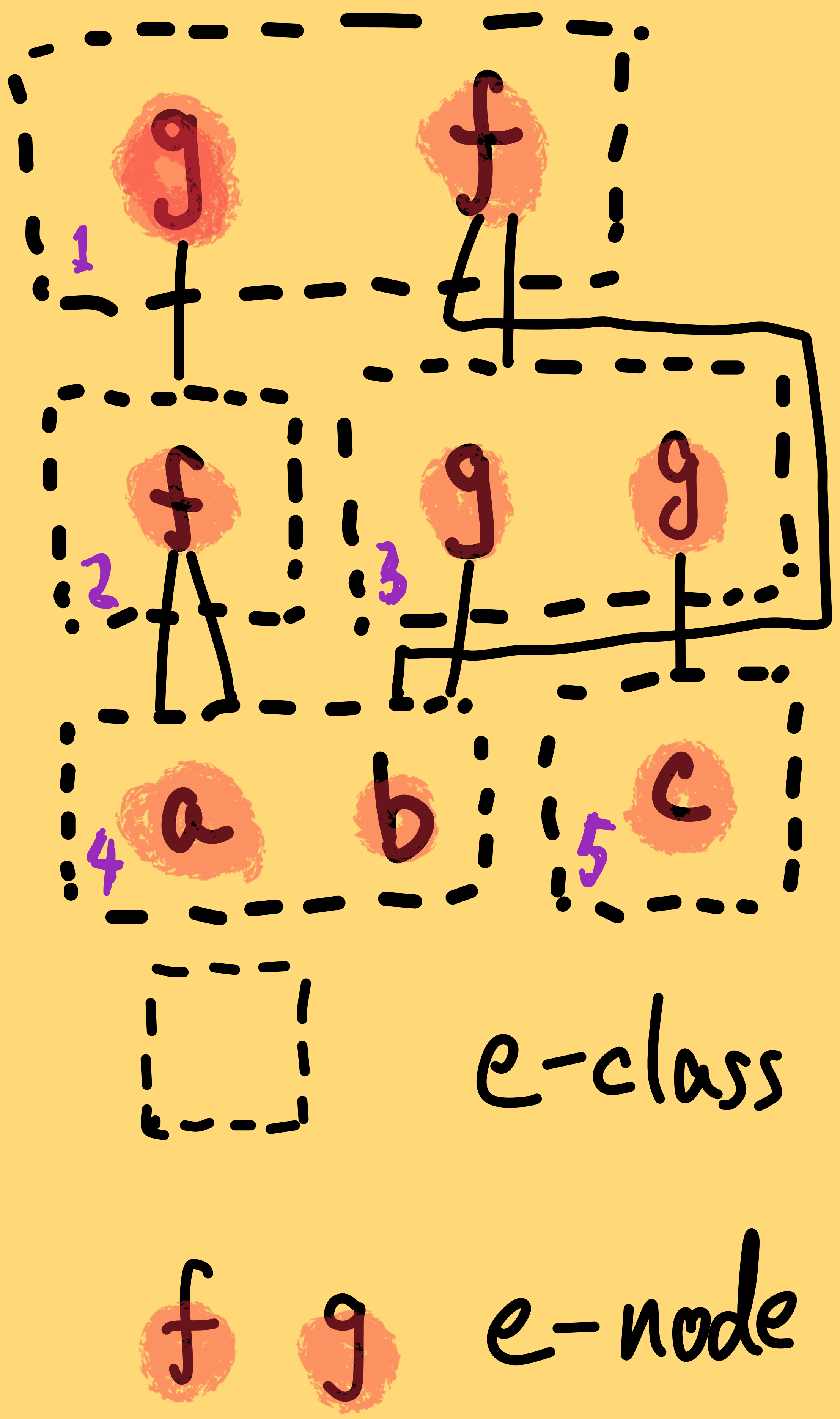

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f(a, g(c))
f(b, g(c))

exponentially many terms!!

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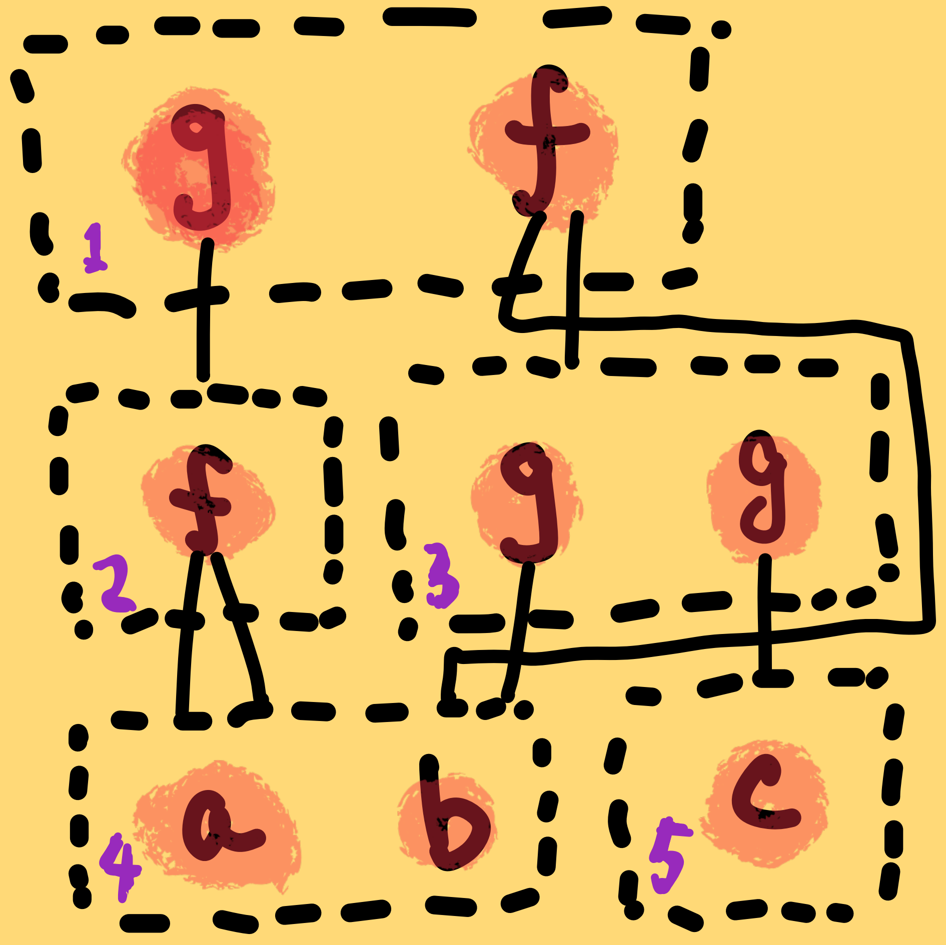

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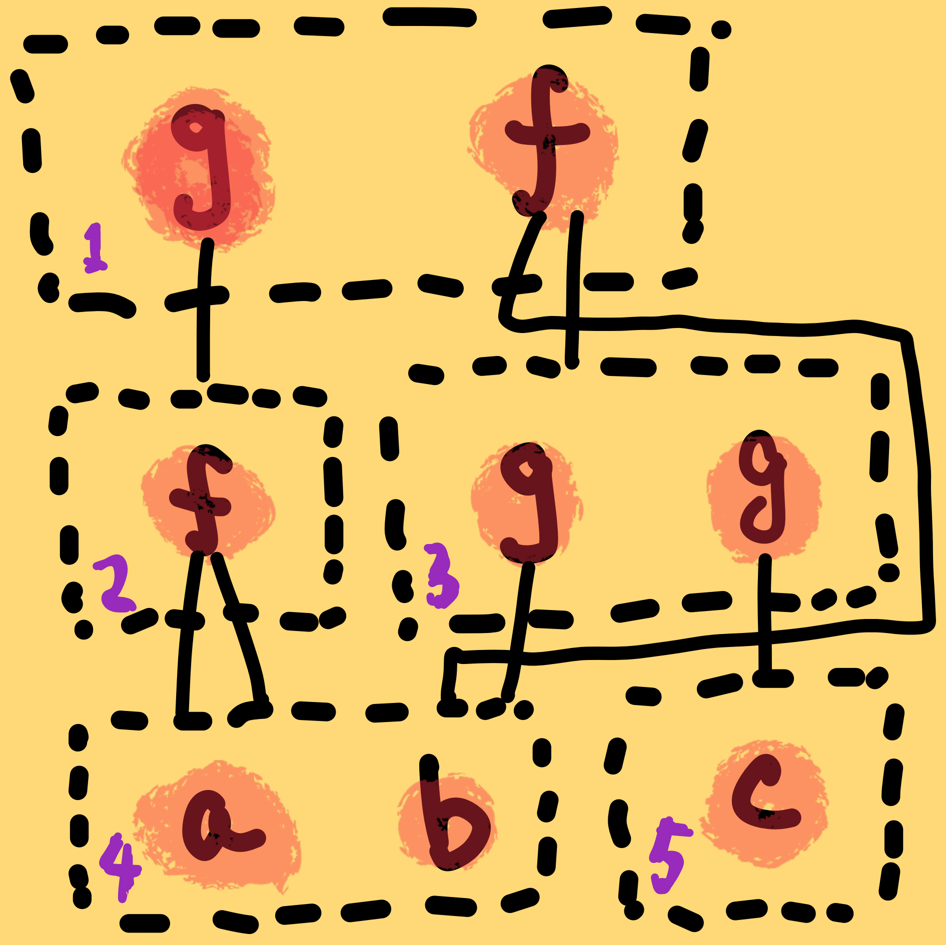

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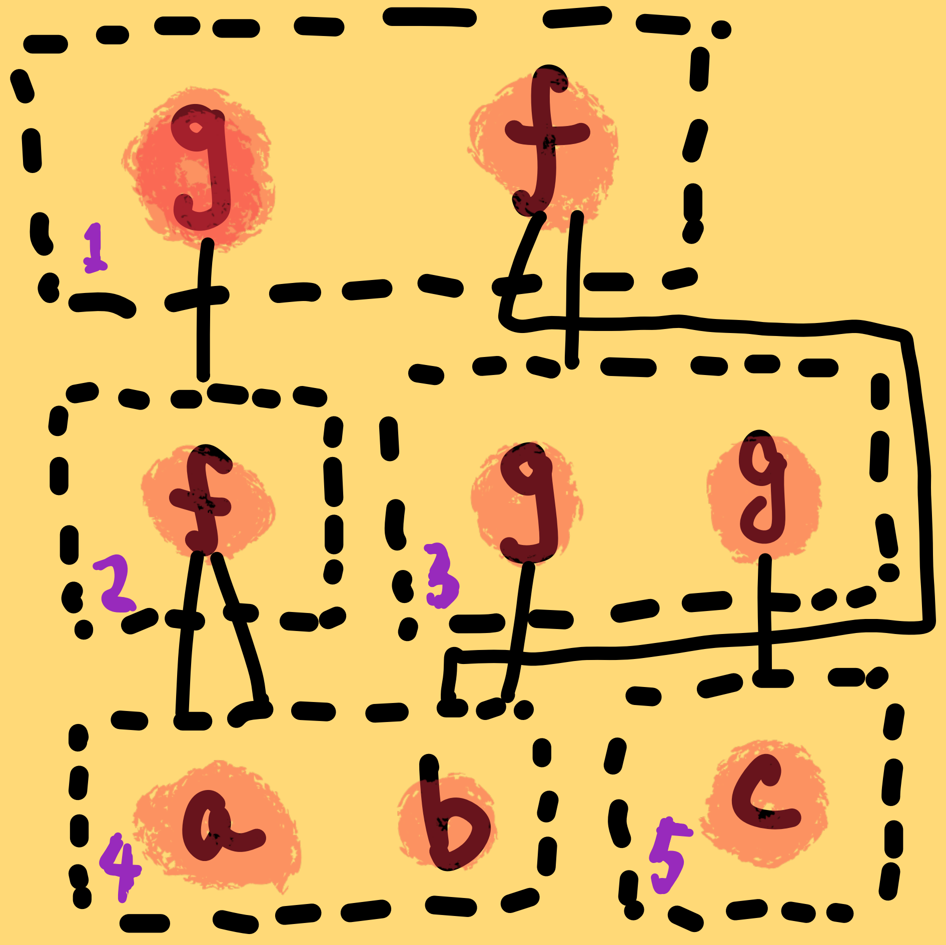

pattern $f(\alpha, g(\alpha))$ corresponds to

$f(a, g(a))$, $f(a, g(b))$

$f(b, g(a))$, $f(b, g(b))$

all witnessed by subst. $\{\alpha \mapsto 4\}$

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Bottleneck!

Existing e-matching algorithms

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Existing e-matching algorithms

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$\underline{\underline{O(n^2)}}$

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We can do better!
(despite the NP-completeness)

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Finding substitutions such that the relational database contains all the instantiated atoms of the query.

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e.g. $Q(a,c) :- R(a,b), S(b,c)$

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$$\subseteq$$

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E-graphs $\subseteq$

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E-graphs	$\subseteq$	Relational databases
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Simpler!!

Relational e-matching

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- Takes an e-graph and a list of patterns.

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- Run the conjuctive queries on the relational database!

Relational e-matching

well suited for equality saturation

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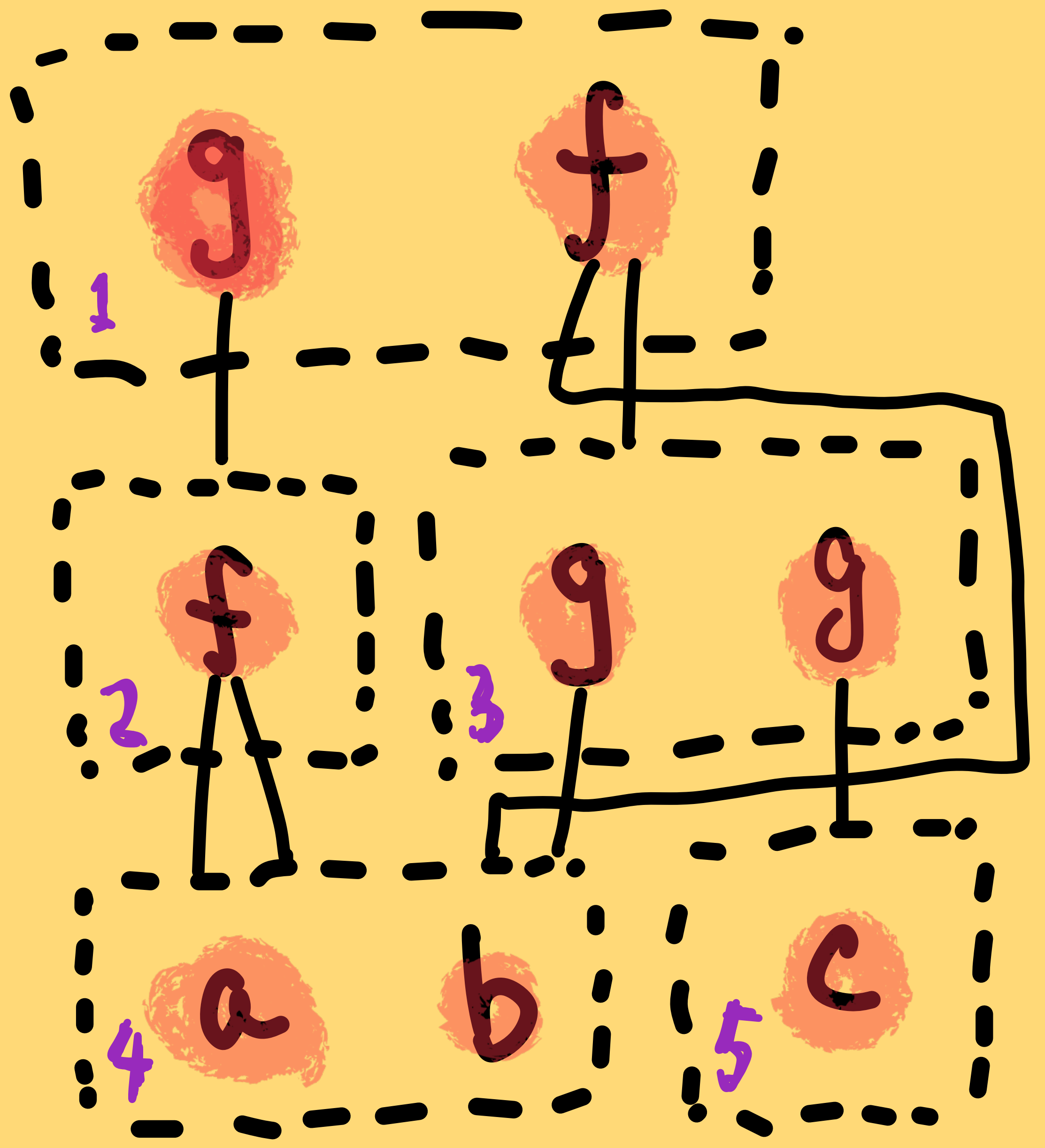
- Run the conjunctive queries on the relational database!

E-graphs $\rightarrow$ Relational database

E-graphs → Relational database



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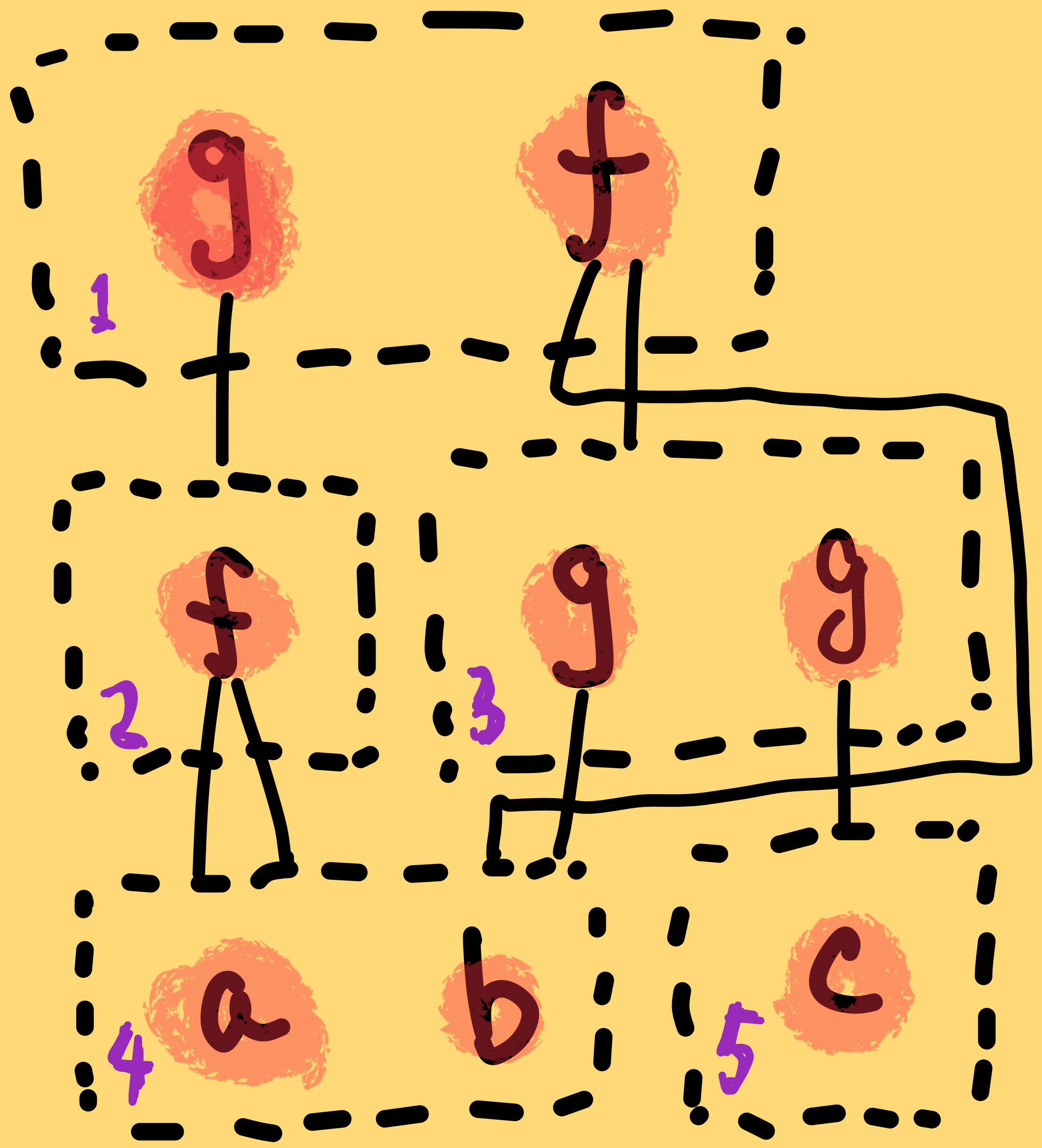

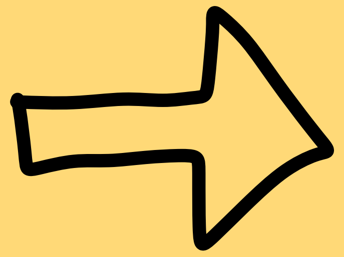

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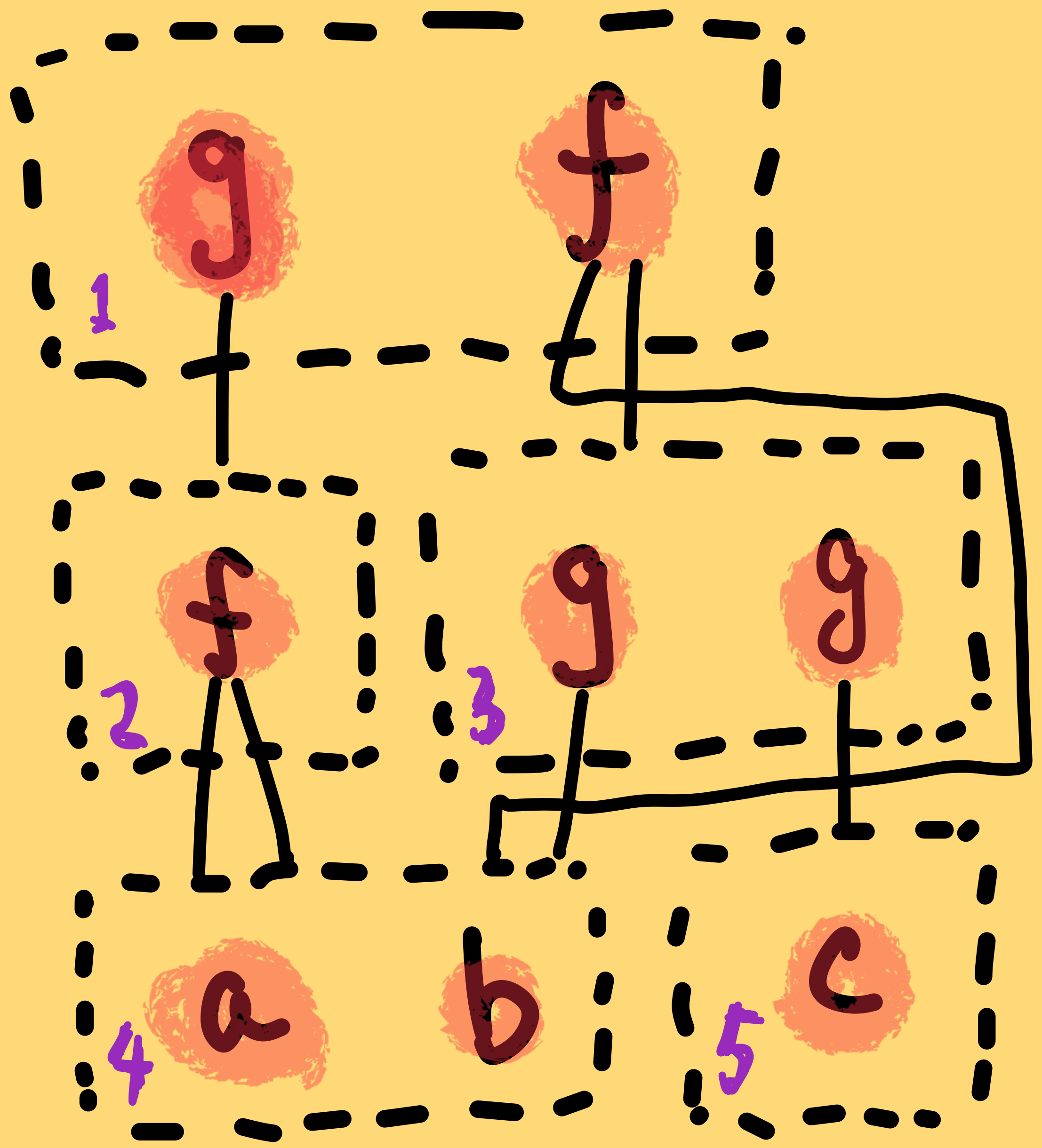

R_f

eclass-id	$child_1$	$child_2$

schema for function symbol f

R_g

eclass-id	$child_1$

schema for function symbol g

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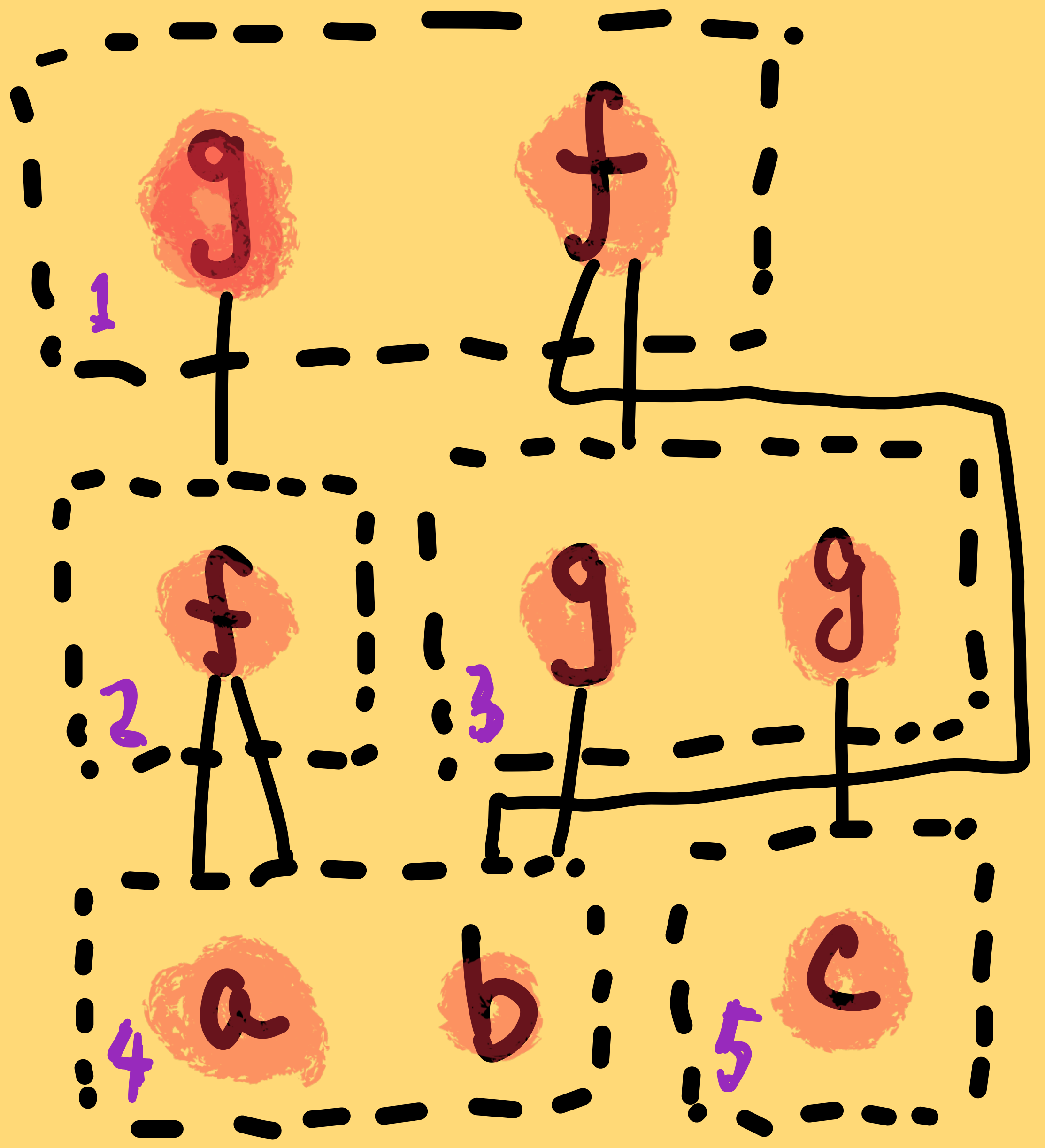

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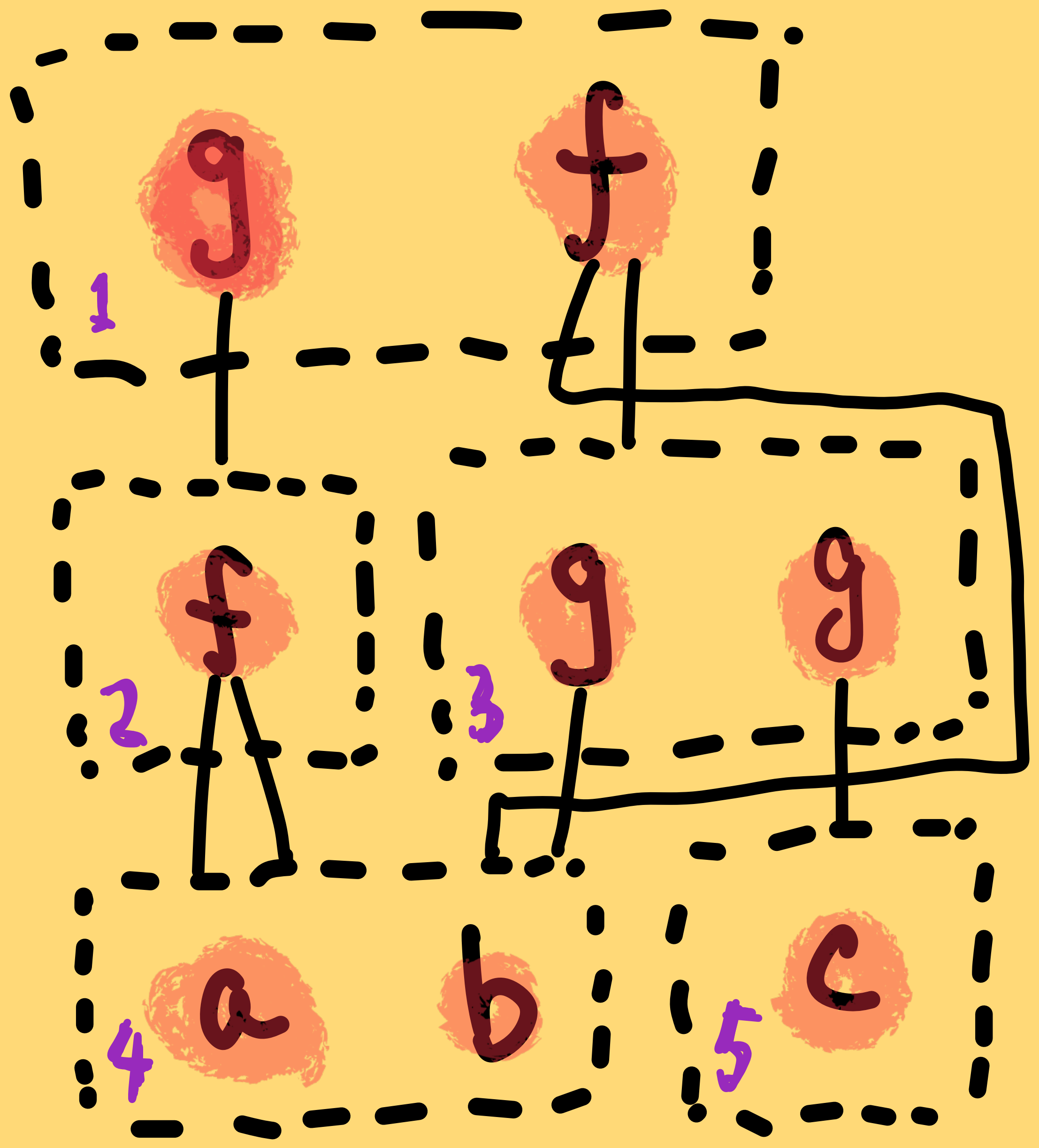

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3	5

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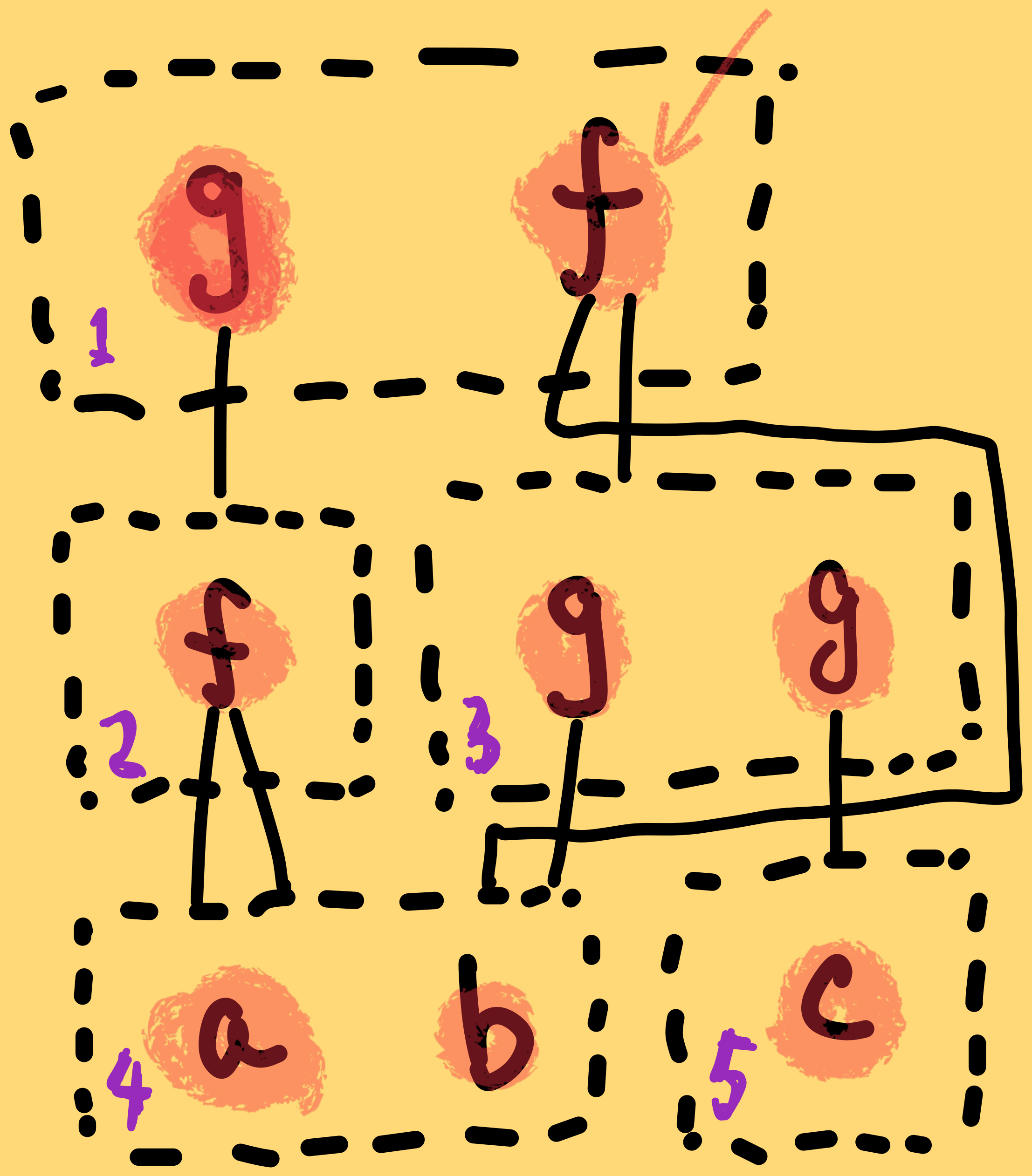

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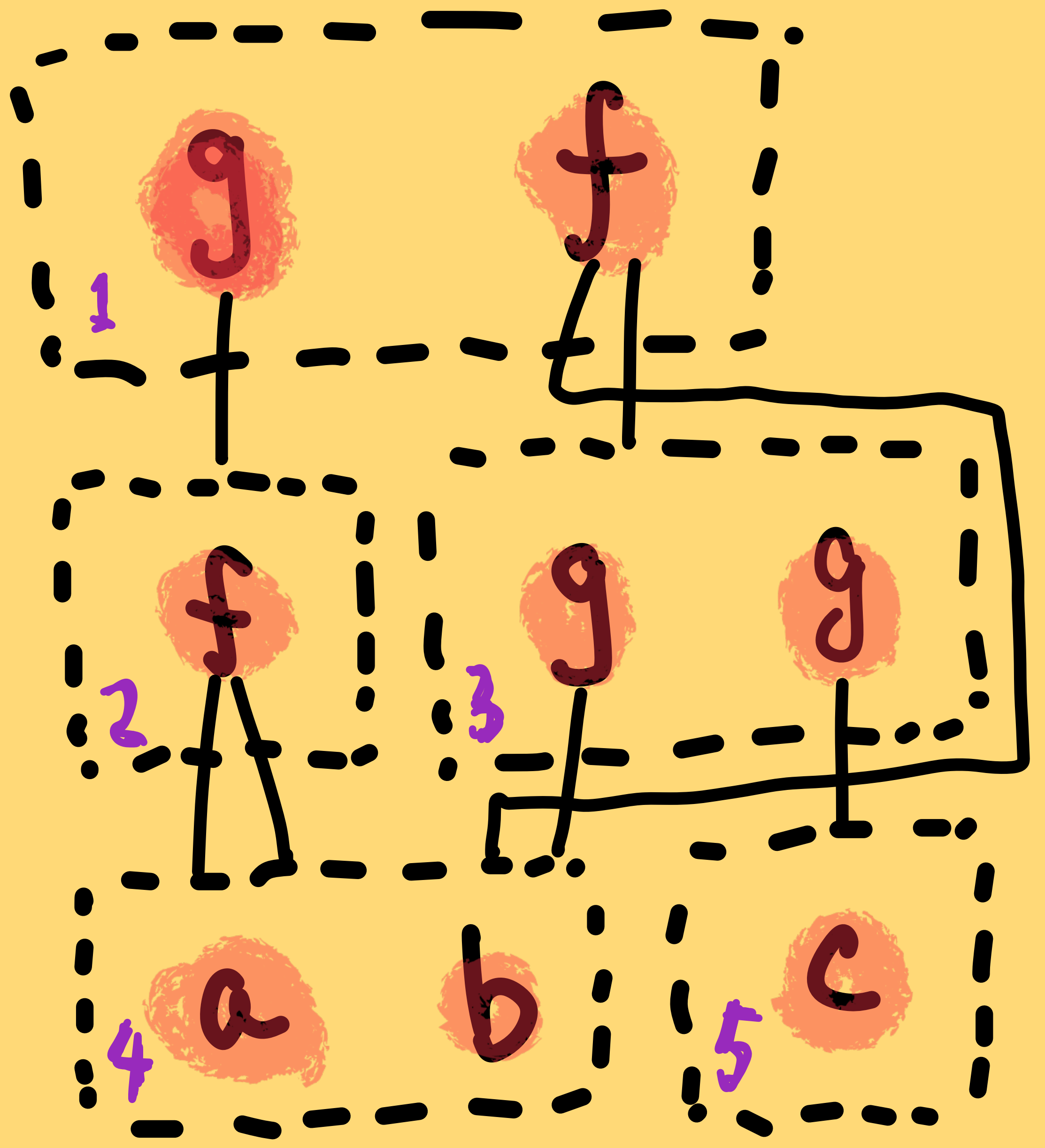

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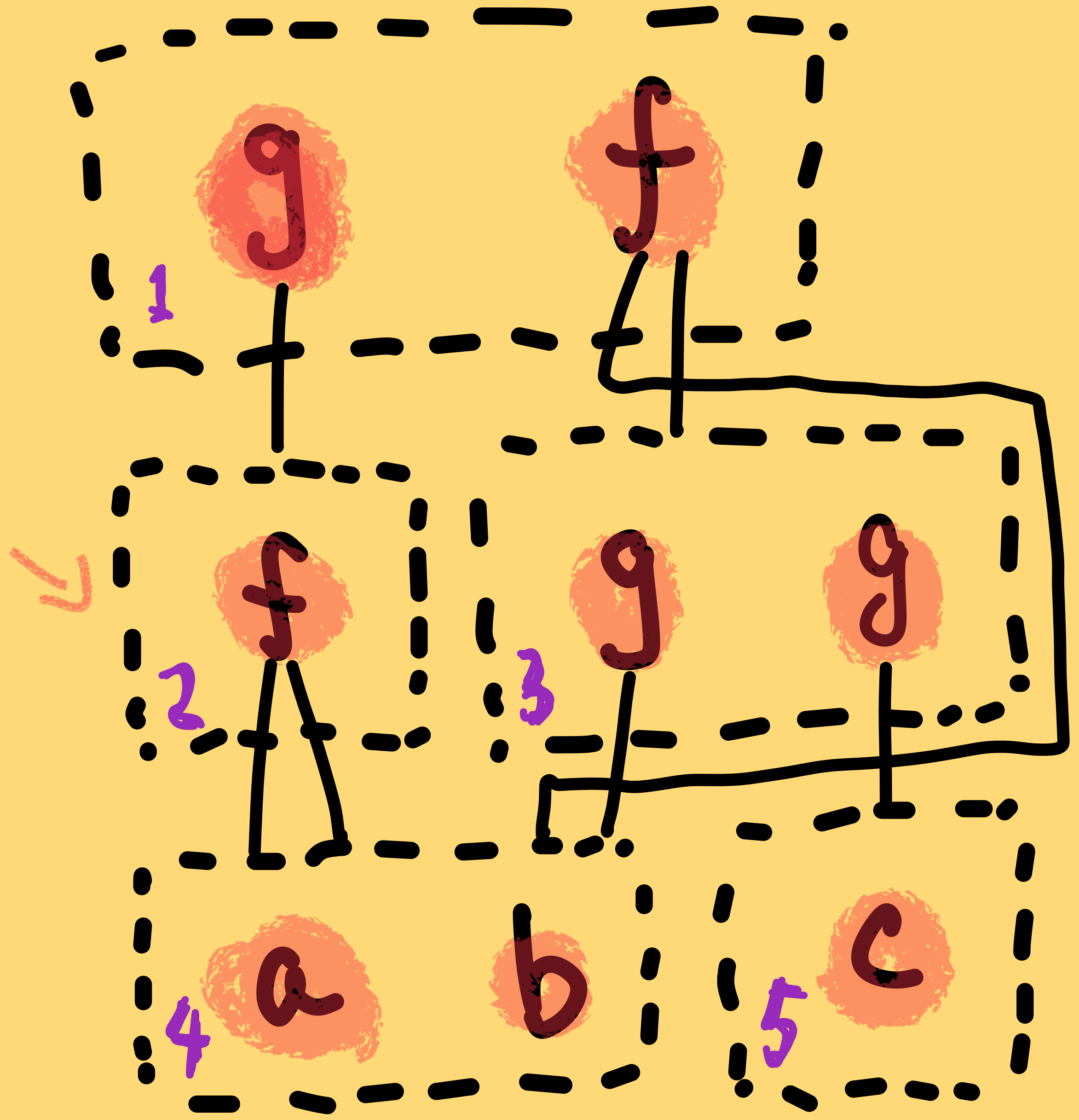

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for all rf in Rf:
  if (rf.child2, rf.child1) in Rg:
    yield {root ↦ Rf.eclass-id,
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Faster!!

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$Q(\text{root}, \alpha)$:-
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equality constraint

structural constraint

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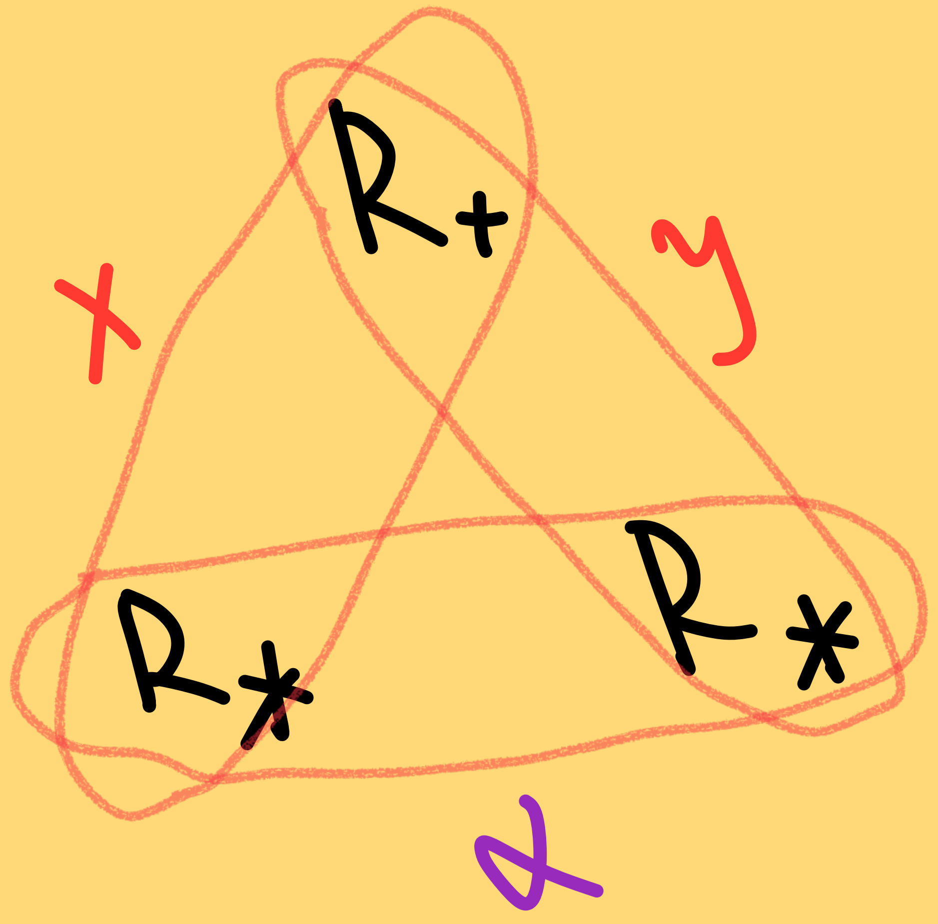

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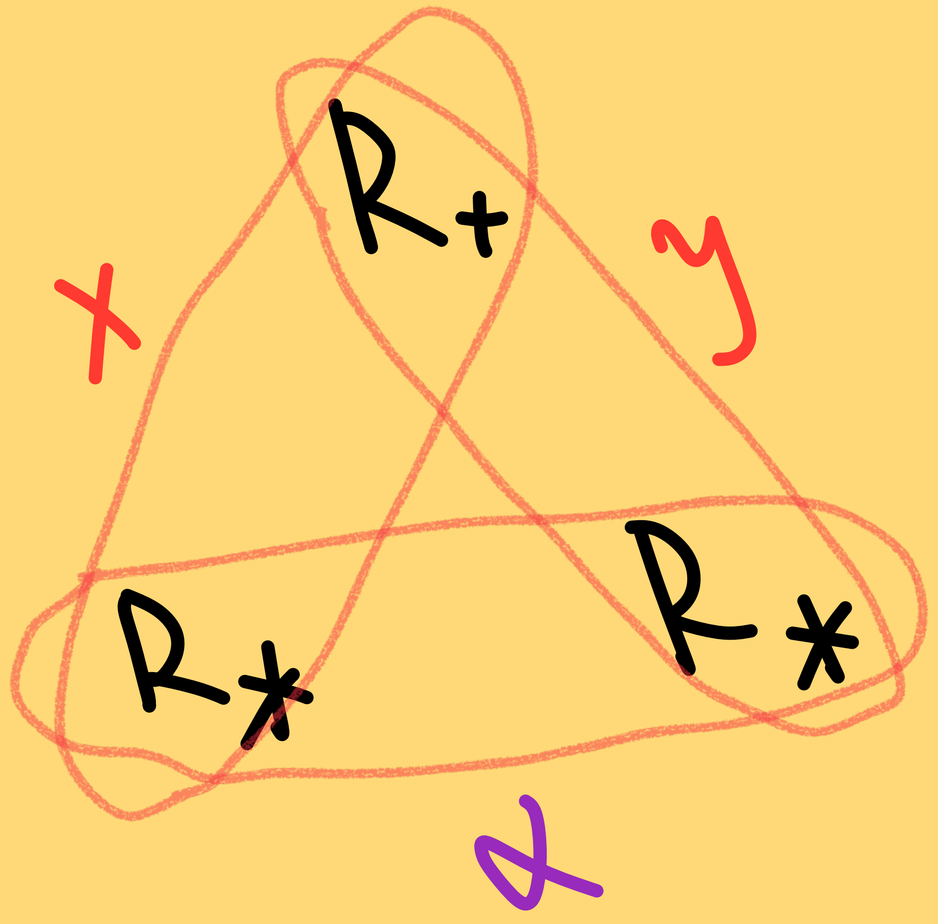

All two-way join plans (e.g. hash join, sort-merge join) are known to be suboptimal on cyclic queries like this triangle query.

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- Worst-case optimal algorithms (The AGM bound).
- Efficient for both cyclic and acyclic queries.

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$\textbf{for all } \alpha \textbf{ in } R_*.\mathrm{child}_1 \cap R_*.\mathrm{child}_1:$

$\quad \textbf{for all } \beta \textbf{ in } R_*[\mathrm{child}_1=\alpha].\mathrm{child}_2:$

$\quad\quad \textbf{for all } \gamma \textbf{ in } R_*[\mathrm{child}_1=\alpha].\mathrm{child}_2:$

$\quad\quad\quad \textbf{for all } x \textbf{ in } R_+.\mathrm{child}_1 \cap R_*[\mathrm{child}_1=\alpha, \mathrm{child}_2=\beta].\text{eclass-id}:$

$\quad\quad\quad\quad \textbf{for all } y \textbf{ in } R_+[\mathrm{child}_1=x].\mathrm{child}_2 \cap R_*[\mathrm{child}_1=\alpha, \mathrm{child}_2=\gamma].\text{eclass-id}:$

$\quad\quad\quad\quad\quad \textbf{for all } \mathrm{root} \textbf{ in } R_+[\mathrm{child}_1=x, \mathrm{child}_2=y]:$

$\quad\quad\quad\quad\quad\quad \textbf{yield } \{\mathrm{root}, \alpha, \beta, \gamma\}$

Generic join algorithms

- The triangle query complies to

```
for all α in R*.child1 ∩ R*.child1:
  for all β in R*[child1=α].child2:
    for all γ in R*[child1=α].child2:
      for all x in R+.child1 ∩ R*[child1=α, child2=β].eclass-id:
        for all y in R+[child1=x].child2 ∩ R*[child1=α, child2=γ].eclass-id:
          for all root in R+[child1=x, child2=y]:
            yield {root, α, β, γ}
```

Process queries variable-by-variable (vs relation-by-relation)

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Optimal !!

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- Singleton pattens: patterns with no nested subpatterns

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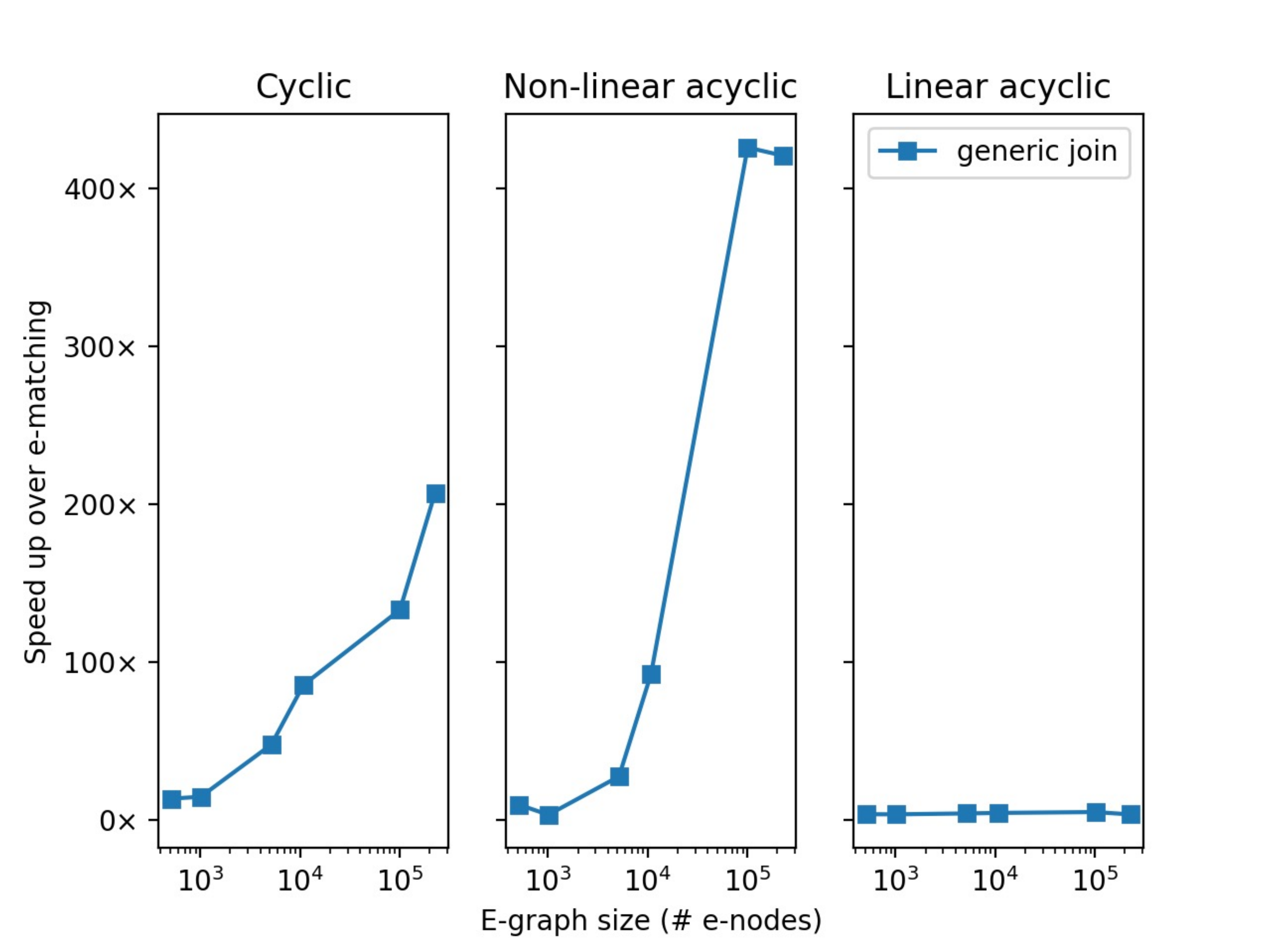
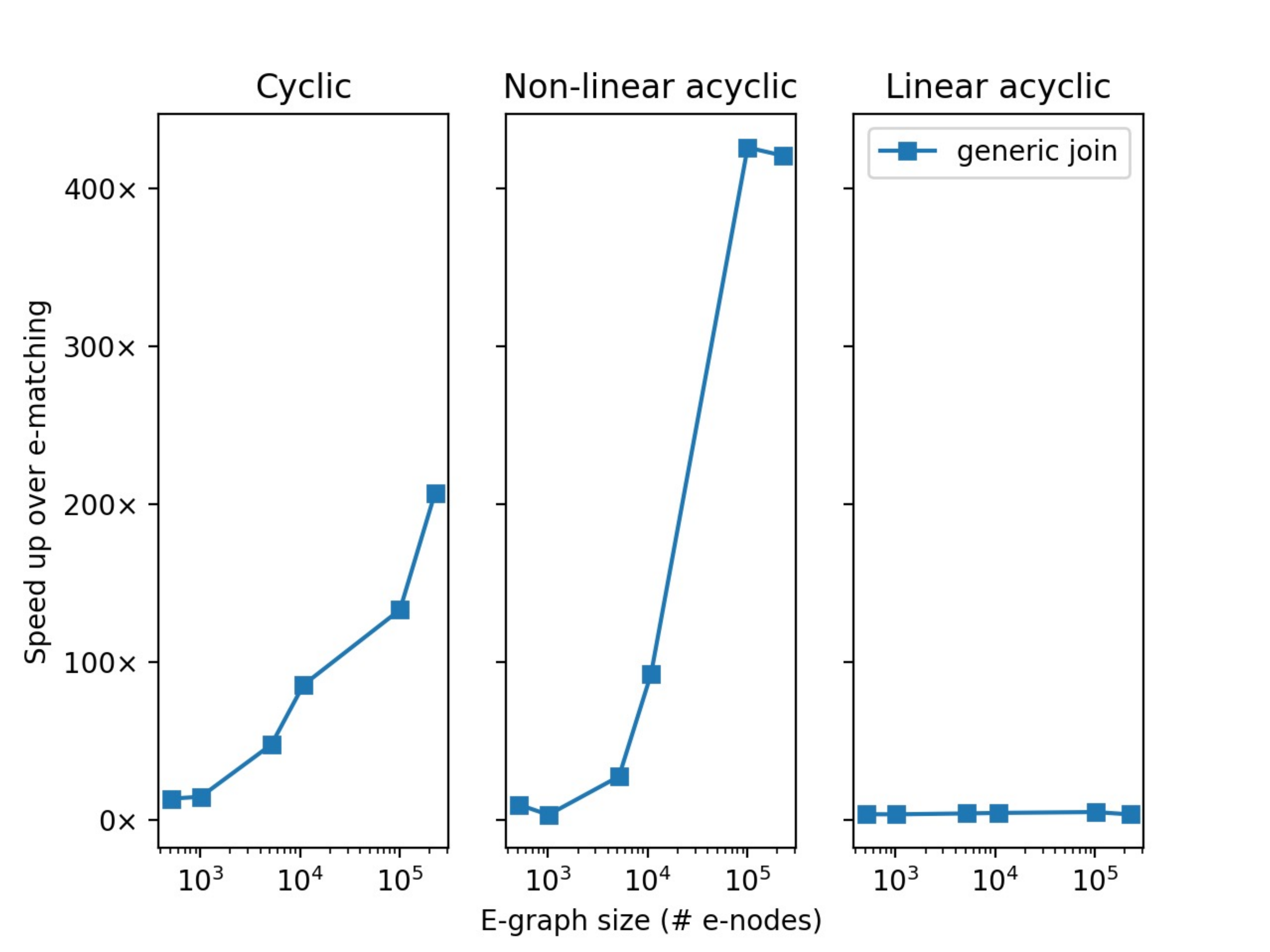


(Preliminary) Evaluations



- Collected from egg's math test suite

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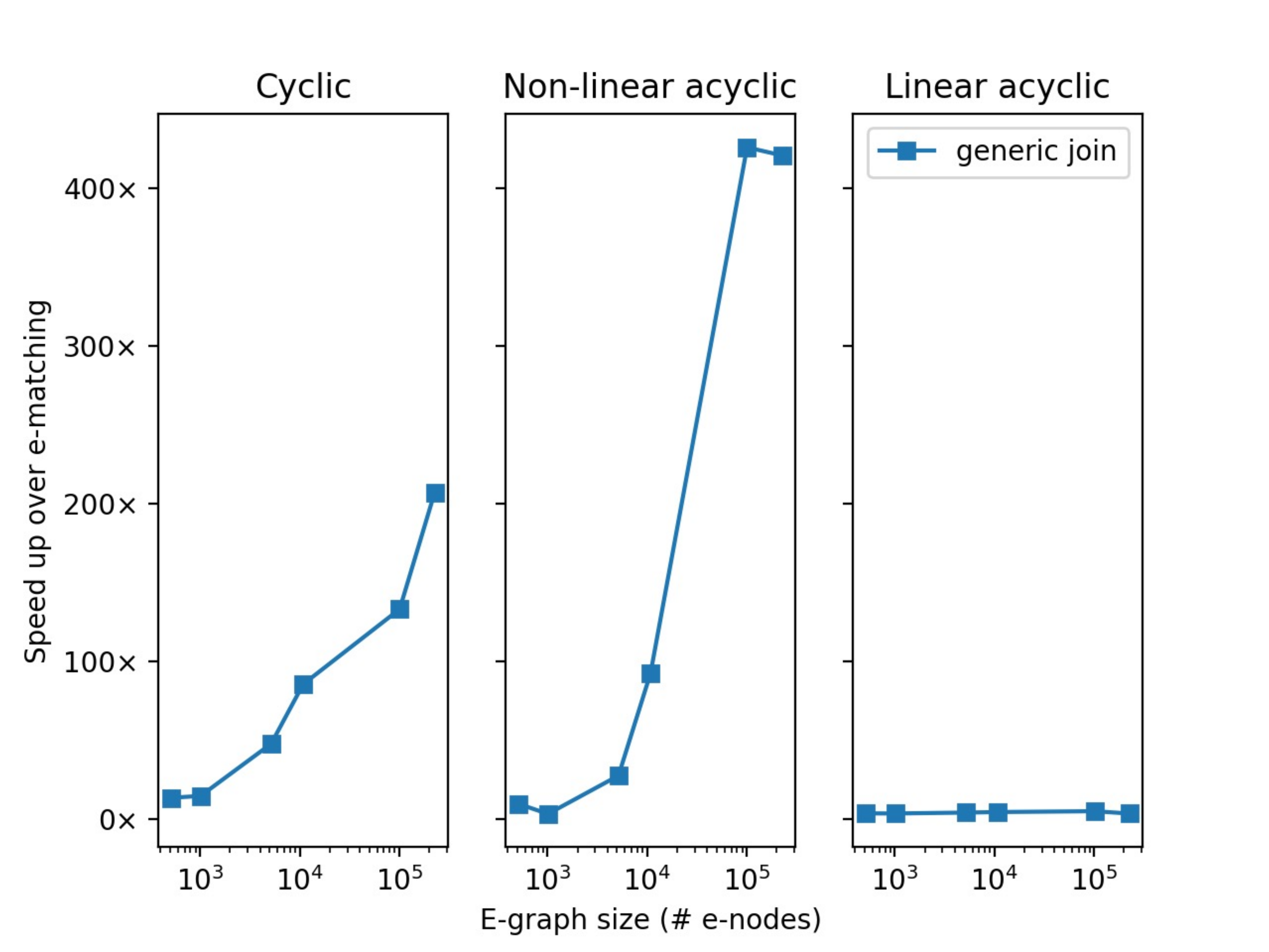

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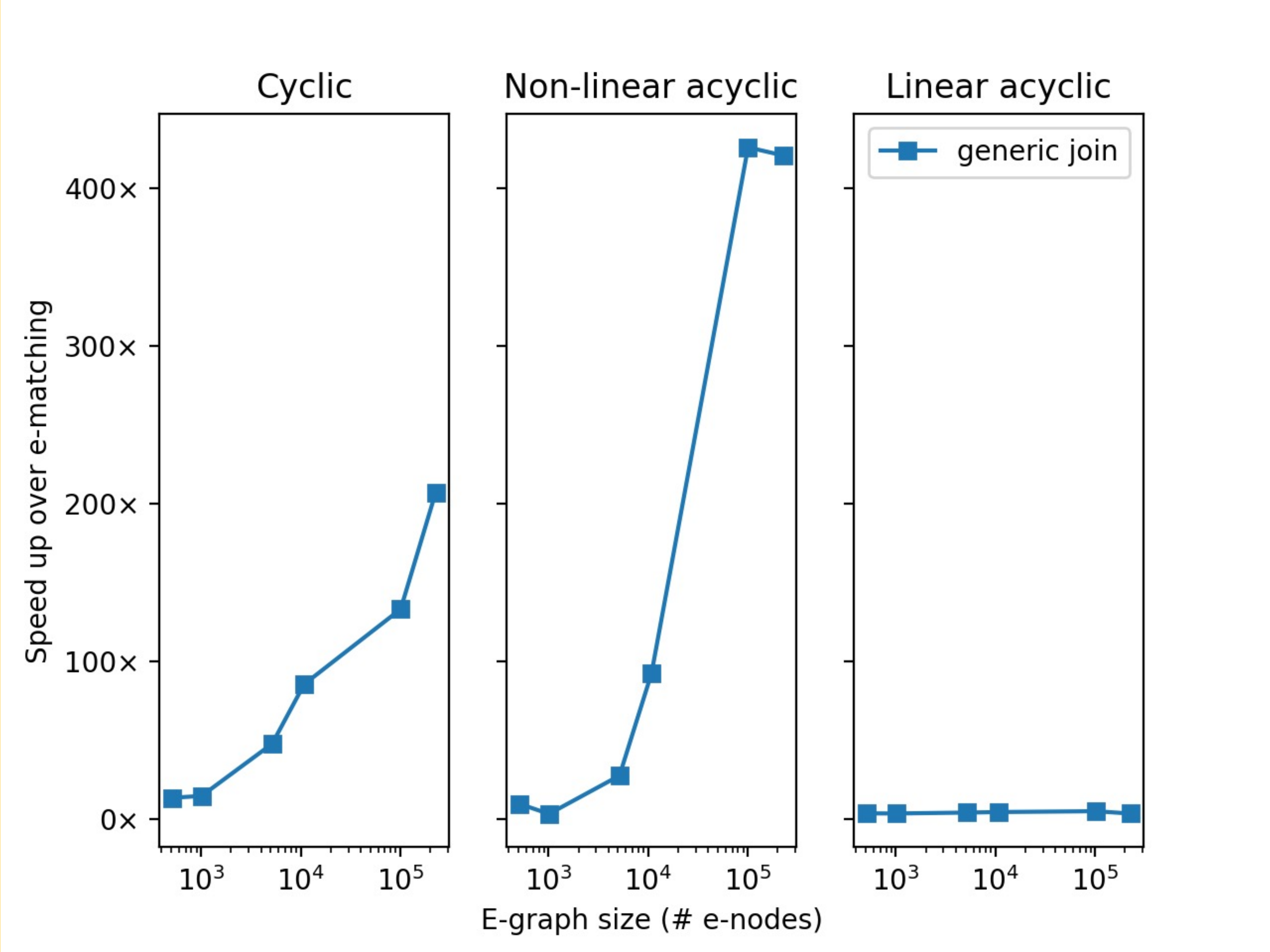

- Collected from egg's math test suite
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- Caveat: relational e-matching is handwritten in this benchmark

(Preliminary) Evaluations - microbenchmarks

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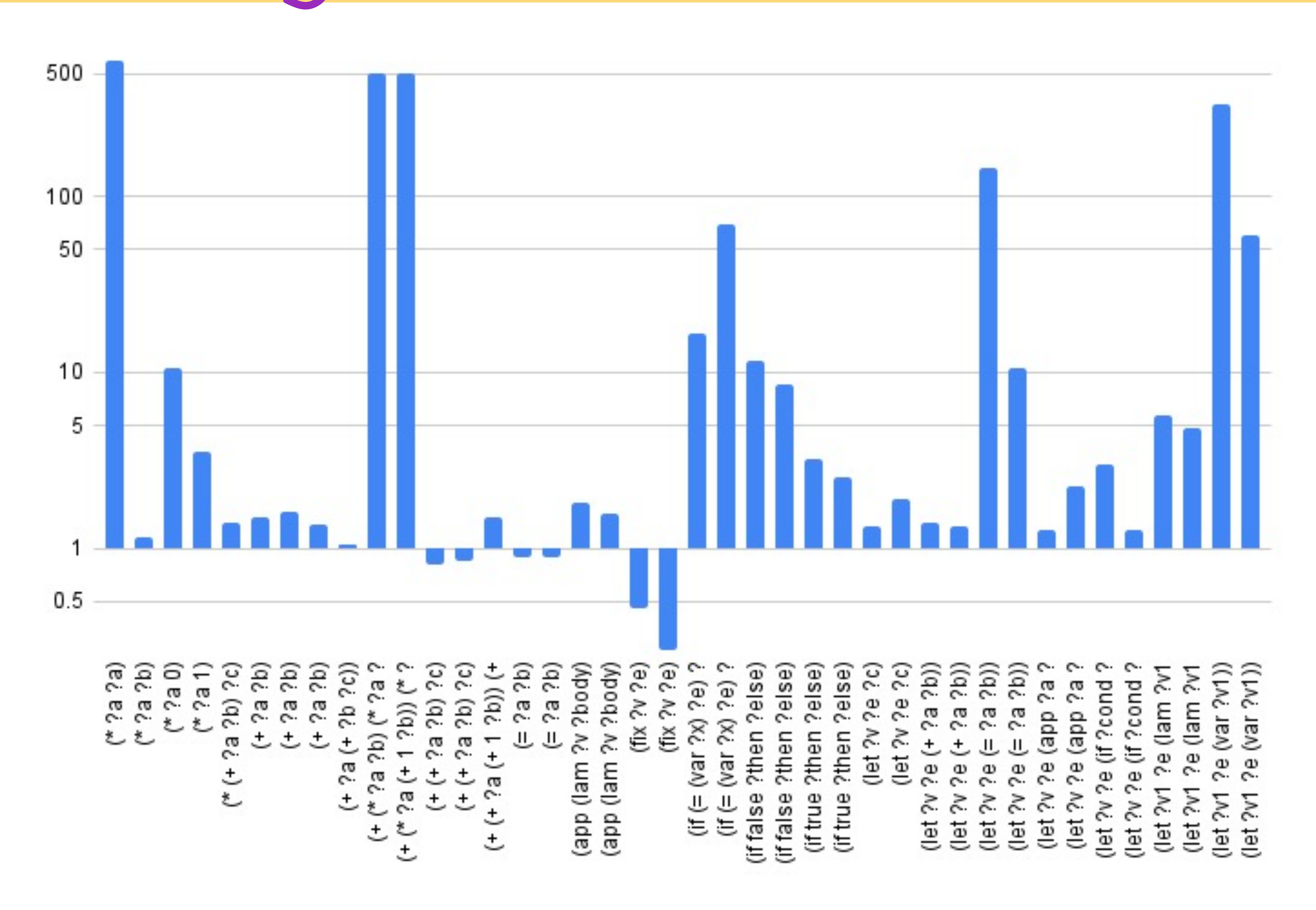

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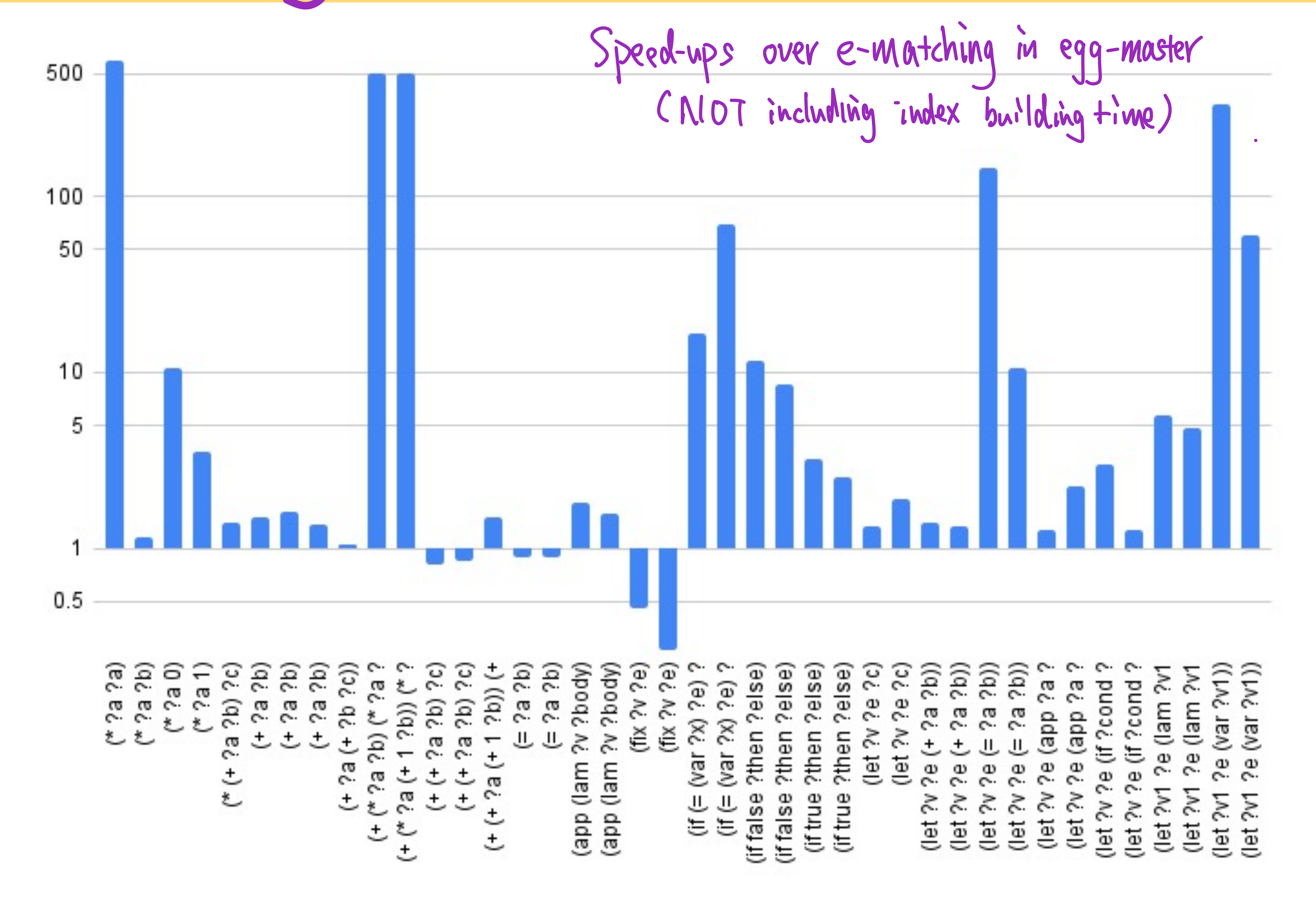

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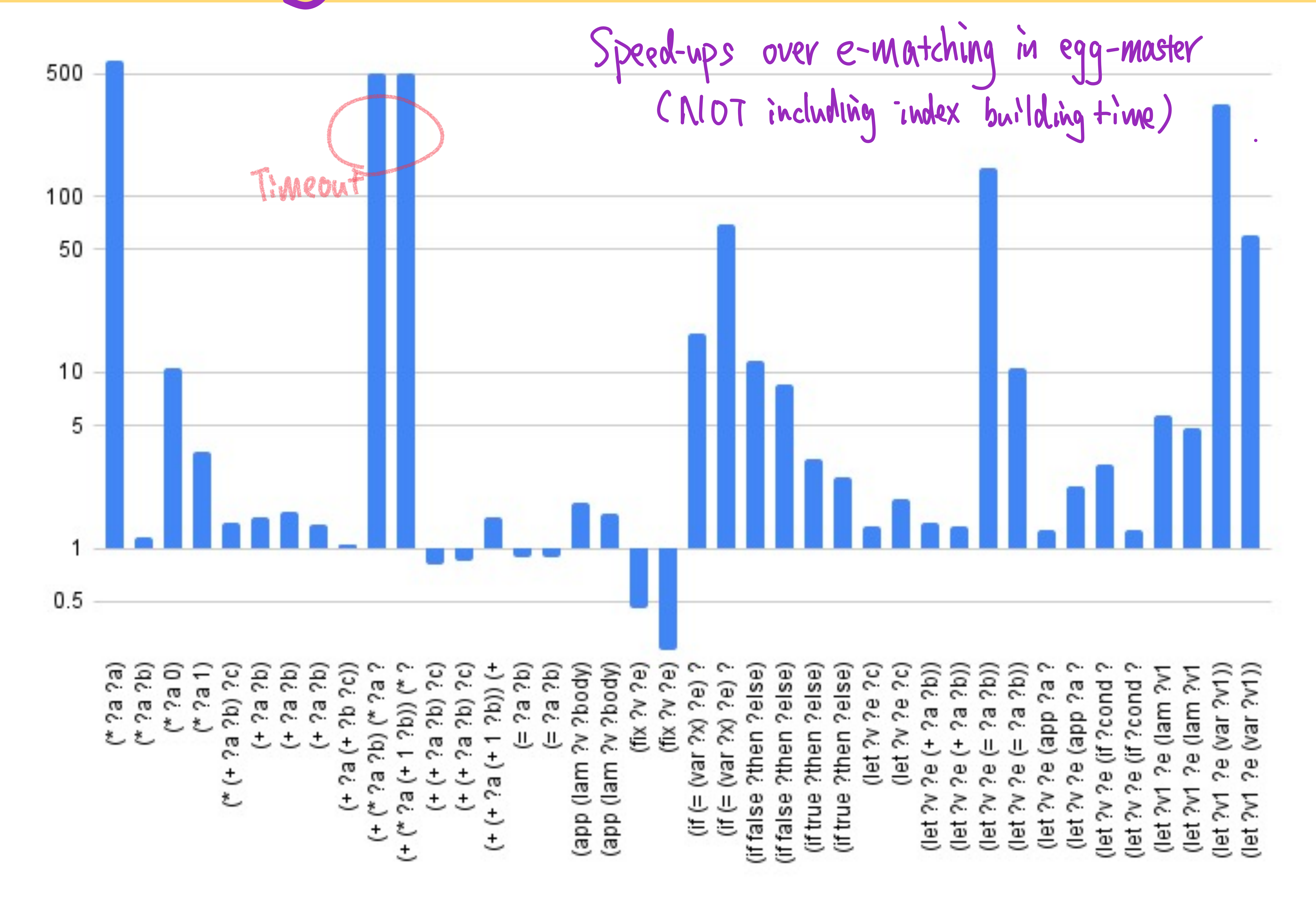

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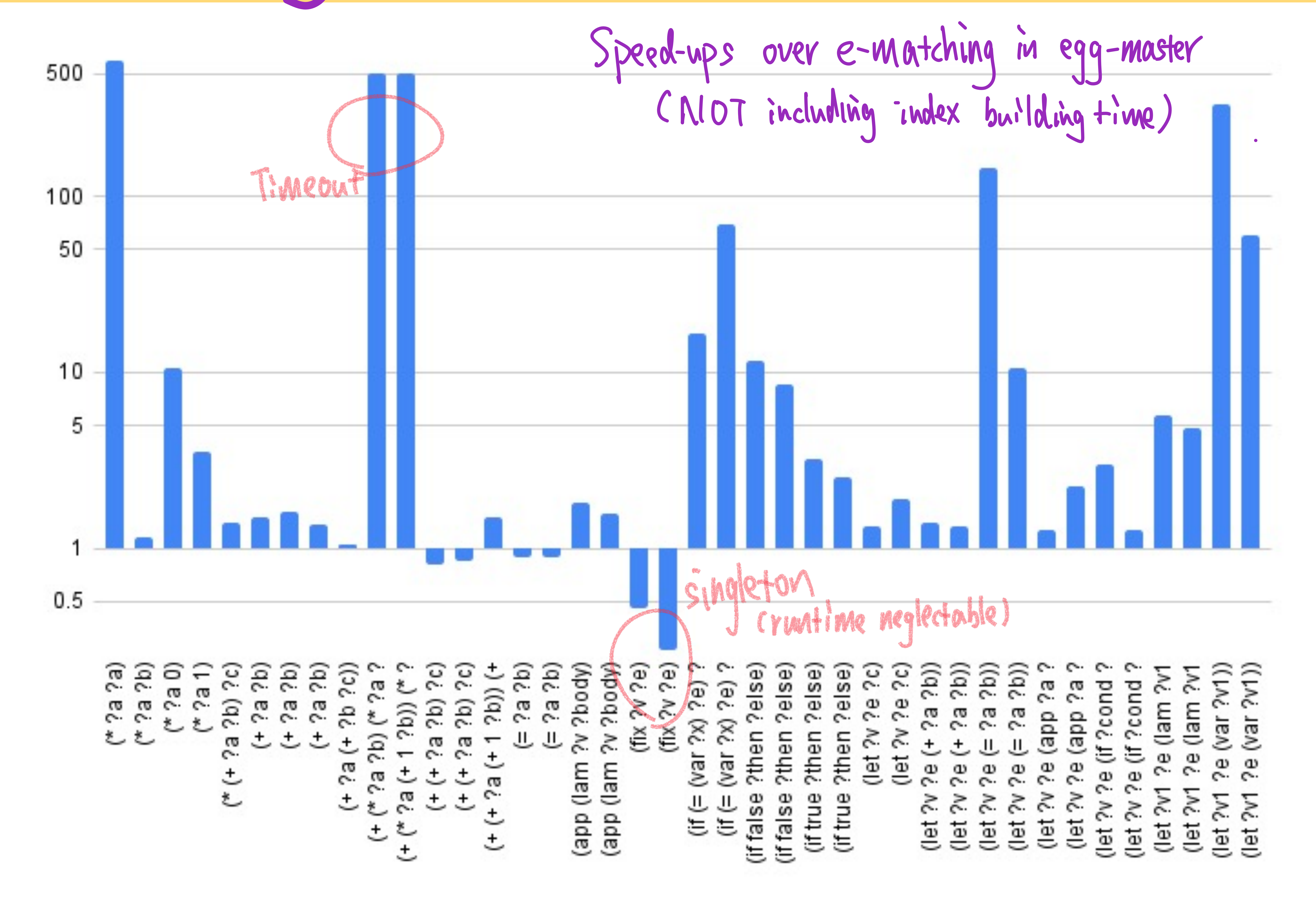

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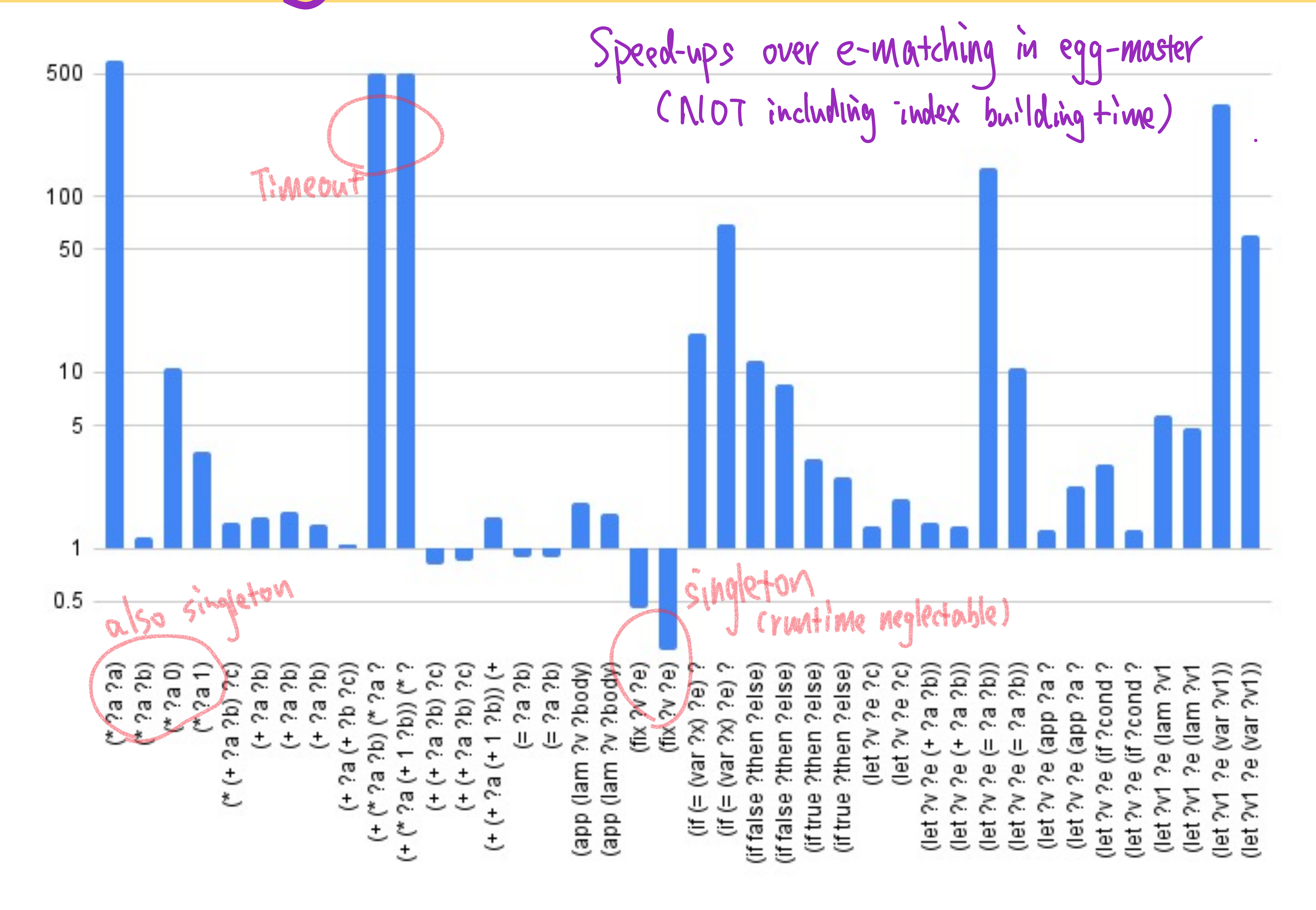

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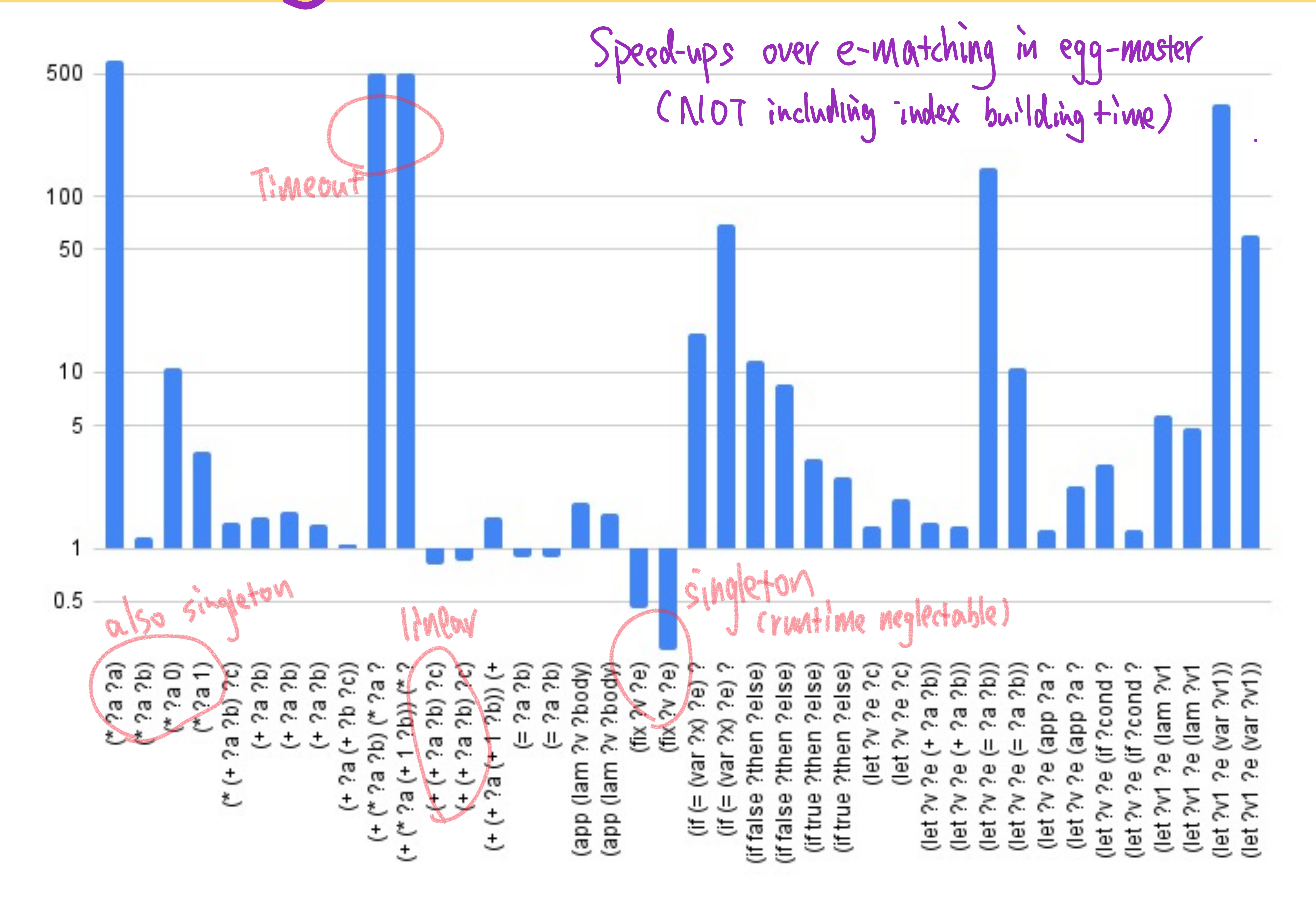

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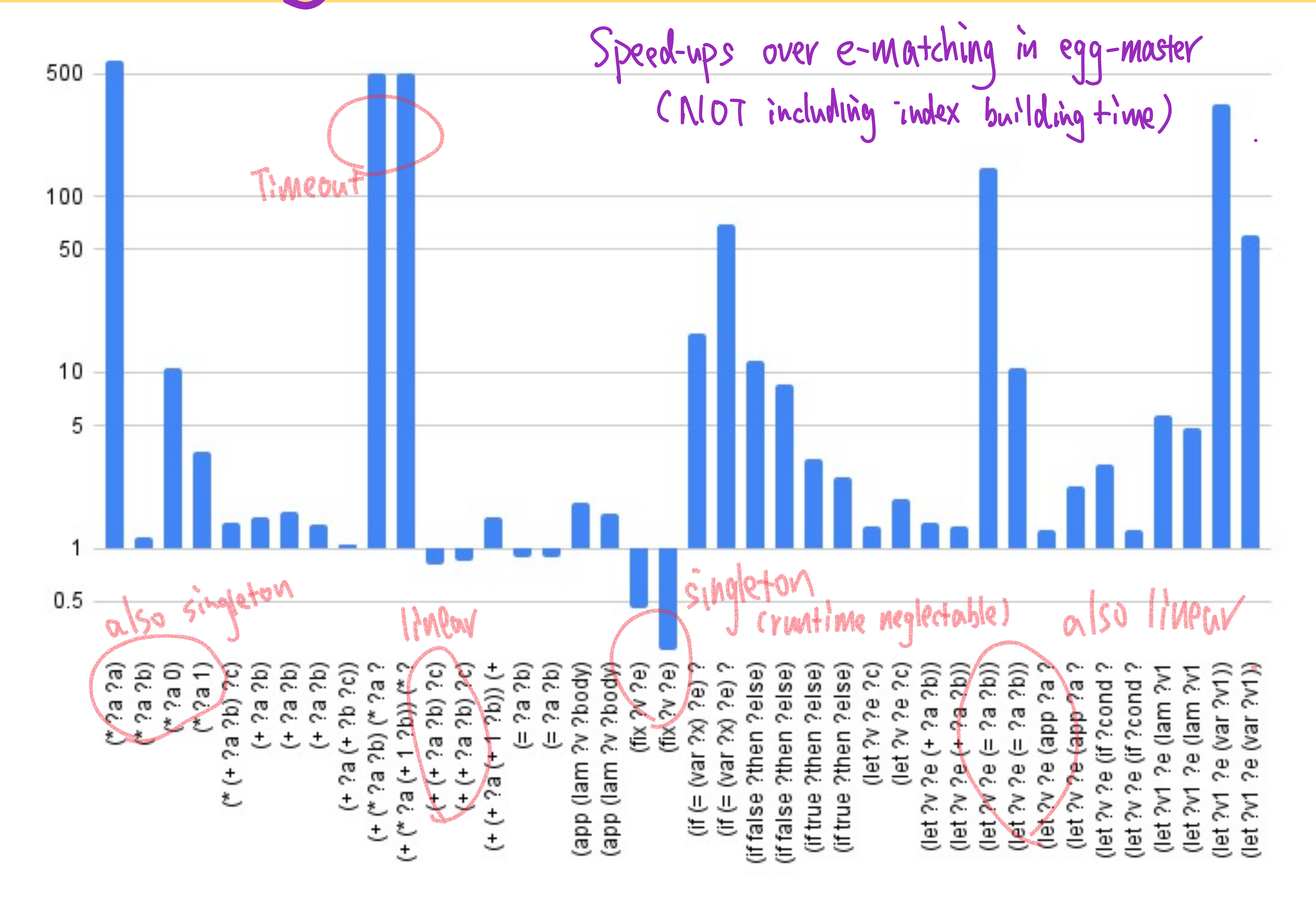

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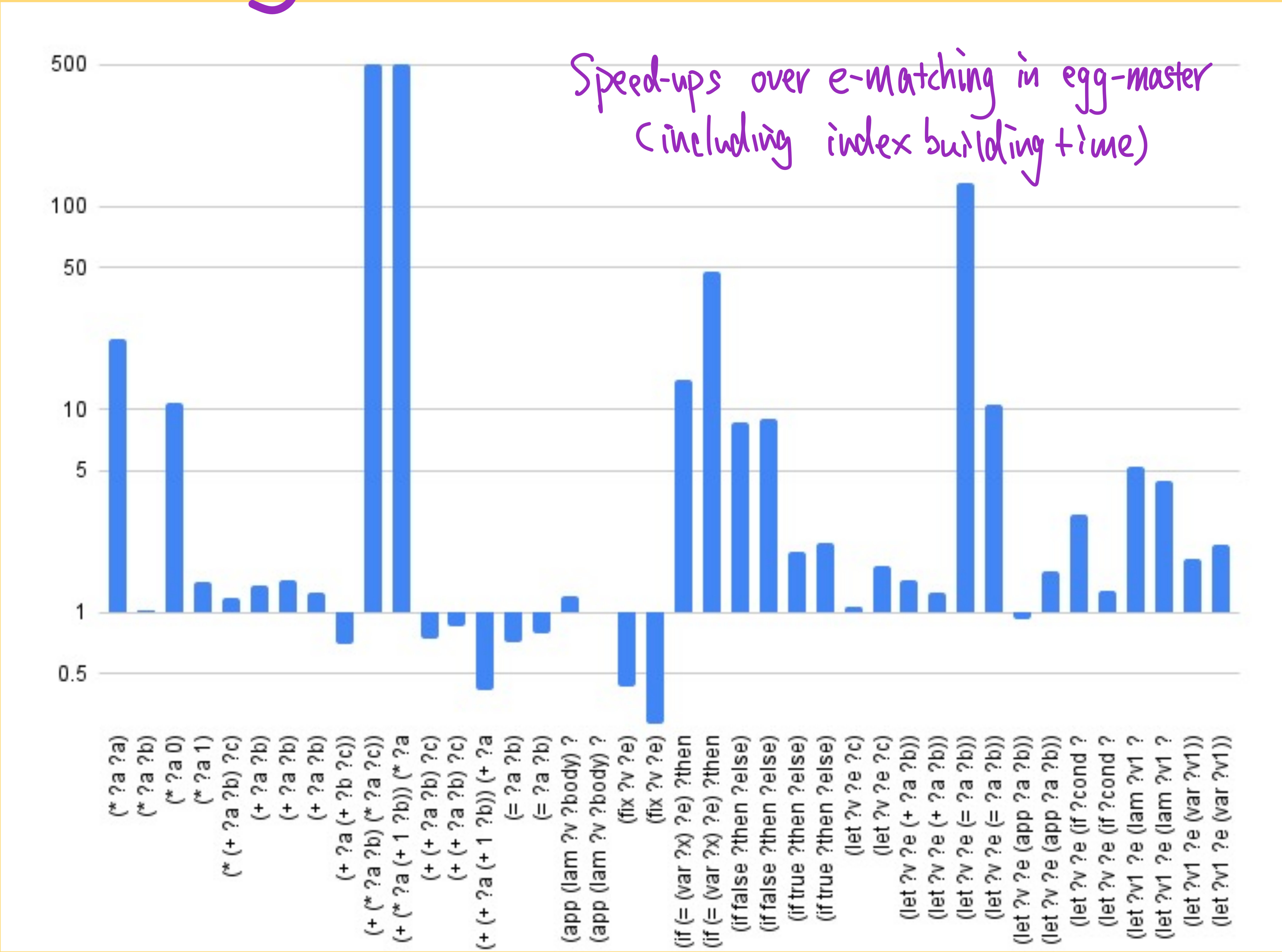

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- Building indices takes time.

How about real-world applications?

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working on fine-tune the performance
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Future work

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  - a tighter worst-case bound

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  - straightforward to support
- e-graph engine fully on top of relational database
  - interactive e-matching
- datalog with internalized congruence
  - a richer language for e-graph (e.g. transitive closure)
  - a faster datalog

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- Backtracking-based e-matchings is bad because they don't exploit the equality constraints during query planning
- Relational database is a very powerful abstraction.
- Needs to fine-tune the performance for easy patterns.