INTERNATIONAL ECONOMIC REVIEW
Vol. 00, No. 00, xxxx 2024 DOI: 10.1111/iere.12733

# CAPITAL AND WAGES*

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Does capital accumulation increase labor demand and wages? Neoclassical production functions, where capital and labor are q-complements, ensure that the answer is yes, so long as labor markets are competitive. This result critically depends on the assumption that capital accumulation does not change the technologies being developed and used. I adapt the theory of endogenous technological change to investigate this question when technology also responds to capital accumulation. I show that there are strong parallels between the relationship between capital and wages and existing results on the conditions under which equilibrium factor demands are upward-sloping (e.g., Acemoglu, *Econometrica* 75(5) (2007), 1371–410). Extending this framework, I provide intuitive conditions and simple examples where a greater capital stock leads to lower wages, because it triggers more automation. I then offer an endogenous growth model with a menu of technologies where equilibrium involves choices over both the extent of automation and the rate of growth of labor-augmenting productivity. In this framework, capital accumulation and technological change in the long run are associated with wage growth, but an increase in the saving rate increases the extent of automation, and initially reduces the wage rate and can subsequently depress its long-run growth rate.

## 1. INTRODUCTION

A mainstay of neoclassical capital theory is that capital accumulation increases wages, at least in the long run. This is one of the key mechanisms via which economic growth generates "shared prosperity" (Acemoglu and Johnson, 2023). Evidence from long-run economic growth is broadly consistent with this prediction: capital–labor ratios have increased rapidly in most industrialized economies in the 20th century, and this was accompanied by growth of labor earnings (Barro and Sala-i-Martin, 2008; Acemoglu, 2009; Gordon, 2016). Nevertheless, the slowdown—or even cessation—of U.S. real wage growth over the last four decades has raised concerns about the relationship between economic growth and wage growth. Several authors have linked the decline in the labor share of national income and the slowdown of wage growth to rapid capital accumulation (Blanchard, 1997; Karabarbounis and Neiman, 2014), while others have suggested that adverse effects for labor have followed from investment in automation technologies (Autor et al., 2003; Acemoglu and Autor, 2011; Acemoglu and Restrepo, 2018 2019; Acemoglu and Johnson, 2023).

Similar questions arise in the context of international capital flows. Neoclassical capital theory implies that an increase in foreign capital flows should benefit labor. This has not always been the case in reality, and ideas going back to the appropriate technology literature of the 1960s and 70s suggest that the consequences of capital flows may be more nuanced because more capital often leads to the adoption of more capital-intensive production tech-

*Manuscript received July 2024;

This article builds on part of the Klein Lecture delivered at Osaka University in 2023. I thank the participants at the Klein Lecture for useful comments and suggestions. I am particularly grateful to Todd Lensman for extraordinary research assistance and to an anonymous referee, an anonymous associate editor, and the editor-in-chief, Masaki Aoyagi, for very helpful suggestions. Please address correspondence to: Daron Acemoglu, Massachusetts Institute of Technology, Cambridge, MA, U.S.A. E-mail: *daron@mit.edu*.

niques, which can have adverse effects for workers (Atkinson and Stiglitz, 1969; Schumacher, 1973; Stewart, 1977; Diwan and Rodrik, 1991; Basu and Weil, 1998; Acemoglu, 2015).

In this article, I start by presenting the foundations of the neoclassical conclusion. When factor markets are competitive and aggregate production exhibits constant returns to scale, more capital always leads to higher wages and greater labor demand. I then explain that this prediction critically depends on the assumption that more capital does *not* change the production possibilities set of firms or the types of techniques they use. When technology responds to capital, greater capital intensity of production could harm labor.

The heart of the article develops a tractable model of endogenous technology choice, building on Acemoglu (2007, 2010). I show how greater capital abundance typically leads to greater *automation*—meaning a change in the organization of production such that capital becomes more important and labor becomes less important, for example, because capital is now used for tasks previously performed by labor (Acemoglu and Restrepo, 2018, 2019).

I use this framework to show how greater capital abundance can reduce wages and labor demand. I establish that this can happen when the menu of technologies enables a strong automation response.

A well-known regularity is that wages have increased in most of the industrialized world for much of the 20th century even as there was rapid capital accumulation and technological change. Does this mean that configurations in which greater capital leads to lower wages are not empirically relevant? In the last part of the article, I show that the answer to this question is no. I embed the ideas developed earlier in the article in an extended endogenous growth model, in which the equilibrium involves both a choice over the extent of automation and over the growth rate of labor-augmenting technology. The economy admits a balanced growth path in which equilibrium wages and income per capita increase because of steady labor-augmenting technological change, but the bias of technology in favor of or against labor is also determined by economic forces. Using this framework, I establish that while equilibrium wages always continue to increase in the long run, a higher saving rate can both reduce wages upon impact and also depress their long-run growth rate.

This article is most closely related to my previous work, Acemoglu (2007, 2010) and Acemoglu and Restrepo (2018). Acemoglu (2007) provided a general framework for the analysis of the equilibrium bias of technology. The microfounded model of technology choice presented here is based on that paper. Acemoglu (2007) develops this framework to investigate the conditions under which an increase in the supply of skilled labor shifts technology in a "skill-biased" direction (this is the *weak equilibrium bias* result), and the more stringent conditions under which a greater abundance of skilled labor *increases* the long-run skill premium (this is the *strong equilibrium bias* result). Acemoglu (2010) uses a similar framework to study the conditions under which the scarcity of labor encourages innovation, as it has been observed in key historical episodes (Habakkuk, 1962; Mokyr, 1990; Allen, 2009). The focus here is on the effects of capital on wages, which has not featured in the previous work. I will show, nonetheless, that there are important parallels between the strong equilibrium bias result in Acemoglu (2007) and the possibility here that greater capital intensity reduces wages via the automation response.

Finally, the modeling of automation in this article builds on Zeira (1998), Acemoglu and Zilibotti (2001), Acemoglu and Autor (2011), and especially Acemoglu and Restrepo (2018, 2019). These papers do not investigate the relationship between the direction of technology and the abundance of capital, though Acemoglu and Restrepo (2018) consider how capital accumulation and equilibrium labor shares impact the balance between automation and new tasks.

The rest of the article is organized as follows: Section 2 demonstrates that with exogenous technology, competitive factor markets and constant returns to scale, increasing the capital stock of the economy always raises wages. This section also highlights the importance of the exogenous technology assumption for this result. Section 3 builds on Acemoglu (1998, 2007, 2010) and provides a simple framework for endogenizing technology and its response to the abundance of capital. Section 4 reviews previous results about how endogenous and directed technology changes the relationship between factor supplies and factor prices. Specifically, it presents simplified versions of the weak and strong bias results of Acemoglu (1998, 2002, 2007). The strong bias results also have intuitions closely related to those I explore in the context of the relationship between capital and wages, as shown in Section 5. The results in this section establish that an increase in the capital stock of the economy can reduce, instead of increase, equilibrium wages when technology responds to this change in factor supplies. Section 6 provides worked-out examples of how a higher capital stock can lead to lower wages in a number of simple economies. Section 7 embeds these ideas into an endogenous growth model with multiple types of technologies—one corresponding to automation and the other to labor-augmenting technology. It establishes that, along the balanced growth path, wages increase together with capital deepening, but a higher saving rate can increase capital abundance and lead to a decline in wage levels. Section 8 concludes.

## 2. CAPITAL AND WAGES IN NEOCLASSICAL GROWTH THEORY

Until Section 7, I focus on static economies, where technology choices are also made statically. In this section, I demonstrate that in the neoclassical world with constant returns to scale, greater capital abundance always increases wages. This result is also exposited in the context of a static model. The terminology introduced in this section will be used throughout the rest of the article.

Consider a neoclassical, constant returns to scale aggregate production function

$$F(L, K, \theta),$$

where $L$ is labor used in production, $K$ is the capital stock of the economy, and $\theta$ is an index of technology. Throughout, factor markets are assumed to be competitive, and the aggregate production function $F$ is continuously differentiable and exhibits constant returns to scale (is linearly homogeneous in $L$ and $K$). Throughout, I use subscripts to denote derivatives.

I also simplify the discussion by assuming that there is a representative household in the background (though its preferences do not play a major role until Section 7). Labor is inelastically supplied, and whenever it is convenient, I normalize its total supply to $\bar{L} = 1$, which simplifies the notation by imposing that the capital stock $K$ and the capital–labor ratio $k \equiv K/L$ are the same and I will use them interchangeably.

Given the differentiability of the production function and competitive markets, the equilibrium wage is given by $w = F_L(L, K, \theta)$. Or exploiting constant returns to scale and defining the per capita production function as $f(k, \theta) \equiv F(L, K, \theta)/L$, the equilibrium wage can be written as

$$w = f(k, \theta) - k f_k(k, \theta).$$

I next prove the following result:

PROPOSITION 1. Holding $\theta$ constant, a higher capital stock always raises equilibrium wages. That is,

$$\frac{dw}{dK} \geq 0.$$

Moreover, this inequality is strict whenever $f_{kk} < 0$.

The proof is straightforward and follows immediately from the fact that

$$\frac{dw}{dK} = -kf_{kk}(k, \theta) \geq 0,$$

since $f_{kk}(k, \theta) \leq 0$ by constant returns to scale (and concavity of the aggregate production function).

Intuitively, with a constant returns to scale production function, capital and labor are q-complements—meaning that an increase in the use of one of these factors raises the marginal product of the other, or $F_{LK} \geq 0$. This feature is implied because, under constant returns to scale, $F_{LK}$ is proportional to $-kf_{kk}$, and $f_{kk} \leq 0$. Hence, more capital always increases the marginal product of labor. So long as the wage is proportional to labor's marginal product (as it is in a perfectly competitive labor market), the conclusion follows.

It is straightforward to embed this result in a standard (exogenous or endogenous) growth model. For example, a higher saving rate starts raising the capital stock immediately and takes the economy toward a new steady state (a balanced growth path) with a higher capital–labor ratio. After the rise in the saving rate, the wage rate starts increasing (relative to the baseline) and the new steady state will have permanently higher wages. I will contrast this result to the pattern that obtains in a dynamic economy with a menu of technologies and directed technological change in Section 5.

The fact that we are holding technology fixed is indeed important for these results. If, instead, we had $\frac{d\theta}{dK} \neq 0$, greater capital abundance would change the aggregate production function (e.g., alter how important capital and labor are in the production process), and this would impact wages. For example, if $\theta$ corresponds to a measure of automation and we have $\frac{d\theta}{dK} > 0$ (more capital and encourages more automation), and $\frac{\partial w}{\partial \theta} < 0$ (automation reduces wages; see Acemoglu and Restrepo, 2019), then the overall impact of greater capital abundance could be to reduce wages.

To explore these issues in detail, we need a way of endogenizing technology choices, which is what I start with in the next section.

## 3. ENDOGENIZING TECHNOLOGY

This section is based on Acemoglu (2007). I provide a brief exposition to avoid repetition and economize on space.

Each firm $i \in \mathcal{F}$ has access to a production function

$$y^i = \alpha^{-\alpha}(1-\alpha)^{-1}F(L^i, K^i, \Theta)^{\alpha}q^i(\Theta)^{1-\alpha}, \tag{1}$$

where $L^i$ is the firm's employment, $K^i$ is its capital stock, $q^i(\Theta)$ is the quantity of intermediate good embedding technology $\Theta$, $\alpha \in (0, 1)$, and $\Theta = (\theta_1, \ldots, \theta_N) \in \mathcal{O} \subset \mathbb{R}^N$ is an $N$-dimensional measure of technology that applies to all firms in the economy. Different dimensions of this measure can capture distinct aspects of technology, such as whether it is augmenting capital or labor or whether it is automating some tasks or changing the substitution patterns between the two factors. I assume throughout that $\mathcal{O}$ is a lattice (see Topkis, 1998), and $F$ is again taken to be a neoclassical production function with constant returns to scale. I use lower case $y^i$ to denote the output of firm $i$, and use $Y$ to denote net aggregate output below. The term $\alpha^{-\alpha}(1-\alpha)^{-1}$ is included as a convenient normalization.

This production structure is similar to models of endogenous technology (e.g., Romer, 1990; Grossman and Helpman, 1991; Aghion and Howitt, 1992), but is somewhat more general since it does not impose that technology necessarily takes a factor-augmenting form.

The monopolist can create (a single) technology $\Theta \in \mathcal{O}$. The cost of inventing this technology is $\Gamma(\Theta)$. I often set this cost to zero, which is without loss of much generality, since $\Theta$ is already part of the production function, so any additional costs can be incorporated into $F$.

In line with Romer's (1990) emphasis that technology has a "nonrivalrous" character and can thus be produced at relatively low cost once invented, I assume that once $\Theta$ is created, the intermediate good embodying technology $\Theta$ can be produced at constant per unit cost normalized to $1-\alpha$ units of the final good (this is also a convenient normalization). The monopolist can then set a (linear) price per unit of the intermediate good of type $\Theta$, denoted by $\chi$.

I continue to assume that labor is inelastically supplied, with total supply denoted by $\bar{L}$, and I take the supply of capital, $\bar{K}$, as given as well. All factor markets are, once again, competitive, and each firm takes the available technology, $\Theta$, and the price of the intermediate good embodying this technology, $\chi$, as given and maximizes

$$(2)\quad \max_{L^i,K^i,q^i(\Theta)} \pi(L^i, K^i, q^i(\Theta) \mid \Theta, \chi) = \alpha^{-\alpha}(1-\alpha)^{-1}F(L^i, K^i, \Theta)^{\alpha} q^i(\Theta)^{1-\alpha} - wL^i - RK^i - \chi q^i(\Theta),$$

which gives the following simple inverse demand for intermediates of type $\Theta$ as a function of its price, $\chi$, and the factor employment levels of the firm as

$$(3)\quad q^i\left(\chi, L^i, K^i \mid \Theta\right) = \alpha^{-1}F(L^i, K^i, \Theta)\chi^{-1/\alpha}.$$

The problem of the monopolist is to maximize its profits:

$$(4)\quad \max_{\Theta,\chi,[q^i(\chi,L^i,K^i\mid\Theta)]_{i\in\mathcal{F}}} \Pi = (\chi - (1-\alpha)) \int_{i\in\mathcal{F}} q^i\left(\chi, L^i, K^i \mid \Theta\right) di - \Gamma(\Theta)$$

subject to (3). Given the supplies of labor and capital, market clearing requires:

$$(5)\quad \int_{i\in\mathcal{F}} L^i di \leq \bar{L} \text{ and } \int_{i\in\mathcal{F}} K^i di \leq \bar{K}.$$

An *equilibrium* is a set of firm decisions $\left\{L^i, K^i, q^i\left(\chi, L^i, K^i \mid \Theta\right)\right\}_{i\in\mathcal{F}}$, technology choice and pricing decisions by the technology monopolist $(\Theta, \chi)$, and factor prices $(w, R)$ such that $\left\{L^i, K^i, q^i\left(\chi, L^i, K^i \mid \Theta\right)\right\}_{i\in\mathcal{F}}$ solve (2) given $(w, R)$ and $(\Theta, \chi)$; (5) holds; and $(\Theta, \chi)$ maximize (4) subject to (3).

This definition emphasizes that factor demands and technology are decided by different agents (the former by the final good producers, the latter by the technology monopolist). This is an important feature both theoretically and as a representation of how technology is determined in practice (see Acemoglu, 2007, for more discussion). Since factor demands and technology are decided by different agents, $F(L^i, K^i, \Theta)$ need not be jointly concave in $(L^i, K^i, \Theta)$. Instead, a well-behaved equilibrium exists, provided that $F$ is concave in $(L^i, K^i)$, which is already assumed given our constant returns to scale restriction, and $F - \Gamma$ is concave in $\Theta$.

To characterize the equilibrium, note that (3) defines a constant elasticity demand curve, so the profit-maximizing price of the monopolist is given by the standard monopoly markup over marginal cost and, given our normalizations, is equal to $\chi = 1$. Consequently, $q^i(\Theta) = q^i\left(\chi = 1, L^i, K^i \mid \Theta\right) = \alpha^{-1}F(L^i, K^i, \Theta)$ for all $i \in \mathcal{F}$. Substituting this into (4), integrating over the set of firms $\mathcal{F}$ and using the fact that $F$ exhibits constant returns to scale and $\chi = 1$, we have that $\Pi = (\chi - (1-\alpha))\alpha^{-1}F(\bar{L}, \bar{K}, \Theta) - \Gamma(\Theta) = F(\bar{L}, \bar{K}, \Theta) - \Gamma(\Theta)$. Therefore, the maximization problem of the monopolist can be expressed as

$$\max_{\Theta\in\mathcal{O}} H(\bar{L}, \bar{K}, \Theta) \equiv F(\bar{L}, \bar{K}, \Theta) - \Gamma(\Theta).$$

This argument establishes:

PROPOSITION 2. *Any equilibrium technology $\Theta^*$ is a solution to*

$$\max_{\Theta \in \mathcal{O}} H(\bar{L}, \bar{K}, \Theta), \tag{6}$$

*and any solution to this problem is an equilibrium technology.*

This proposition shows that the equivalence between equilibrium technology and the maximizers of $H(\bar{L}, \bar{K}, \Theta) \equiv F(\bar{L}, \bar{K}, \Theta) - \Gamma(\Theta)$. Notice also that the function $H$ inherits all the properties of the production function $F$ with respect to $L$ and $K$. However, because of the monopoly markup, there are distortions, and equilibrium technology is not at the level that maximizes net output.

I now use the fact that the profit-maximizing monopoly price is $\chi = 1$ and substitute (3) into the production function (1), and then subtract the cost of technology choice, $\Gamma(\Theta)$, and the cost of production of the machines, $(1-\alpha)\alpha^{-1}F(L, K, \Theta)$, from gross output. This gives net output in this economy as

$$Y(\bar{L}, \bar{K}, \Theta) \equiv \frac{2-\alpha}{1-\alpha} F(\bar{L}, \bar{K}, \Theta) - \Gamma(\Theta). \tag{7}$$

Observe that the coefficient in front of $F(\bar{L}, \bar{K}, \Theta)$ is always greater than one, so equilibrium technology will generally fail to maximize net output. A more systematic discussion of distortions in the direction of innovation is provided in Acemoglu (2023).

Finally, it can be verified that the equilibrium wage is now

$$w = \frac{1}{1-\alpha} F_L(\bar{L}, \bar{K}, \Theta),$$

and thus proportional to the standard wage equation. This wage equation can be rewritten as

$$\begin{aligned} w &= \frac{1}{1-\alpha}[f(k, \Theta) - f_k(k, \Theta)k] \\ &= \frac{1}{1-\alpha}[h(k, \Theta) + \Gamma(\Theta) - h_k(k, \Theta)k], \end{aligned}$$

where $h(k, \Theta) \equiv f(k, \Theta) - \Gamma(\Theta)$.

REMARK 1. This section focused on what was referred to as "Economy M" in Acemoglu (2007), where technology is supplied by a single monopolist. This is for simplicity, and all of the results in this section and the rest of the article also apply in "Economy O," where different components of technology are supplied by oligopolistic technologists, and in "Economy E," where firms themselves choose their own technology from the menu given by $O$, but there are externalities on other firms from these decentralized technology choices. The other environment considered in Acemoglu (2007) is "Economy D," where each firm chooses its own technology and there are no externalities (and thus, the equilibrium is Pareto-efficient). The characterization in this section also applies to this economy, but some of the later results do not hold in this case, and I will comment on those at the end of the next section.

## 4. REVIEW OF WEAK AND STRONG BIAS RESULTS

Here, I review the results presented in Acemoglu (2007), which, in turn, draw on and significantly generalize those in Acemoglu (1998, 2002). The focus will be on how the change in the supply of labor changes the demand for labor and the equilibrium wage. For this purpose, I consider a setting, as in Acemoglu (2007), where there are potentially several factors, which would include capital, summarized by the vector $Z$, and additionally, labor $L$. I assume for simplicity that all of these factors are inelastically supplied, with supplies denoted by $(\bar{L}, \bar{Z})$. Hence, with analogy to the previous section, equilibrium technology is now given by the solution to

$$\max_{\Theta \in \mathcal{O}} H(L, Z, \Theta).$$

The next two definitions introduce the notions of bias and weak equilibrium bias.

DEFINITION 1. An increase in technology $\theta_j$ for $j = 1, \ldots, N$ is (absolutely) biased toward $L$ at supplies $(\bar{L}, \bar{Z})$ if $\partial w / \partial \theta_j \geq 0$.

This definition only considers small changes in technology at the *current* factor proportions $(\bar{L}, \bar{Z})$, which simplifies the results. The more general case is presented in Acemoglu (2007). I also assume throughout that all the relevant functions are continuously differentiable and the relevant first-order conditions hold (and I discuss this issue further in footnote 1).

DEFINITION 2. Denote the equilibrium technology at factor supplies $(\bar{L}, \bar{Z})$ by $\Theta^* (\bar{L}, \bar{Z})$ and assume that $\partial \theta_j^* / \partial L$ exists at $(\bar{L}, \bar{Z})$ for all $j = 1, \ldots, N$.[1] Then, there is weak (absolute) equilibrium bias at $(\bar{L}, \bar{Z}, \Theta^* (\bar{L}, \bar{Z}))$ if

$$\sum_{j=1}^{N} \frac{\partial w}{\partial \theta_j} \frac{\partial \theta_j^*}{\partial L} \geq 0. \tag{8}$$

This definition requires the (total) induced change in technology resulting from an increase in $L$ to raise the marginal product of labor. The summation ensures that the distinct effects of different components of technology are taken into account on the demand for labor and thus the bias of technology.

PROPOSITION 3. *Let the equilibrium technology at factor supplies $(\bar{L}, \bar{Z})$ be $\Theta^* (\bar{L}, \bar{Z})$ and assume that $\Theta^* (\bar{L}, \bar{Z})$ is in the interior of $\mathcal{O}$ and that $\partial \theta_j^* / \partial Z$ exists at $(\bar{L}, \bar{Z})$ for all $j = 1, \ldots, N$. Then, there is weak (absolute) equilibrium bias at all $(\bar{L}, \bar{Z})$, meaning that*

$$\sum_{j=1}^{N} \frac{\partial w}{\partial \theta_j} \frac{\partial \theta_j^*}{\partial L} \geq 0 \text{ for all feasible } (\bar{L}, \bar{Z}), \tag{9}$$

*with strict inequality if $\partial \theta_j^* / \partial L \neq 0$ for some $j = 1, \ldots, N$.*

This proposition is proved in Acemoglu (2007), and I omit the proof to save space and avoid repetition.

Proposition 3 establishes an unambiguous and at first surprising result: without any further assumptions on the production technology, an increase in the supply of a factor, say labor, always induces technology to become more complementary to that factor. Intuitively, an increase in the supply of a factor raises the benefit of technologies that make better use of that factor, and this leads to change in the bias of technology in favor of that factor.

[1] The assumption that $\partial \theta_j^* / \partial L$ exists at $(\bar{L}, \bar{Z})$ entails two restrictions. The first is the usual nonsingularity requirement to enable an application of the Implicit Function Theorem, that is, that the Hessian of $H$ with respect to $\Theta$, is nonsingular at the point $\Theta^*$. Second, a small change may shift the technology choice from one local optimum to another, in which case $\partial \theta_j^* / \partial L$ is undefined. This possibility is also ruled out by this assumption. The assumption that $\partial \theta_j / \partial L$ exists at $(\bar{L}, \bar{Z})$ can be replaced by an assumption on primitives as shown in Acemoglu (2007). Here, I omit the details.

There is a parallel between Proposition 3 and Samuelson's LeChatelier principle, which states that "long-run" factor demand curves are more elastic than "short-run" factor demand curves that hold some factors constant. Proposition 3, on the other hand, states that long-run changes in marginal products (and factor prices) will be less than those in the short run because of induced technological change. However, there are also some major differences. First, this proposition concerns how marginal products change as a result of technological responses to factor supplies—instead of the elasticity of short-run and long-run demand curves. Second, the result here applies to the equilibrium of an economy, not to the maximization problem of a single firm. This last distinction is central for the next result I present, which shows that labor demand curves can be upward-sloping, a possibility that is ruled out for price-taking firms.

While weak (absolute) bias is about how the technology changes, *strong bias* is about how a change in supplies affects factor prices. In competitive factor markets with exogenous technology, the increase in the supply of a factor, say labor, always reduces its price. Hence, labor demand curves are downward-sloping. Strong bias applies when this result no longer holds—that is, the increase in the supply of labor increases the equilibrium wage. This is now defined more formally.

DEFINITION 3. Denote the equilibrium technology at factor supplies $\left(\bar{L}, \bar{Z}\right)$ by $\Theta^*\left(\bar{L}, \bar{Z}\right)$ and suppose that $\partial\theta_j^*/\partial L$ exists at $\left(\bar{L}, \bar{Z}\right)$ for all $j = 1, \ldots, N$. Then, there is strong (absolute) equilibrium bias at $\left(\bar{L}, \bar{Z}\right)$ if

$$\frac{dw}{dL} = \frac{\partial w}{\partial L} + \sum_{j=1}^{N} \frac{\partial w}{\partial \theta_j}\frac{\partial \theta_j^*}{\partial L} > 0.$$

In this definition, $dw/dL$ denotes the total derivative, while $\partial w/\partial L$ denotes the partial derivative holding $\Theta = \Theta^*\left(\bar{L}, \bar{Z}\right)$. Recall also that if $H$ is jointly concave in $(L, \Theta)$ at $\left(\bar{L}, \Theta^*\left(\bar{L}, \bar{Z}\right)\right)$, its Hessian with respect to $(L, \Theta)$, $\nabla^2 H_{(L,\Theta)(L,\Theta)}$, is negative semidefinite at this point (though negative semidefiniteness is not sufficient for local joint concavity).

PROPOSITION 4. *Assume that $\Theta^*$ is in the interior of $\mathcal{O}$ and that $\partial\theta_j^*\left(\bar{L}, \bar{Z}\right)/\partial L$ exists at $\left(\bar{L}, \bar{Z}\right)$ for all $j = 1, \ldots, N$. Then, there is strong (absolute) equilibrium bias at $\left(\bar{L}, \bar{Z}\right)$ if and only if $H(L, Z, \Theta)'$s Hessian in $(L, \Theta)$, $\nabla^2 H_{(L,\Theta)(L,\Theta)}$, is not negative semidefinite at $\left(\bar{L}, \bar{Z}, \Theta^*\left(\bar{L}, \bar{Z}\right)\right)$.*

Proposition 4 is proved in Acemoglu (2007), and I omit the proof to save space and avoid repetition.

This result shows that with endogenous technology, labor (more generally, factor) demand curves can be upward-sloping, and provides the necessary and sufficient condition for upward-sloping factor demands. With exogenous technology and price-taking firms, downward-sloping factor demands follow from cost minimization when firms take factor prices as given. But the response of technology to factor supplies implies that demand for a factor increases with its supply (Proposition 3), and this induced response can be larger than the direct impact of the supply increase. Acemoglu (1998, 2002) and Acemoglu (2023) discuss a number of applications of this result and the empirical evidence from health care, agriculture, the energy sector, robotics, and economic history, documenting the response of technology to factor supplies and the possibility of upward-sloping factor demand curves.

The necessary and sufficient conditions in the proposition turn on a form of "nonconvexity": factor demand curves are downward-sloping when the production possibilities set is convex, or the maximization problem is concave, in the relevant factor and technology (e.g., in labor and $\Theta$); conversely, they are upward-sloping when there is a nonconvexity at $\left(\bar{L}, \bar{Z}\right)$.

REMARK 2. Acemoglu (2007) establishes that Propositions 3 and 4 hold in the oligopolistic Economy O and in Economy E with externalities, briefly outlined in Remark 1. However, only Proposition 3 holds in Economy D, where each firm chooses its own technology without any interactions with others or externalities. This is intuitive. Proposition 3 relies on properties that apply both in a decentralized equilibrium and in the socially-optimal allocation. In contrast, Proposition 4 requires that there is no (local) concavity in the choice of $(L, \Theta)$. This result thus crucially relies on the fact that technology and factor demands are being decided by different agents. If the same firms chose technology and factor demands and there are no external effects, as in Economy D, then the second-order conditions of these firms' maximization problem would imply that all factor demands are downward-sloping. However, when labor is chosen by final good producers and technology is decided by different firms, as in the setting here, then this type of nonconvexity is not ruled out. This potential nonconvexity is essential for upward-sloping factor demand curves, and we will see that it is also important for the relationship between capital and wages. In summary, Proposition 4 requires some deviation from a fully competitive economy in which firms choose their factor demands as well as technology, without any restrictions and any external effects on others.

## 5. CAPITAL AND WAGES WITH ENDOGENOUS TECHNOLOGY

In this section, I specialize the economy with endogenous technology to one with a single dimension of technology, so that $\Theta = \theta$, and assume that the equilibrium technology $\theta^*$ is interior. I again normalize labor supply to $\bar{L} = 1$. I also set $\Gamma \equiv 0$, so that there are no external costs of choosing different values of $\theta$. This implies that there is no difference between $F$ and $H$ in this section, and the first-order condition for technology choice is

$$F_\theta(\bar{L}, \bar{K}, \theta) = H_\theta(\bar{L}, \bar{K}, \theta) = 0. \tag{10}$$

Acemoglu (2007) provides sufficient conditions for this to be the case and for the second-order conditions to hold with strict inequality (i.e., $h_{\theta\theta}(k, \theta) < 0$). I omit these details here.

We can write the total impact of a greater capital stock on equilibrium wages as

$$\frac{dw}{dK} = \frac{\partial w}{\partial K} + \frac{\partial w}{\partial \theta}\frac{d\theta^*}{dK}, \tag{11}$$

where $\partial w/\partial K$ is the partial derivative, holding technology constant at $\theta = \theta^*$, and thus is identical to the expression derived above:

$$\frac{\partial w}{\partial K} = -\frac{1}{1-\alpha} f_{kk}(k, \theta^*)k > 0.$$

Following the same steps as in Acemoglu (2007), we have

$$\begin{aligned}\frac{\partial w}{\partial \theta} &= \frac{1}{1-\alpha}[f_\theta(k, \theta^*) - f_{\theta k}(k, \theta^*)k] \\ &= -\frac{1}{1-\alpha} f_{\theta k}(k, \theta^*)k,\end{aligned}$$

since in the technology equilibrium, $f_\theta(k, \theta^*) = 0$.

Now, $d\theta^*/dK$ can be obtained from the Implicit Function Theorem applied to the first-order condition (10):

$$\frac{d\theta^*}{dK} = -\frac{f_{\theta k}(k, \theta^*)}{f_{\theta\theta}(k, \theta^*)}. \tag{12}$$

Substituting this into (11), we obtain

$$\frac{dw}{dK} = \frac{1}{1-\alpha}\left[-f_{kk}(k,\theta^*)k + f_{\theta k}(k,\theta^*)\frac{f_{k\theta}(k,\theta^*)k}{f_{\theta\theta}(k,\theta^*)}\right]$$
$$= -\frac{k}{(1-\alpha)f_{\theta\theta}(k,\theta^*)}\left[f_{kk}(k,\theta^*)f_{\theta\theta}(k,\theta^*) - (f_{\theta k}(k,\theta^*))^2\right].$$

Since $f_{\theta\theta}(k,\theta^*) < 0$, we have $\frac{dw}{dK} > 0$ if and only if the square bracketed term is positive. This is, of course, nothing but the condition for the joint concavity of the function $f$ in $(k,\theta)$. If technology were chosen by the same agent as labor demand, this would have to be satisfied by the second-order conditions. But since technology is chosen by a monopolist, while labor demand is decided by final good producers, there is no guarantee that it is satisfied, as explained in the context of the strong bias result above. This discussion establishes:

Proposition 5. *With endogenous technology ($\theta$ responding to capital), a greater capital stock can reduce equilibrium wages. That is, $\frac{dw}{dK} < 0$ is possible.*

*In particular, we have $\frac{dw}{dK} \geq 0$ whenever the per capita production function $f$ is jointly concave in $(k,\theta)$, and $\frac{dw}{dK} < 0$ whenever $f$ is not locally jointly concave (its Hessian is not negative semidefinite) in $(k,\theta)$.*

This proposition also highlights that capital can have either a positive or negative effect on wages, and whether it is the former or the latter depends on whether the indirect effects working via the response of technology to greater capital is greater than the direct impact of capital on wages.

Proposition 5 is a new result relative to those presented in Acemoglu (2007), but has clear parallels to Proposition 4. Both propositions require a local failure of convexity (or the relevant maximization problem not to be locally concave). This is because, as just remarked, in both cases, the new (paradoxical) result obtains when the indirect effects are more powerful than the direct impact.

It is also interesting to consider the conditions under which this type of indirect technology effect can be sufficiently powerful. Clearly, when $f_{\theta k}(k,\theta^*)$ is small, the technology's response to capital in (12) will be limited and consequently, the response of the equilibrium wage to capital will be similar to the exogenous technology case. Hence, we need a strong "complementarity" between technology and capital, which is closely linked to the presence of "automation-type" technologies and underpins the failure of joint concavity of $f$ in $(k,\theta)$. As emphasized in Acemoglu and Autor (2011) and Acemoglu and Restrepo (2018, 2019), automation corresponds to technologies enabling capital to take over tasks previously performed by labor. This increases the importance of capital, raising its marginal product all else equal, and tends to have a negative impact on the marginal product of labor.

In this context, one might conjecture that the direction of change represented by $\theta$ is important. This is not the case, however: the case in which $\theta$ corresponds to automation that complements capital and substitutes for labor, and the one in which it complements labor and substitutes for capital will have similar properties. This is because in one case, a higher capital stock will increase $\theta$, while in the other, it will decrease it, and the implications of $\theta$ for wages are symmetric. [2] Hence, what matters is that the available technologies feature strong complementarities to either capital or labor, which will be the case when $\left|f_{\theta k}(k,\theta^*)\right|$ is high. In the examples discussed in the next section, I will always include automation-type technologies and for specificity, will adopt the convention that higher $\theta$ corresponds to more automation (though this is not important as explained in this paragraph).

[2] This is, in fact, related to the reason why the weak bias result of Acemoglu (2002, 2007) holds regardless of the elasticity of substitution and the exact details of technology—the direction of change is always dictated by the (relative) abundance of factors. See the discussion in Acemoglu (2007).

REMARK 3. The key new insights in Proposition 5 depend on the capital–labor ratio, and I have so far kept the supply of labor fixed to highlight these new insights. There are many ways in which the supply of labor can be endogenized in the context of a model of endogenous technology (e.g., via education choices at birth, as in Acemoglu, 1998, or from a neoclassical utility function defined over consumption and leisure, as in Acemoglu and Restrepo, 2018). Regardless of how labor supply responds, the ultimate effects depend on how much the equilibrium capital–labor ratio changes and the conditions highlighted in Proposition 5. The response of labor supply then matters for the impact on output per worker and dynamics, as explored in Acemoglu (1998) and Acemoglu and Restrepo (2018) in the context of growth models with directed technological change.

## 6. WHEN CAPITAL REDUCES WAGES

In this section, I provide several example economies in which a higher capital stock reduces equilibrium wages.

6.1. *An Example with Linear Technology.* The simplest economy is one in which the marginal product of labor is independent of capital and there is an automation-type technology, $\theta$, which affects the importance of capital and labor. Consider the production function

$$F(L, K, \theta) = (1-\theta)L + \theta K,$$

and assume that $K > L$.

Suppose that the cost of producing this technology is given by $\Gamma(\theta)$, which is assumed to be nondecreasing and convex. The technology equilibrium will then maximize

$$H(L, K, \theta) = (1-\theta)L + \theta K - \Gamma(\theta).$$

Suppose we have an interior solution (which can be guaranteed if we assume that the derivative of $\Gamma(\theta)$ satisfies Inada-type boundary conditions; in particular, $\Gamma'(0) = 0$ and $\lim_{\theta\to 1}\Gamma'(\theta) = \infty$). Then, we have

$$H_\theta(L, K, \theta) = 0 \Longleftrightarrow K - L = \Gamma'(\theta),$$

with the second-order condition

$$H_{\theta\theta} < 0 \Longleftrightarrow \Gamma''(\theta) > 0$$

always satisfied by assumption.

We also have

$$\frac{d\theta}{dK} = \frac{1}{\Gamma''(\theta)} > 0,$$

meaning that a greater stock of capital induces more automation. The equilibrium wage rate is simply $w = 1 - \theta$, because, with the linear production technology, factor usage does not affect the marginal product of labor. The impact of $\theta$ on the wage can then be computed as $\frac{\partial w}{\partial \theta} = -1$, meaning that more automation always reduces the equilibrium wage.

Moreover, we have $\frac{\partial w}{\partial K} = 0$, given the linear technology, and thus

$$\frac{dw}{dK} = \frac{\partial w}{\partial \theta}\frac{d\theta}{dK}$$

$$= -\frac{1}{\Gamma''(\theta)} < 0.$$

This establishes:

PROPOSITION 6. *With a linear aggregate production function and endogenous technology, a higher capital stock always (strictly) reduces the equilibrium wage.*

Intuitively, with a linear production function, the positive effect of the capital stock on the wage is removed, and, given the response of automation to capital, a higher capital stock always reduces the equilibrium wage, establishing Proposition 6.

6.2. *Constant Elasticity of Substitution.* This example generalizes the previous one to a constant elasticity setting: [3] where

$$F(L, K, \theta) = \left[(1-\theta)L^{\frac{\sigma-1}{\sigma}} + \theta K^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}.$$

The cost of choosing the technology is again $\Gamma(\theta) > 0$, where $\Gamma$ is increasing, differentiable, and strictly convex, and its derivative $\Gamma'(\theta)$ satisfies the same boundary conditions as in the previous subsection: $\Gamma'(0) = 0$ and $\lim_{\theta\to 1}\Gamma'(\theta) = \infty$. I also assume that $K > L$. The maximization problem of the technology monopolist is

$$\max_{\theta\in[0,1]} H(L, K, \theta) = \left[(1-\theta)L^{\frac{\sigma-1}{\sigma}} + \theta K^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} - \Gamma(\theta).$$

**The Technology** optimality condition is now:

$$H_\theta = 0 \Longleftrightarrow$$
$$\frac{\sigma}{\sigma-1}\left(K^{\frac{\sigma-1}{\sigma}} - L^{\frac{\sigma-1}{\sigma}}\right)F(L, K, \theta)^{\frac{1}{\sigma}} = \Gamma'(\theta)$$

(which is always satisfied as an equality given $K > L$ and the Inada-type conditions on $\Gamma'(\theta)$ imposed above).

The second-order condition is $H_{\theta\theta} < 0$, and can be written as

$$\frac{\sigma}{(\sigma-1)^2}\left(K^{\frac{\sigma-1}{\sigma}} - L^{\frac{\sigma-1}{\sigma}}\right)^2 F(L, K, \theta)^{\frac{2-\sigma}{\sigma}} - \Gamma''(\theta) < 0.$$

In what follows, I assume that this condition is satisfied, and note that for large values of $\sigma$ (on which I will focus below), the first term disappears and this second-order condition is equivalent to $-\Gamma''(\theta) < 0$, which is satisfied by assumption.

[3] This is related to, but simplified from, the constant elasticity of substitution representation that Acemoglu and Restrepo (2019) derived from a task-based model. The simplification is adopted for expositional clarity and does not have any substantive implications. In particular, Acemoglu and Restrepo (2019) show that aggregate output can be written as $Y = \Pi(\theta)(\zeta(\theta)^{\frac{1}{\sigma}}(A^L L)^{\frac{\sigma-1}{\sigma}} + (1-\zeta(\theta))^{\frac{1}{\sigma}}(A^K K)^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}}$, where $\zeta(\theta) = \int_\theta^1 \psi^L(z)^{\sigma-1}/\Pi(\theta)$ and $\Pi(\theta) = \int_0^\theta \psi^K(z)^{\sigma-1}dz + \int_\theta^1 \psi^L(z)^{\sigma-1}$, with the $\psi$ terms capturing the productivity of capital and labor in different tasks. It can be verified that $\Pi(\theta)$ can be decreasing or increasing in $\theta$, and we are simplifying the exposition here by assuming that it is constant and by approximating $\zeta(\theta)$ by $\theta$.

The response of technology to capital is given by

$$\frac{d\theta}{dK} = \frac{K^{-1/\sigma}F(L,K,\theta)^{\frac{1}{\sigma}} + \frac{\theta}{\sigma-1}K^{-1/\sigma}\left(K^{\frac{\sigma-1}{\sigma}} - L^{\frac{\sigma-1}{\sigma}}\right)F(L,K,\theta)^{\frac{2-\sigma}{\sigma}}}{\Gamma''(\theta) - \frac{\sigma}{(\sigma-1)^2}\left(K^{\frac{\sigma-1}{\sigma}} - L^{\frac{\sigma-1}{\sigma}}\right)^2 F(L,K,\theta)^{\frac{2-\sigma}{\sigma}}}.$$

Although the sign of this expression is in general ambiguous, when $\sigma$ is large, the second term in the numerator and the second term in the denominator become small, and consequently, we have $\frac{d\theta}{dK} > 0$. In other words, for a sufficiently high elasticity of substitution between capital and labor, a greater capital stock always induces more automation.

The equilibrium wage rate is

$$w = (1-\theta)L^{-\frac{1}{\sigma}}F(L,K,\theta)^{\frac{1}{\sigma}},$$

and the impact of $\theta$ on the wage can be written as

$$\frac{\partial w}{\partial \theta} = -L^{-\frac{1}{\sigma}}F(L,K,\theta)^{\frac{1}{\sigma}} + \frac{1-\theta}{\sigma-1}L^{-\frac{1}{\sigma}}\left(K^{\frac{\sigma-1}{\sigma}} - L^{\frac{\sigma-1}{\sigma}}\right)F(L,K,\theta)^{\frac{2-\sigma}{\sigma}}.$$

Moreover, we have

$$\frac{\partial w}{\partial K} = \frac{(1-\theta)\theta}{\sigma}L^{-\frac{1}{\sigma}}K^{-\frac{1}{\sigma}}F(L,K,\theta)^{\frac{2-\sigma}{\sigma}}.$$

Then,

$$\frac{dw}{dK} = \frac{\partial w}{\partial K} + \frac{\partial w}{\partial \theta}\frac{d\theta}{dK}.$$

The sign of this expression is in general ambiguous. But notice that as $\sigma$ gets large, $\frac{\partial w}{\partial K} \to 0$, and $\frac{\partial w}{\partial \theta} \to -1$ (as in the previous example). Moreover, in this case, we also have $\frac{d\theta}{dK} > 0$, as noted above. Therefore, there exists $\sigma^*$ such that for all $\sigma > \sigma^*$, a greater capital stock reduces wages. This discussion establishes:

PROPOSITION 7. *With a constant elasticity of substitution aggregate production function, there exists $\sigma^* < \infty$, such that whenever the elasticity of substitution between capital and labor $\sigma$ is greater than $\sigma^*$, a higher capital stock reduces the equilibrium wage.*

6.3. *A Cobb–Douglas Economy.* I now discuss a simple Cobb–Douglas production function, which will be a key ingredient of the full growth model in the next section. Suppose now that

$$F(L,K,\theta) = (1-\theta)L^{1-\theta}K^{\theta}, \tag{13}$$

and also set $\Gamma = 0$ and assume that $K/L > e$, which, as I show below, ensures that $\ln k > 1$. This functional form is also related to automation. Acemoglu and Restrepo (2018, 2019) establish that when there is automation and tasks are combined with a unit elasticity of substitution, the equilibrium representation of aggregate output takes a Cobb–Douglas form, with exponents corresponding to the extent of automation, as in (13), and I take the term in front to be decreasing in $\theta$ to focus on the more interesting case (and this can be justified with the same arguments as in footnote 3).

The equilibrium wage in this case is

$$w = (1-\theta)^2 k^\theta.$$

Moreover, the first-order condition for technology choice is $-k^\theta + (1-\theta)k^\theta \ln k = 0$, which is always satisfied at an interior solution provided that $\ln k > 1$. The second-order condition, $-\ln k < 0$, is always satisfied. Rearranging the first-order condition, we can conclude that equilibrium technology satisfies

$$1-\theta^* = \frac{1}{\ln k}.$$

Next, observe that

$$\frac{\partial w}{\partial k} = \theta(1-\theta)^2 k^{\theta-1} > 0,$$

while

$$\frac{\partial w}{\partial \theta} = -2(1-\theta)k^\theta + (1-\theta)^2 k^\theta \ln k,$$

which can be negative or positive. Nevertheless, using the equilibrium technology relationship to substitute out $\ln k$, we obtain

$$\frac{\partial w}{\partial \theta} \propto -2(1-\theta^*) + (1-\theta^*) = -(1-\theta^*),$$

and thus, higher $\theta$ always reduces wages.

Finally, we have

$$\frac{d\theta^*}{dK} = \frac{1}{k(\ln k)^2} > 0.$$

Therefore, a higher capital stock always induces greater automation, increasing $\theta$. Moreover, this indirect effect of capital on wages becomes negative and dominates the direct effect, provided that the capital–labor ratio is not too high. Specifically,

$$\begin{aligned}
\frac{dw}{dK} &= \frac{\partial w}{\partial k} + \frac{\partial w}{\partial \theta^*}\frac{d\theta^*}{dK} \\
&= k^{\theta^*-1}\left[\theta^*(1-\theta^*)^2 - 2(1-\theta^*)\frac{1}{(\ln k)^2} + (1-\theta^*)^2\frac{1}{\ln k}\right] \\
&= k^{\theta^*-1}(1-\theta^*)^2[\theta^* - (1-\theta^*)] \\
&= -k^{\theta^*-1}(1-\theta^*)^2(1-2\theta^*).
\end{aligned}$$

This expression is negative if and only if

$$\theta^* < \frac{1}{2} \Longleftrightarrow \ln k < 2.$$

Hence we have:

PROPOSITION 8. *With the Cobb–Douglas production function given in (13), a higher capital stock reduces the equilibrium wage provided that* $\ln k < 2$.

This proposition establishes that, consistent with Proposition 5, the total impact of greater capital abundance on the equilibrium wage can be negative or positive. When $\ln k < 2$, a greater capital stock generates a powerful automation response, and in this range, automation reduces wages. Consequently, despite the direct positive effect of capital on wages, the total impact is negative. In contrast, when $\ln k > 2$, the direct effect is more powerful than the indirect effect, and the total impact is positive, as in the neoclassical benchmark.

To understand the intuition, first note that the more novel, paradoxical case (when $\ln k < 2$) requires that the negative displacement effect of automation-type technologies is stronger than their positive productivity effect (see Acemoglu and Restrepo, 2018, 2019). If the productivity effect, driven by the substitution of cheaper capital for more expensive labor, were more powerful than the displacement effect, the impact of technology on wages would be positive and could not reverse the positive direct effect of capital on wages. In addition, we also need the response of technology to capital to be sufficiently pronounced. The condition $\ln k < 2$ ensures both of these.

Overall, we obtain a negative relationship between capital and wages, when (i) the direct positive effect of the capital stock on the wage is not too large; (ii) the automation response is strong; and (iii) the productivity effect from automation is not too large. This intuition also sheds light on why there is no negative relationship between capital and wages when $\ln k$ is greater than 2. In this case, $\theta$ is already high, so a further increase does not create much displacement but raises productivity, and via this channel, it tends to boost the equilibrium wage.

## 7. ENDOGENOUS GROWTH WITH A MENU OF TECHNOLOGIES

For much of the 20th century, capital accumulation in the industrialized world went hand-in-hand with rising wages (e.g., Barro and Sala-i-Martin, 2008; Acemoglu, 2009; Gordon, 2016). Hence, the simple negative relationship highlighted in the previous two sections cannot account for the long-run association between capital and wages. In this section, I develop an endogenous growth model with a menu of technologies—automation and labor-augmenting technological change—and show that secular increases in the equilibrium wage can coexist together with a negative relationship between capital intensity and labor demand.

Specifically, I consider a dynamic economy in discrete time with two technologies, $\Theta_t = (\theta_t, A_t)$ and assume that the aggregate production function is

$$Y_t = (1-\theta_t)K_t^{\theta_t}(A_tL)^{1-\theta_t}. \tag{14}$$

I assume, to simplify the analysis of dynamics, that the economy is inhabited by a representative household with a constant saving rate $s \in (0,1)$ and normalize labor supply to $\bar{L} = 1$. Hence, aggregate and per capita consumption is

$$C_t = (1-s)(1-\theta_t)K_t^{\theta_t}A_t{}^{1-\theta_t}.$$

I also simplify the notation by assuming that capital does not depreciate. The evolution of the capital stock is then given by

$$K_{t+1} = K_t + s(1-\theta_t)K_t^{\theta_t}A_t{}^{1-\theta_t}. \tag{15}$$

The cost of choosing technology combination $(\theta_t, A_t)$ at time $t$ is assumed to be

$$\gamma\left(\frac{A_t}{A_{t-1}}\right)Y_t,$$

where $\gamma$ is differentiable, increasing and strictly convex, with derivative denoted by $\gamma'$. I also impose the following Inada-type condition: $\gamma'(1) = 0$. This specification implies that current increases in the labor-augmenting technology $A_t$ build on past advances. Additionally, $Y_t$ is included in the cost so that the cost of increasing $A_t$ over time is proportional to current output, and I simplify the analysis by assuming that the technology monopolist takes $Y_t$ as given in evaluating costs.[4]

Note finally that varying $\theta_t$ has no external technology costs beyond its impact via (14), as in the Cobb–Douglas example in the previous section.

With a slight abuse of notation, I now define $k_t$ to be the effective capital–labor ratio, given by

$$k_t \equiv \frac{K_t}{A_t}.$$

I take the initial endowment of capital $K_0$ and the initial labor-augmenting technology $A_0$ to be such that $\ln k_0 > 1$, as imposed in the previous section in the context of the analysis of the Cobb–Douglas economy.

With this notation, the equilibrium wage is

$$w_t = (1-\theta_t)^2 A_t k_t^{\theta_t}. \tag{16}$$

Moreover, the first-order conditions for the two components of technology are given as

$$\frac{(1-\theta_t)Y_t}{A_t} = \frac{1}{A_{t-1}}\gamma'\left(\frac{A_t}{A_{t-1}}\right)Y_t. \tag{17}$$

and $-k_t^{\theta_t} + (1-\theta_t)k_t^{\theta_t}\ln k_t = 0$, which can again be simplified to:

$$1-\theta_t = \frac{1}{\ln k_t}. \tag{18}$$

This equation clarifies why $\ln k_t$ has to be greater than 1, since otherwise the technology first-order condition would have a corner solution. Moreover, as in the previous section, I will typically restrict $\ln k_t < 2$, to focus on the more interesting case where an increase in the capital stock reduces the equilibrium wage rate.

Combining the two first-order conditions gives:

$$1-\theta_t = \frac{A_t}{A_{t-1}}\gamma'\left(\frac{A_t}{A_{t-1}}\right).$$

Therefore, $1-\theta_t = (1+g_t)\gamma'(1+g_t)$, where $1+g_t = \frac{A_t}{A_{t-1}}$. This equation always holds in equilibrium given the assumption that $\gamma'(1) = 0$. For notational convenience, I define:

$$1-\theta = Z(1+g) \equiv (1+g)\gamma'(1+g), \text{ and}$$
$$1+g = G(1-\theta) \equiv Z^{-1}(1-\theta).$$

[4] This is just for simplicity in the text. Consider the alternative, slightly more complicated cost function: $\left[1-\exp\left(-\gamma\left(\frac{A_t}{A_{t-1}}\right)\right)\right]Y_t$. Then, the maximization problem becomes $\max Y_t \exp\left(-\gamma\left(\frac{A_t}{A_{t-1}}\right)\right)$, which is also equivalent to maximizing $\ln Y_t - \gamma\left(\frac{A_t}{A_{t-1}}\right)$, because log is a monotone function and thus its maximizers coincide with the maximizers of the original function. The first-order conditions of this maximization are identical to (17) and (18).

Notice that because $\gamma$ is convex, $Z$ is increasing, and so is $G$. Moreover, in what follows, I assume that $G$ is strictly concave.[5] With this notation, we can write

$$1 + g_t = G(1 - \theta_t). \tag{19}$$

Finally, rewriting (15) in terms of the effective capital–labor ratio $k_t$, equilibrium dynamics satisfy

$$k_{t+1} = \frac{1}{1 + g_{t+1}} \Big[ k_t + s(1 - \theta_t) k_t^{\theta_t} \Big], \tag{20}$$

where I used the fact that $A_{t+1}/A_t = 1 + g_{t+1}$. Given the initial conditions $k_0$ and $A_0$, a dynamic equilibrium path $\{k_t, A_t, \theta_t\}_{t=0}^{\infty}$ is characterized by Equations (18)–(20).[6] Equilibrium aggregate output and consumption can be obtained from these variables. Finally, the equilibrium wage rate is given by (16).

Let us define a balanced growth path (BGP) as an equilibrium in which $\theta_t^*$ and $k_t$ are constant, and $A_t^*$ grows at a constant rate $g_t = g^*$. Then, a BGP, represented by $(k^*, \theta^*, g^*)$, satisfies:[7]

$$G\left(\frac{1}{\ln k^*}\right) - 1 = \frac{s}{e} \frac{1}{\ln k^*}, \tag{21}$$

with

$$1 - \theta^* = \frac{1}{\ln k^*}, \text{ and} \tag{22}$$

$$g^* = G\left(\frac{1}{\ln k^*}\right) - 1. \tag{23}$$

Notice also that the BGP growth rate of wages is given by $g^*$ in view of (16) and the fact that $k_t$ and $\theta_t$ are constant.

I will also assume that

$$\begin{gathered} G(1) < 1 + \frac{s}{e}, \text{ and} \\ G\left(\frac{1}{2}\right) > 1 + \frac{1}{2}\frac{s}{e}. \end{gathered} \tag{24}$$

The conditions in (24) ensure that the (unique) solution to (21) satisfies $\ln k^* \in (1, 2)$, as I explain below. [8]

[5] The elasticity of the $\gamma''$ function being less than 2, that is, $-(1+g)\gamma'''(1+g)/\gamma''(1+g) < 2$ for all $g > 0$, is sufficient for the concavity of $G$.

[6] There is no initial condition for $\theta$, which is not a state variable and adjusts immediately. Hence, $\theta_0$ is an equilibrium object.

[7] To obtain this, substitute from (18) and (19) into (20), use the fact that, for any $x > 0$, $x^{-\frac{1}{\ln x}} = e^{-1}$, and then impose $k_t = k_{t+1} = k^*$.

[8] In terms of $\gamma$, (24), can be written as

$$\begin{gathered} 1 < \left(1 + \frac{s}{e}\right)\gamma'\left(1 + \frac{s}{e}\right) \text{ and} \\ \frac{1}{2} > \left(1 + \frac{1}{2}\frac{s}{e}\right)\gamma'\left(1 + \frac{1}{2}\frac{s}{e}\right). \end{gathered}$$

Proposition 9. *Suppose that $G$ is strictly concave and satisfies (24). Then, there exists a unique BGP where labor-augmenting technology $A_t$, the capital stock $K_t$, GDP $Y_t$, and aggregate consumption $C_t$ all grow at the rate $g^*$ and the effective capital–labor ratio $k^*$ is constant and satisfies (21) with $\ln k^* \in (1, 2)$. Given $k^*$, $\theta^*$ is constant and satisfies (22), with $\theta^* < 1/2$, while $g^*$ is given by (23).*

*This unique BGP is asymptotically stable, meaning that starting with any initial conditions $k_0 \in (e, e^2)$ and $A_0$, the dynamic equilibrium converges to the unique BGP $(k^*, \theta^*, g^*)$. Moreover, this convergence is monotone.*

Proof. First, I prove that a BGP $(k^*, \theta^*, g^*)$ satisfies (21), (22), and (23) and exists. A preliminary step is to observe that $G(1/\ln k)\,k$ is strictly increasing in $k$ whenever $\gamma$ is convex, as assumed. This follows because

$$\begin{aligned}\frac{d\ln\left(G\left(\frac{1}{\ln k}\right)k\right)}{d\ln k} &= 1 - \frac{G'\left(\frac{1}{\ln k}\right)}{G\left(\frac{1}{\ln k}\right)}\left(\frac{1}{\ln k}\right)^2\\ &= 1 - \frac{1}{\ln k}\frac{1}{G\left(\frac{1}{\ln k}\right)\frac{\gamma''\left(G\left(\frac{1}{\ln k}\right)\right)}{\gamma'\left(G\left(\frac{1}{\ln k}\right)\right)}+1} > 0,\end{aligned}$$

where the last inequality exploits the fact that, given the convexity of $\gamma$ and $\ln k > 1$, the second term in the second line is always less than 1.

I next establish that (18)–(20) **define a well-defined** dynamical system. With the same steps as outlined before the proposition, Equation (20), combined with (18), can be written as

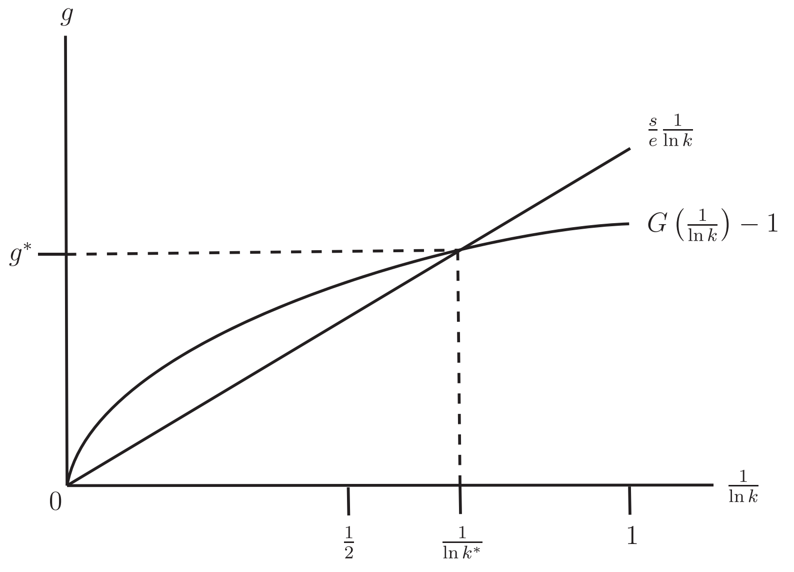
$$G\left(\frac{1}{\ln k_{t+1}}\right)k_{t+1} = \left(1 + \frac{s}{e}\frac{1}{\ln k_t}\right)k_t \text{ for all } k_t > 0. \tag{25}$$

The left-hand side of (25) satisfies $G(1)e < e + s$ by (24), and $\lim_{k\to\infty} G(1/\ln k)k = \infty$, and thus, for any $k_t$, there exists a solution $k_{t+1} \in (e, \infty)$ to this equation by the Intermediate Value Theorem. Moreover, because $G(1/\ln k)k$ is strictly increasing, this solution is unique, establishing that this relationship defines a first-order difference equation. The effective capital–labor ratio $k$ must be constant in BGP, and hence, its BGP value $k^*$ can be written as a fixed point of this difference equation, which gives (21).

Given (24), the Intermediate Value Theorem ensures the existence of a solution $\ln k^* \in (1, 2)$. Since $G$ is concave and given (24), (21) can have at most one solution on $k \in (0, \infty)$, as illustrated in Figure 1, and (24) guarantees that this solution is between $\ln k = 1$ and $\ln k = 2$.

Next, I verify that this BGP is asymptotically stable under (24). The preceding argument establishes that (25) is a well-defined difference equation and has a single fixed point, and moreover, as implied by Figure 1, the left-hand side of (25) intersects the right-hand side from above. First recall that, as established above, $G(1/\ln k)\,k$ is strictly increasing in $k$. Moreover, the right-hand side of (25), $\left(1 + \frac{s}{e}\frac{1}{\ln k}\right)k$, is also increasing in $k$, which follows by direct differentiation. Now consider any $k_t \in (e, k^*)$. Then, (25) implies that $k_{t+1} < k^*$. To see this, observe that:

$$\begin{aligned}G\left(\frac{1}{\ln k_{t+1}}\right)k_{t+1} &= \left(1 + \frac{s}{e}\frac{1}{\ln k_t}\right)k_t\\ &< \left(1 + \frac{s}{e}\frac{1}{\ln k^*}\right)k^*\\ &= G\left(\frac{1}{\ln k^*}\right)k^*,\end{aligned}$$




NOTES: $G\left(\frac{1}{\ln k}\right)-1$ is the left-hand side of (21), while $\frac{s}{e}\frac{1}{\ln k}$ is the right-hand side. The two curves both start at 0, and given the concavity of $G$, there can be at most one intersection where the left-hand side intersects the right-hand side from above. The first condition in (24) ensures that there is always such an intersection and hence a unique BGP.

FIGURE 1

EXISTENCE AND UNIQUENESS OF THE BGP

where the first line repeats (25), while the second line uses the fact that the right-hand side of (25) is strictly increasing, as stated above, and $k_t < k^*$.

Next, also observe that $k_{t+1} > k_t$. This follows because

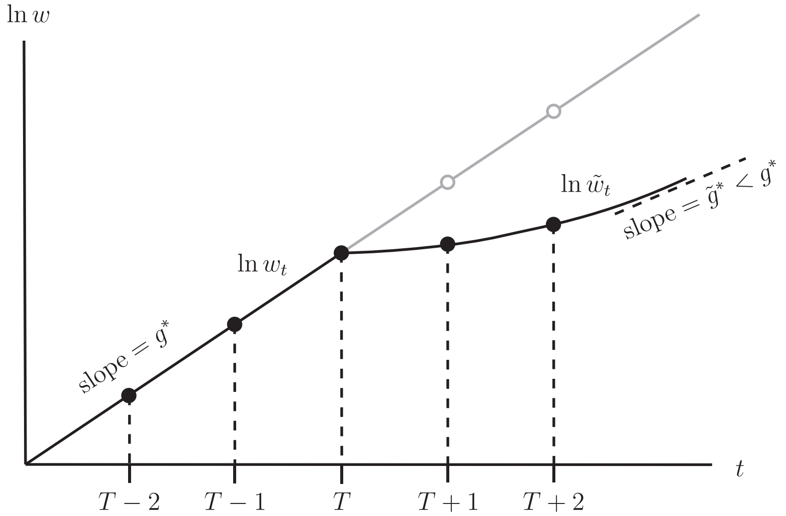
$$G\left(\frac{1}{\ln k_{t+1}}\right)k_{t+1} = \left(1+\frac{s}{e}\frac{1}{\ln k_t}\right)k_t > G\left(\frac{1}{\ln k_t}\right)k_t,$$

in view of the fact that $k_t < k^*$. Since the left-hand side of (25) is strictly increasing, as established above, the outer inequality implies that $k_{t+1} > k_t$.

These two steps together imply $k_{t+1} \in (k_t, k^*)$, and thus, the effective capital–labor ratio monotonically converges to the BGP $k^*$ from below. With the same argument, starting with any $k_t > k^*$, we have $k_{t+1} \in (k^*, k_t)$, guaranteeing that this time there will be monotonic convergence from above to the unique BGP $k^*$. Given this, $\theta_t$ and $g_t$ also monotonically converge to their unique BGP values $(\theta^*, g^*)$. The proof for the case $k_t \in (k^*, e^2)$ is analogous, completing the proof.

Proposition 9 establishes the existence of a unique and asymptotically stable BGP, in which there is constant labor-augmenting technological change, but in addition, the extent to which tasks are automated is also endogenously determined. In addition, the assumption that $G\left(\frac{1}{2}\right) > 1+\frac{1}{2}\frac{s}{e}$ (imposed in (24)) ensures that this unique BGP involves $\ln k^* < 2$, placing us in the range where a greater capital stock reduces the equilibrium wage.

Let us next consider the comparative dynamics of this BGP in response to an increase in the saving rate $s$. It is straightforward to verify that this raises the BGP effective capital–labor ratio $k^*$ given by (21). Consequently, the BGP value of $\theta^*$ also increases from (22), but the growth rate of labor-augmenting technology $A_t$ decreases from (23). Hence, even though wages continue to grow in the BGP, their growth rate is reduced. In addition, we can trace the full equilibrium response of wages to this increase in the saving rate of the economy. As soon as the saving rate increases, $\theta_t$ also increases with $k_t$ as dictated by (18). What about $A_t$? From




NOTES: This reduces the capital stock at $T+1$, and reduces the new equilibrium wage $\tilde{w}_{T+1}$ below what it would have been without the change in saving rate. The growth rate of the new equilibrium wage, $\tilde{g}^*$, converges to a lower value than its growth rate before the change in the saving rate, $g^*$.

FIGURE 2

WAGE DYNAMICS AFTER A PERMANENT INCREASE IN THE SAVING RATE *s AT TIME* $T$.

(19), greater $\theta_t$ leads to a lower rate of increase of $A_t$. Hence, the immediate impact of the higher saving rate is to reduce the equilibrium wage relative to the counterfactual of a constant saving rate.[9]

We summarize this discussion in the next proposition and Figure 2 (proof in the text).

PROPOSITION 10. *Consider a permanent increase in the saving rate s. This immediately reduces the equilibrium wage relative to the baseline of constant saving rate and also depresses the rate of labor-augmenting productivity growth. In the long run, the economy converges to a new BGP in which technology involves greater automation and the growth rate of the equilibrium wage is lower.*

The consequences of Proposition 10 are illustrated in Figure 2. Until time $T$, the saving rate is constant at $s$ and the economy is assumed to be in BGP, so the equilibrium wage grows at the rate $g^*$. At $T$, the saving rate increases to $s' > s$. This leads to a larger increase in the capital stock at time $T+1$, $K_{T+1}$, and the new equilibrium wage $\tilde{w}_{T+1}$ drops below $w_{T+1}$. Thereafter, the rate of labor-augmenting technological change slows down, so the growth rate of the equilibrium wage $\tilde{w}_t$ converges to $\tilde{g}^* < g^*$. Hence, this proposition shows that the economic forces highlighted in our static model are present in this dynamic setup. In particular, greater capital abundance induces further automation, potentially harming workers. Moreover, this possibility does not contradict the growth of equilibrium wages together with technological change and capital accumulation along the BGP.

Finally, this section focused on the case in which $\theta$—as an automation technology—has a potentially large negative effect on wages. Alternative formulations, for example, where the impact of $\theta$ on wages is more muted, can lead to situations in which an increase in the saving rate can temporarily reduce the equilibrium wage, without affecting its long-run growth

[9] Whether the equilibrium wage actually declines depends on how strong the response of $A_t$ is. Holding $A_t$ constant, the increase in $\theta$ between $T$ and $T+1$ would lead to a lower equilibrium wage, since we are in the range where $\ln k^* \in (1,2)$. However, between these two dates, $A_t$ increases as well. If $\gamma'$ is high and the equilibrium $g_T$ is low, the effect through $\theta$ can dominate and we may first get a decline in the equilibrium wage. But in any case, the equilibrium wage always falls below its counterfactual trajectory under the constant saving rate.

rate. I chose the aggregate production function in (14) to highlight the most novel results of the framework.

## 8. CONCLUSION

A celebrated result in neoclassical growth theory maintains that a greater capital stock—and hence capital accumulation—raises labor demand and equilibrium wages. This is a critical channel for "shared prosperity" working via "trickle-down" —the market process ensuring that higher wealth or capital income translates into greater wages. In this case, any process of capital accumulation (e.g., because investment has become more profitable, firms have greater retained earnings, or capitalists have greater wealth) will create a powerful force toward some of these gains being shared with workers, whose incomes depend on the labor market wage.

The last several decades during which wages have stagnated in the United States and have increased only slowly in many other industrialized nations pose a challenge for this perspective, however.

In this article, I argued that the impact of a greater stock of capital on wages is more complex, because technology responds to the availability of more abundant capital. Under reasonable conditions, a greater capital stock induces further automation, and automation could reduce (real) wages.

I showed that the conditions under which more capital reduces equilibrium wages are strongly tied to the conditions under which the demand for labor (or other factors) are upward-sloping, because of technology responses (e.g., Acemoglu, 2002, 2007). I then illustrated how simple economies with broadly neoclassical features can reverse this result as soon as the direction of technology is endogenized.

The last part of the article shows that the economic forces I have emphasized do not imply that wages should fall steadily along the process of economic growth with capital accumulation. I constructed a model of endogenous growth with a menu of technologies, whereby firms decide both the extent of automation and the pace of labor-augmenting technological change. The long-run equilibrium of the economy involves constant wage growth. Nevertheless, an increase in the saving rate has both a negative impact effect and leads to lower long-run growth rate of wages, because it induces greater automation.

Future interesting directions of research include more detailed analysis of the interplay between capital accumulation, technology choices, and wages in models in which there are noncompetitive elements in product or labor markets, as well as even richer menus of technologies available to firms. For example, the issues raised in this article become particularly important in cases where sustained capital accumulation associated with certain types of technological advances, such as those triggered by artificial intelligence or robotics, can drive economic growth, but whether this will benefit workers and wages remains an open question. An extended version of the framework developed here could be used to study the two-way interactions between capital accumulation and the direction of technological change.

An even more important area for future research is the empirical exploration of the channels highlighted by the current framework. Important empirical directions include the estimation of how the direction of technology responds to greater abundance of capital at the aggregate, local economy and firm level. A variety of drivers of capital investment could be leveraged to explore how an unanticipated increase in the abundance of capital (or a decline in the cost of capital) changes firms' incentives concerning the direction of technology adoption or innovation. Ultimately, it would be worthwhile to systematically explore the joint dynamics of capital, technology (automation), and equilibrium wages.

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