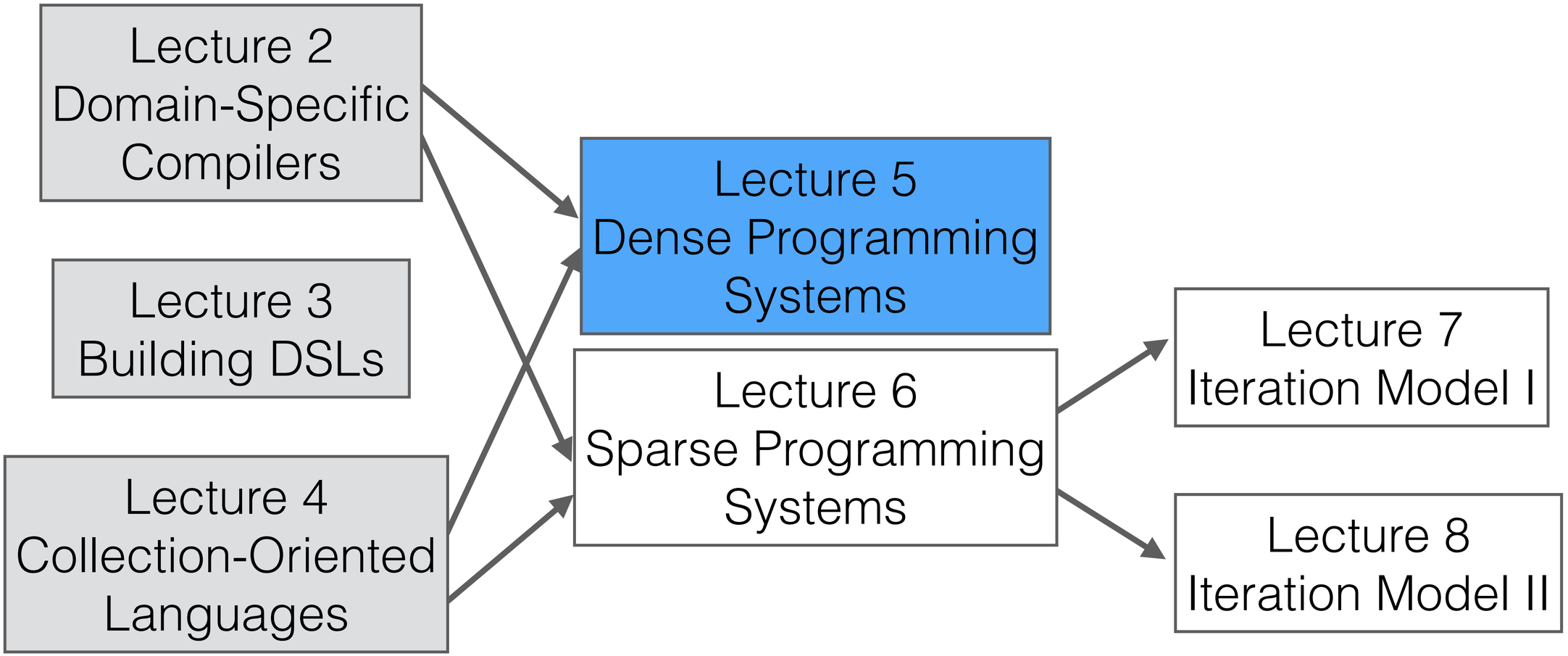
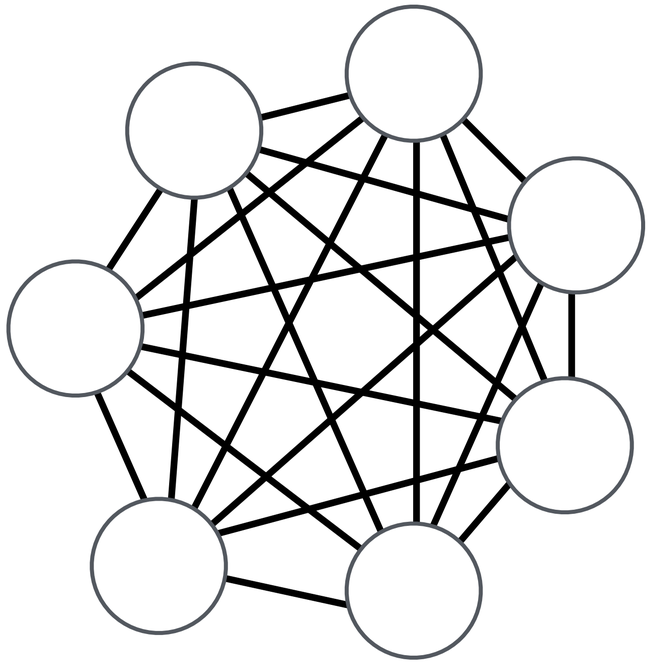
Lecture 5 — Dense Programming Systems

Stanford CS343D (Winter 2026)
Fred Kjolstad

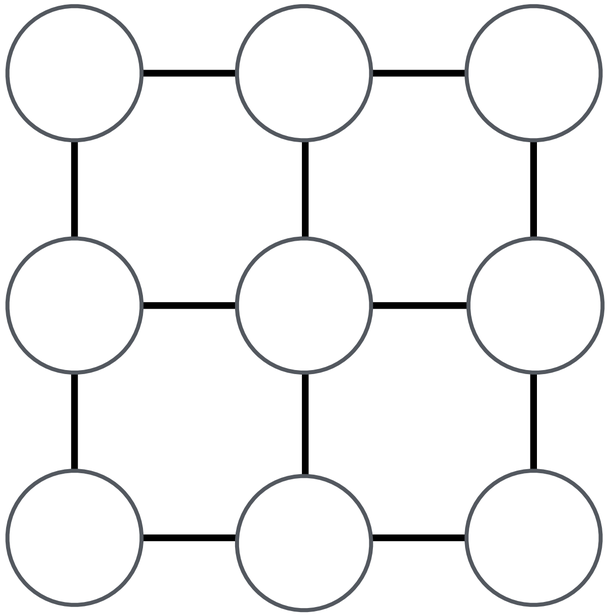

Lecture 2
Domain-Specific
Compilers
Lecture 3
Building DSLs
Lecture 4
Collection-Oriented
Languages
Lecture 5
Dense Programming
Systems
Lecture 6
Sparse Programming
Systems
Lecture 7
Iteration Model I
Lecture 8
Iteration Model II

Terminology: Regular and Irregular

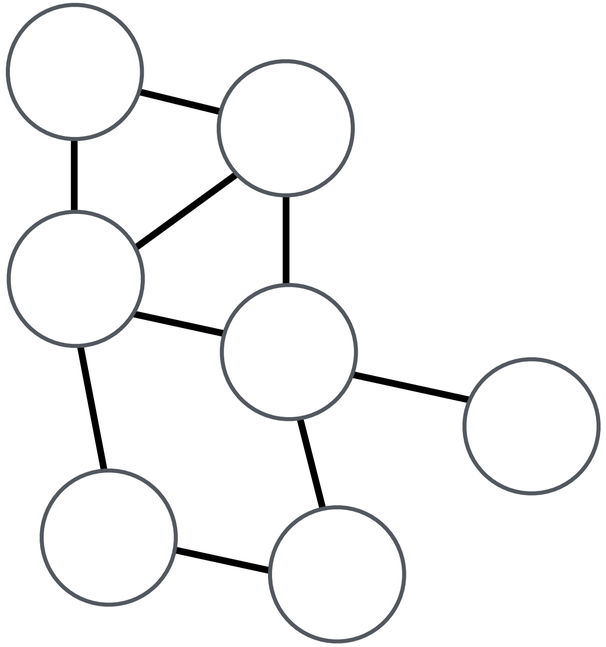
Fully Connected Regular System



Regular System

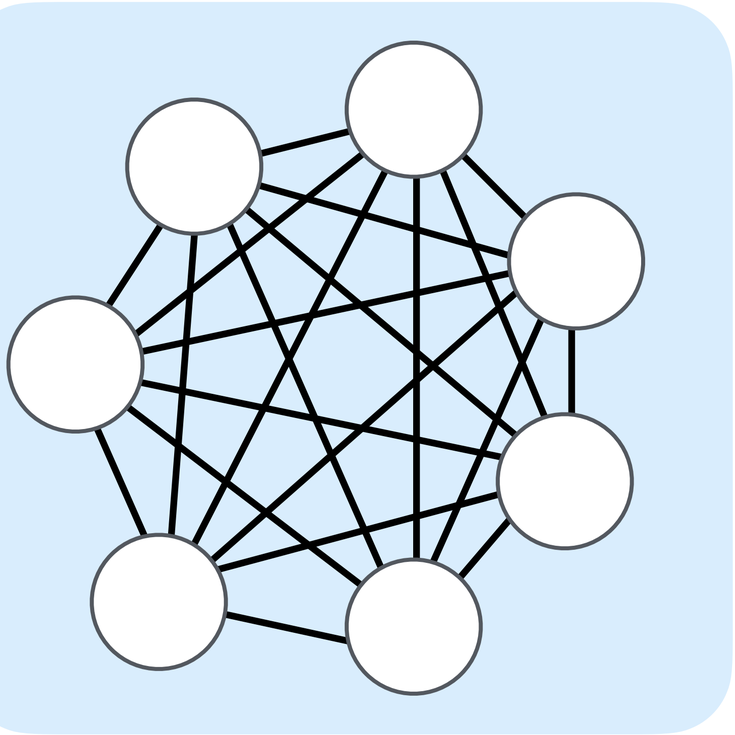


Irregular System

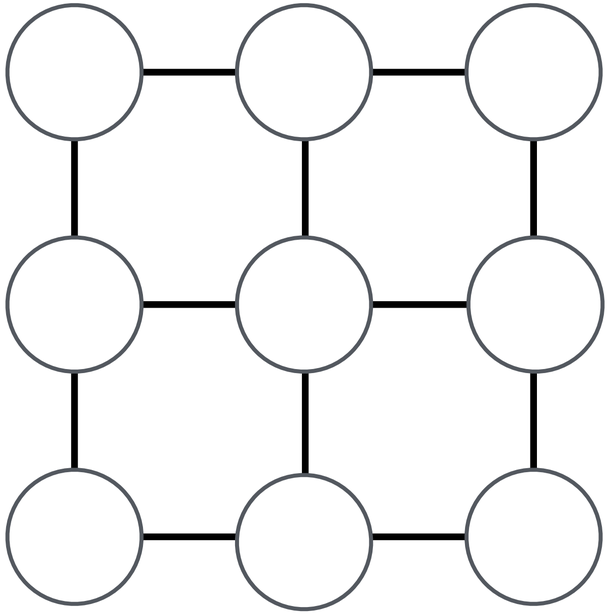


Terminology: Regular and Irregular

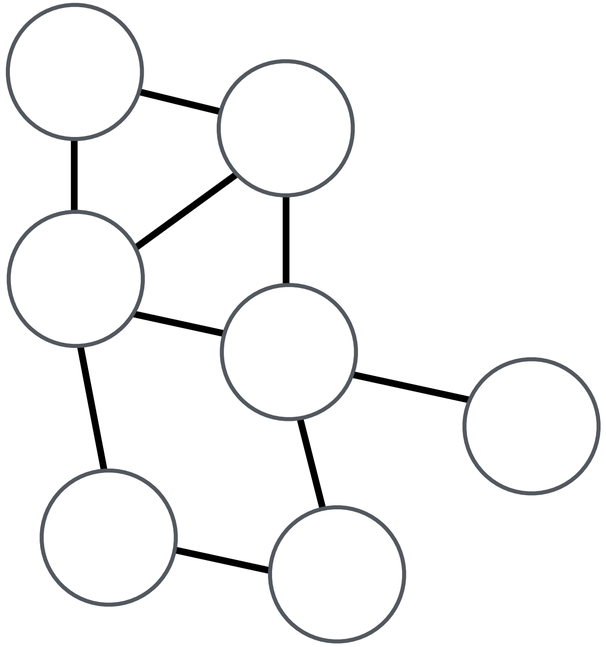
Fully Connected Regular System



Regular System

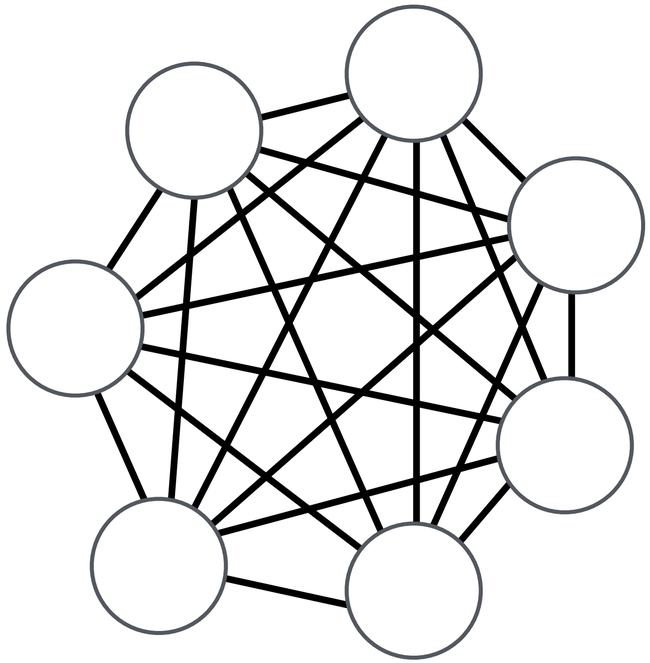


Irregular System

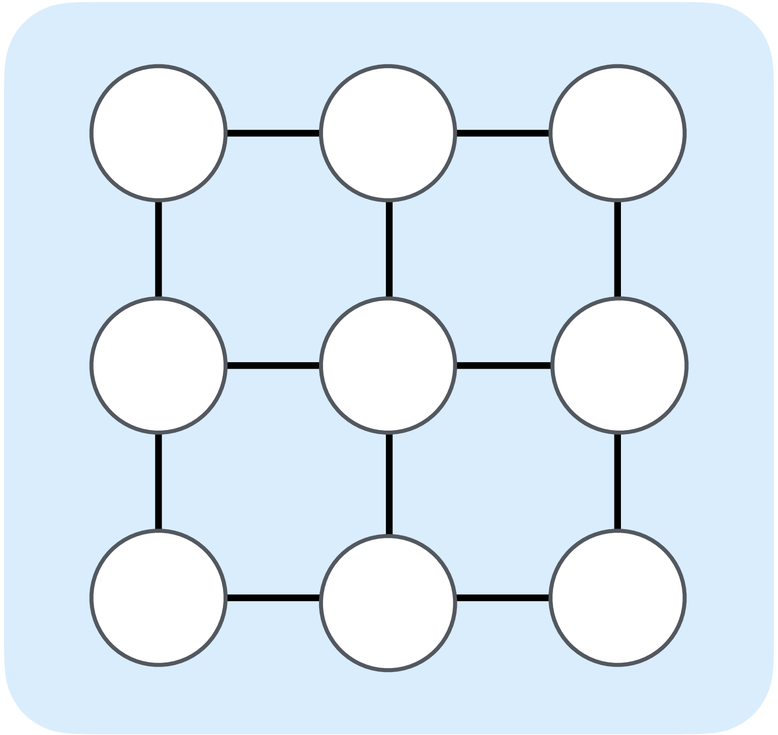


Terminology: Regular and Irregular

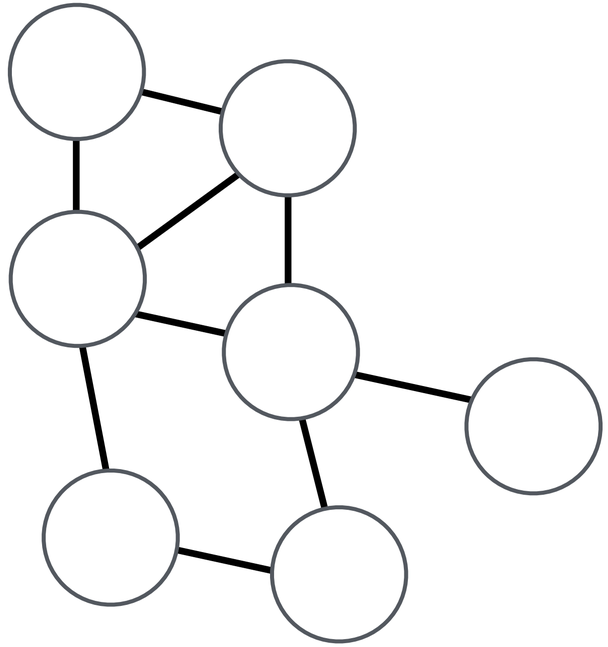
Fully Connected Regular System



Regular System

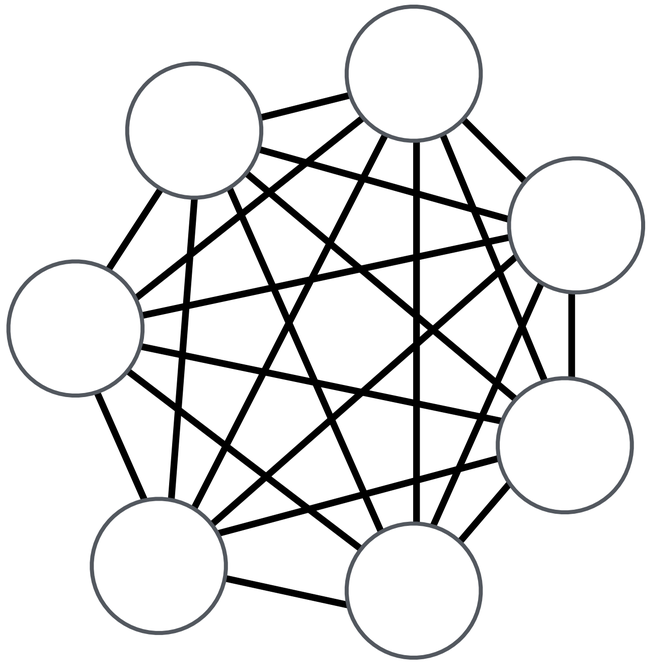


Irregular System

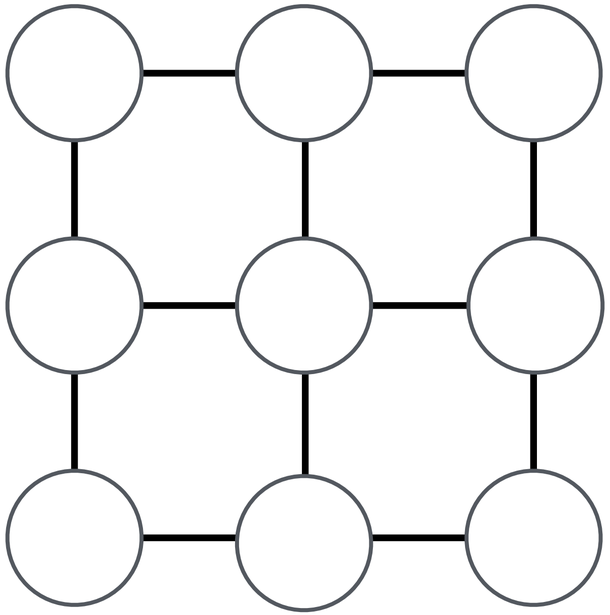


Terminology: Regular and Irregular

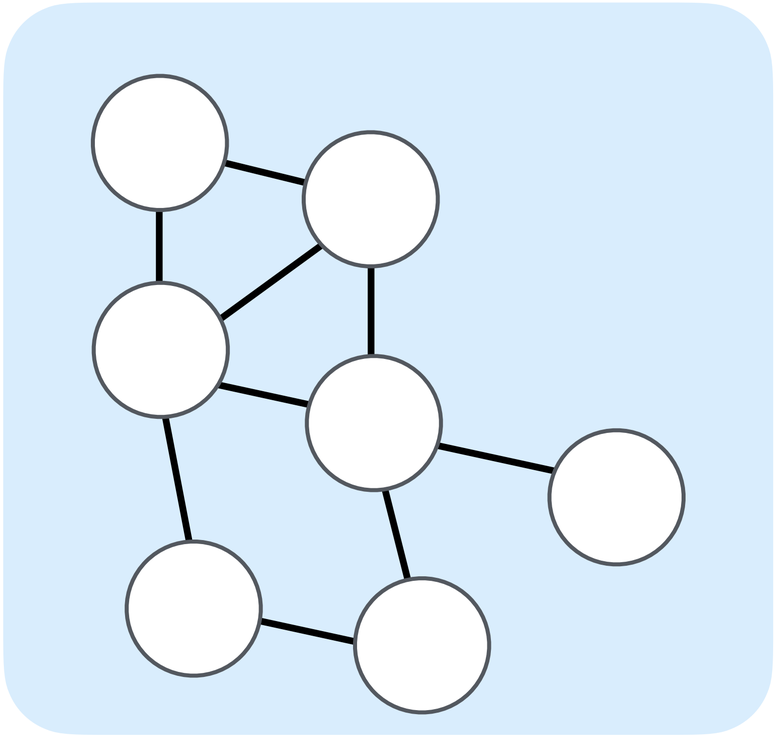
Fully Connected Regular System



Regular System

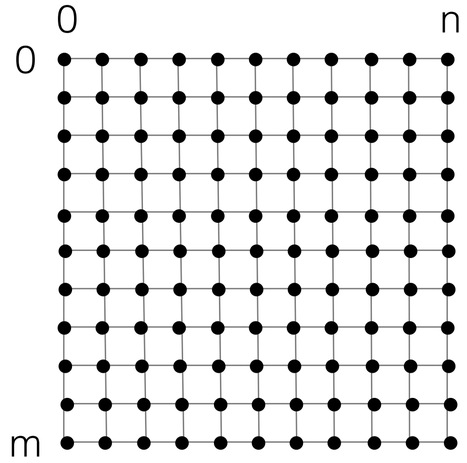


Irregular System



Terminology: Dense and Sparse

Dense loop iteration space

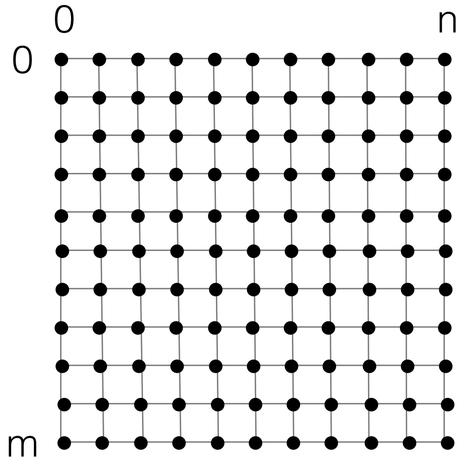



```
for (int i = 0; i < m; i++) {
  for (int j = 0; j < n; j++) {
    y[i] += A[i*n+j] * x[j];
  }
}
```

$$y = Ax$$

Terminology: Dense and Sparse

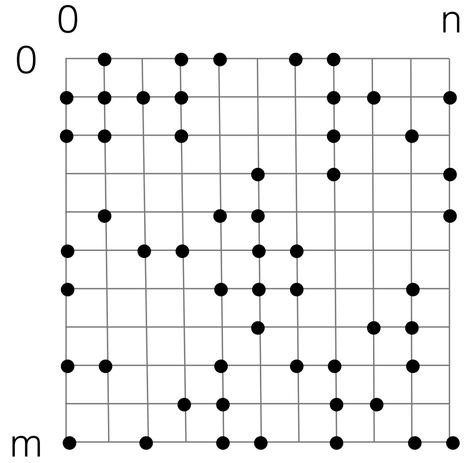
Dense loop iteration space




```
for (int i = 0; i < m; i++) {
  for (int j = 0; j < n; j++) {
    y[i] += A[i*n+j] * x[j];
  }
}
```

$$y = Ax$$

Sparse loop iteration space

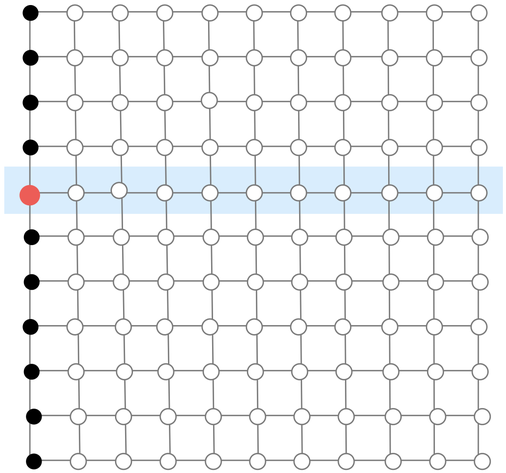



```
for (int i = 0; i < m; i++) {
 for (int pA = A2_pos[i]; pA < A_pos[i+1]; pA++) {
    int j = A_crd[pA];
    y[i] += A[pA] * x[j];
  }
}
```

$$y = Ax$$

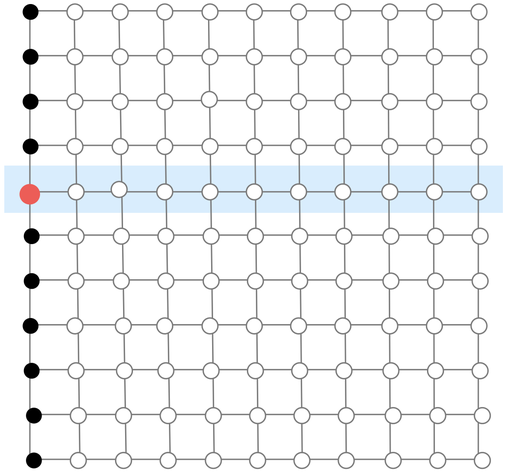
Dense operations

Dense Matrix-Vector Multiplication

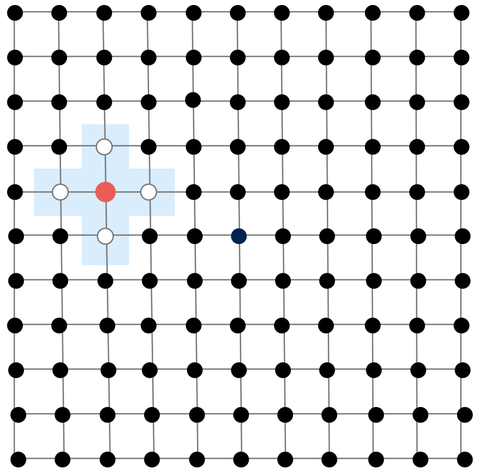


Dense operations

Dense Matrix-Vector Multiplication

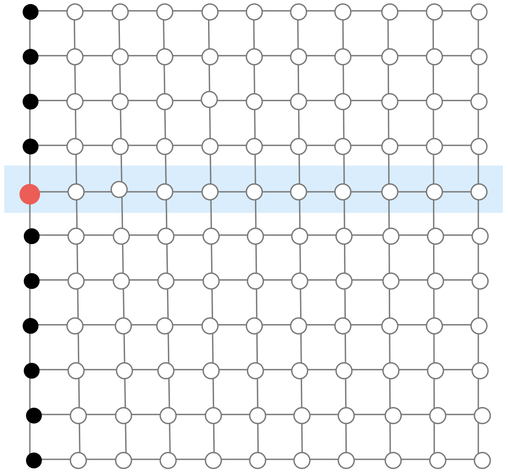


Stencils

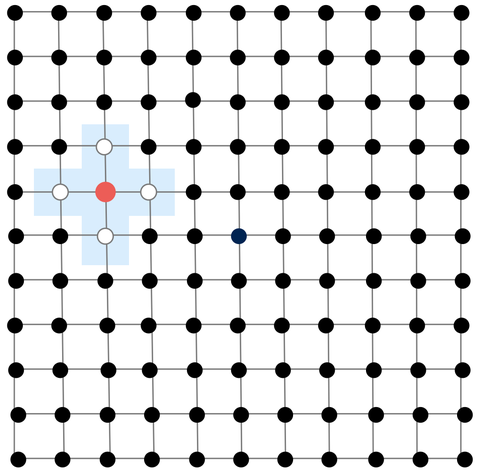


Dense operations

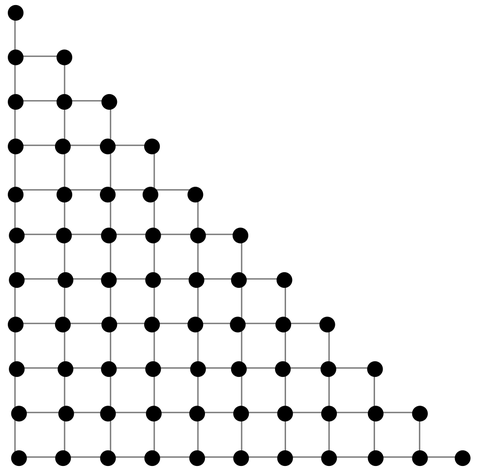
Dense Matrix-Vector Multiplication



Stencils

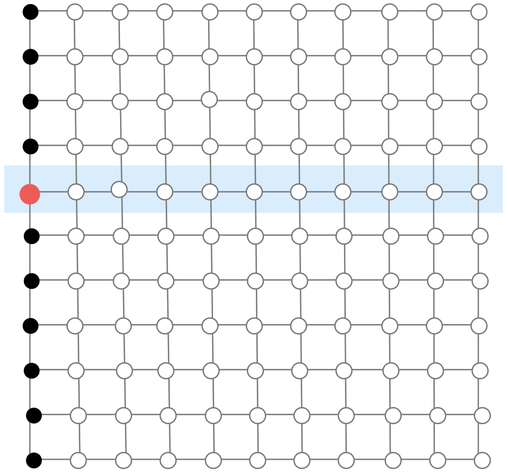


Triagonal Solve

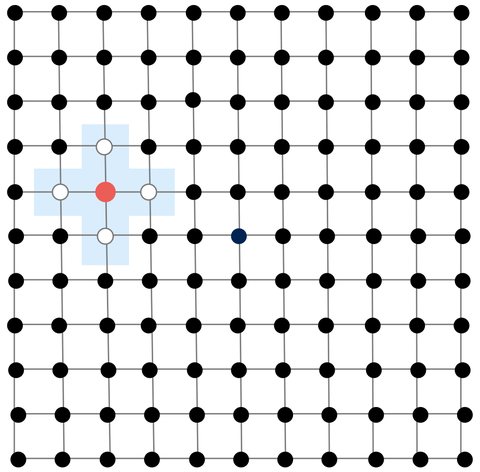


Dense operations

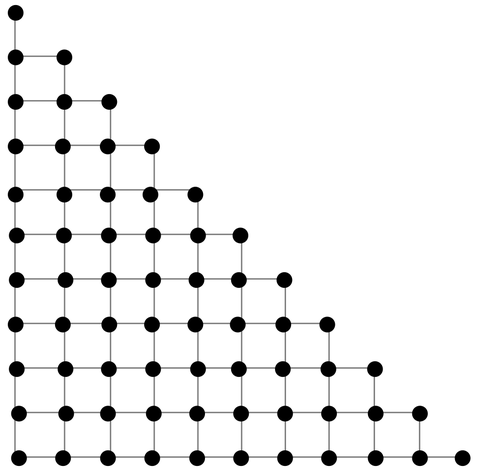
Dense Matrix-Vector Multiplication



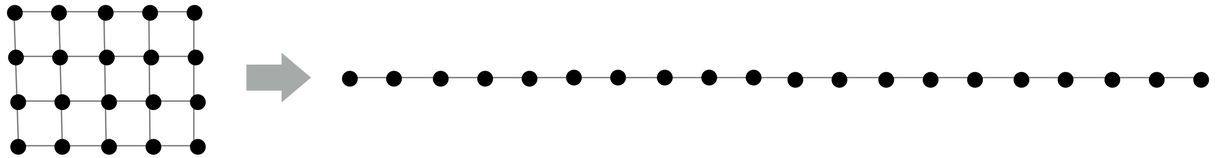
Stencils



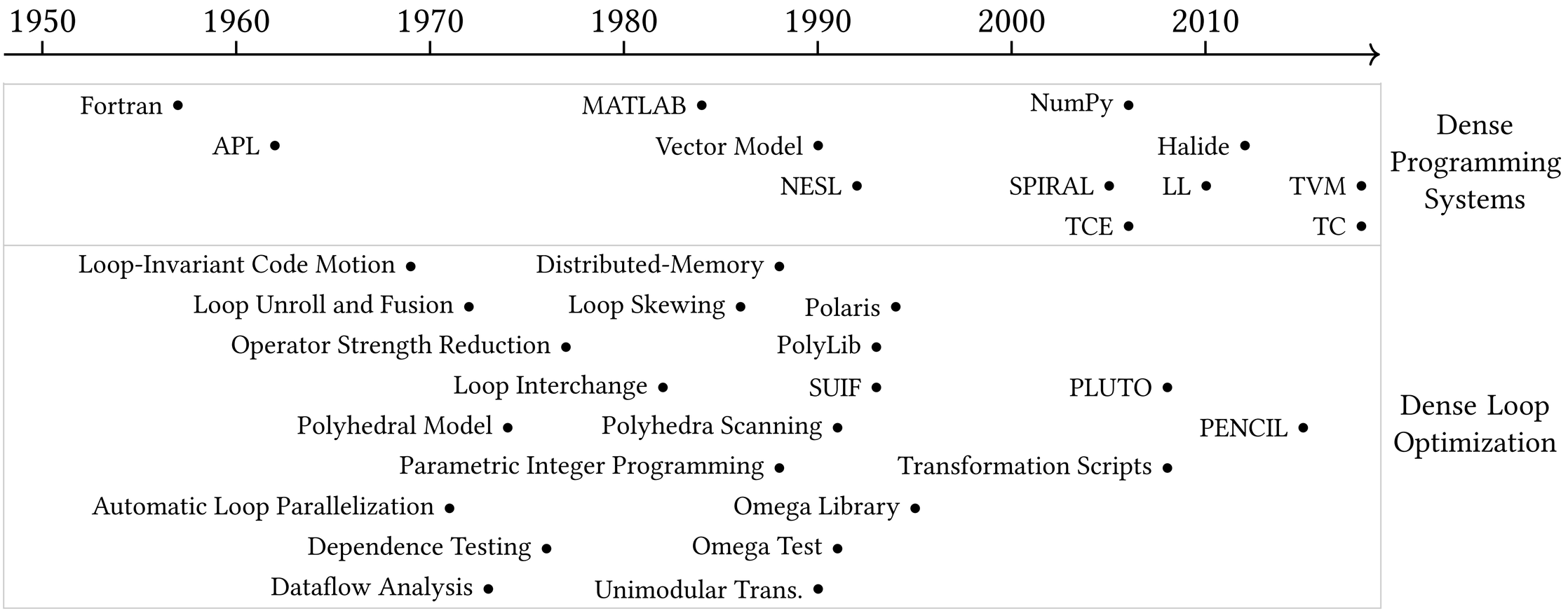
Triagonal Solve



Reshaping



Timeline of some important developments in compilers and programming languages for dense compilers



Traditional compiler loop transformations

Reorder (interchange)



```
for (int i=0; i<m; i++)
  for (int j=0; j<n; j++)
    A[i][j] = B[i][j] + C[i][j];
```

```
for (int j=0; j<n; j++)
  for (int i=0; i<m; i++)
    A[i][j] = B[i][j] + C[i][j];
```

Traditional compiler loop transformations

Split (Stripmine)



```
for (int i=0; i<m; i++)
  a[i] = b[i] + c[i];
```

```
for (int k=0; k<m; k+=4)
  for (int i=k; i<k+4; i++)
    a[i] = b[i] + c[i];
```

Traditional compiler loop transformations

Vectorize



```
for (int k=0; k<m; k+=4)
  for (int i=k; i<k+4; i++)
    a[i] = b[i] + c[i];
```

```
for (int k=0; k<m; k+=4)
  a[k:k+4] = b[k:k+4] + c[k:k+4];
```

Traditional compiler loop transformations

Fusion



```
for (int i=0; i<m; i++)
  a[i] = b[i] + c[i];

for (int i=0; i<m; i++)
  d[i] = -b[i];
```

```
for (int i=0; i<m; i++)
  a[i] =  b[i] + c[i];
  d[i] = -b[i];
```

Traditional compiler loop transformations

Collapse (flatten)



```
for (int i=0; i<m; i++)
  for (int j=0; j<n; j++)
    A[i*m+j] = -B[i*m+j];
```

```
for (int ij=0; ij<m*n; ij++)
  A[ij] = -B[ij];
```

Two models of loop optimization: source code rewrite and mathematical frameworks

Source Code Rewrite

```
for (int i=0; i<m; i++) {
  a[i] = b[i] + c[i];
}
```




```
for (int k=0; k<m; k+=4) {
  for (int i=k; i<k+4; i++) {
    a[i] = b[i] + c[i];
}
```

Mathematical loop optimization frameworks include the polyhedral model

Two models of loop optimization: source code rewrite and mathematical frameworks

Source Code Rewrite

```
for (int i=0; i<m; i++) {
  a[i] = b[i] + c[i];
}
```

split/stripmine(4)

```
for (int k=0; k<m; k+=4) {
  for (int i=k; i<k+4; i++) {
    a[i] = b[i] + c[i];
}
```

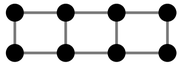
Mathematical Frameworks

```
for (int i=0; i<m; i++) {
  a[i] = b[i] + c[i];
}
```

convert to integer domain



split/stripmine(4)



code generation

```
for (int k=0; k<m; k+=4) {
  for (int i=k; i<k+4; i++) {
    a[i] = b[i] + c[i];
}
```

Mathematical loop optimization frameworks include the polyhedral model

Optimizing dense codes require complex tradeoffs between parallelism, locality, and work efficiency

Clean C++: 9.94 ms per megapixel

```
void blur(const Image &in, Image &blurred) {
 Image tmp(in.width(), in.height());

 for (int y = 0; y < in.height(); y++)
  for (int x = 0; x < in.width(); x++)
   tmp(x, y) = (in(x-1, y) + in(x, y) + in(x+1, y))/3;

 for (int y = 0; y < in.height(); y++)
  for (int x = 0; x < in.width(); x++)
   blurred(x, y) = (tmp(x, y-1) + tmp(x, y) + tmp(x, y+1))/3;
}
```

Fast x86 C++: 0.9 ms per megapixel

```
void fast_blur(const Image &in, Image &blurred) {
 __m128i one_third = _mm_set1_epi16(21846);
 #pragma omp parallel for
 for (int yTile = 0; yTile < in.height(); yTile += 32) {
  __m128i a, b, c, sum, avg;
  __m128i tmp[(256/8)*(32+2)];
  for (int xTile = 0; xTile < in.width(); xTile += 256) {
   __m128i *tmpPtr = tmp;
   for (int y = -1; y < 32+1; y++) {
    const uint16_t *inPtr = &(in(xTile, yTile+y));
    for (int x = 0; x < 256; x += 8) {
     a = _mm_loadu_si128((__m128i*)(inPtr-1));
     b = _mm_loadu_si128((__m128i*)(inPtr+1));
     c = _mm_load_si128((__m128i*)(inPtr));
     sum = _mm_add_epi16(_mm_add_epi16(a, b), c);
     avg = _mm_mulhi_epi16(sum, one_third);
     _mm_store_si128(tmpPtr++, avg);
     inPtr += 8;
   }}
   tmpPtr = tmp;
   for (int y = 0; y < 32; y++) {
    __m128i *outPtr = (__m128i *)(&(blurred(xTile, yTile+y)));
    for (int x = 0; x < 256; x += 8) {
     a = _mm_load_si128(tmpPtr+(2*256)/8);
     b = _mm_load_si128(tmpPtr+256/8);
     c = _mm_load_si128(tmpPtr++);
     sum = _mm_add_epi16(_mm_add_epi16(a, b), c);
     avg = _mm_mulhi_epi16(sum, one_third);
     _mm_store_si128(outPtr++, avg);
}}}}}
```

Decoupling Algorithms from Schedules for Easy Optimization of Image Processing Pipelines. *Ragan-Kelley et al.* (2012)

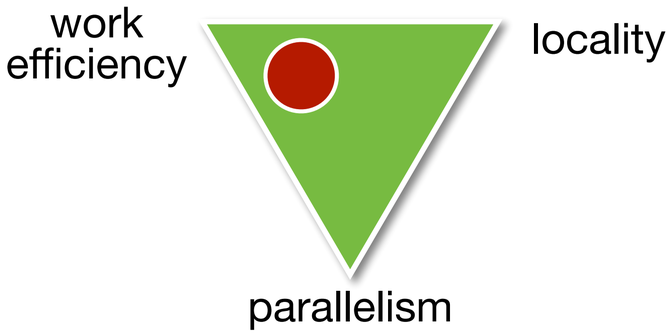
Optimizing dense codes require complex tradeoffs between parallelism, locality, and work efficiency

Clean C++: 9.94 ms per megapixel

```
void blur(const Image &in, Image &blurred) {
 Image tmp(in.width(), in.height());

 for (int y = 0; y < in.height(); y++)
  for (int x = 0; x < in.width(); x++)
   tmp(x, y) = (in(x-1, y) + in(x, y) + in(x+1, y))/3;

 for (int y = 0; y < in.height(); y++)
  for (int x = 0; x < in.width(); x++)
   blurred(x, y) = (tmp(x, y-1) + tmp(x, y) + tmp(x, y+1))/3;
}
```

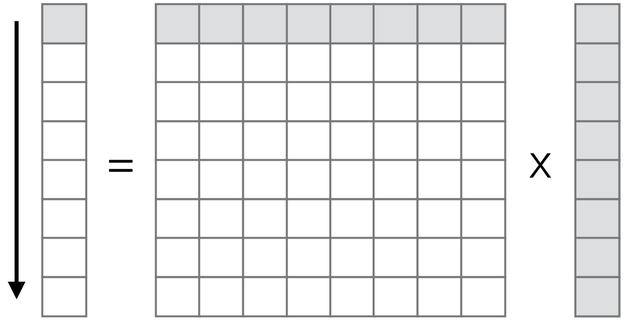


Fast x86 C++: 0.9 ms per megapixel

```
void fast_blur(const Image &in, Image &blurred) {
 __m128i one_third = _mm_set1_epi16(21846);
 #pragma omp parallel for
 for (int yTile = 0; yTile < in.height(); yTile += 32) {
  __m128i a, b, c, sum, avg;
  __m128i tmp[(256/8)*(32+2)];
  for (int xTile = 0; xTile < in.width(); xTile += 256) {
   __m128i *tmpPtr = tmp;
   for (int y = -1; y < 32+1; y++) {
    const uint16_t *inPtr = &(in(xTile, yTile+y));
    for (int x = 0; x < 256; x += 8) {
     a = _mm_loadu_si128((__m128i*)(inPtr-1));
     b = _mm_loadu_si128((__m128i*)(inPtr+1));
     c = _mm_load_si128((__m128i*)(inPtr));
     sum = _mm_add_epi16(_mm_add_epi16(a, b), c);
     avg = _mm_mulhi_epi16(sum, one_third);
     _mm_store_si128(tmpPtr++, avg);
     inPtr += 8;
   }}
   tmpPtr = tmp;
   for (int y = 0; y < 32; y++) {
    __m128i *outPtr = (__m128i *)(&(blurred(xTile, yTile+y)));
    for (int x = 0; x < 256; x += 8) {
     a = _mm_load_si128(tmpPtr+(2*256)/8);
     b = _mm_load_si128(tmpPtr+256/8);
     c = _mm_load_si128(tmpPtr++);
     sum = _mm_add_epi16(_mm_add_epi16(a, b), c);
     avg = _mm_mulhi_epi16(sum, one_third);
     _mm_store_si128(outPtr++, avg);
}}}}}
```

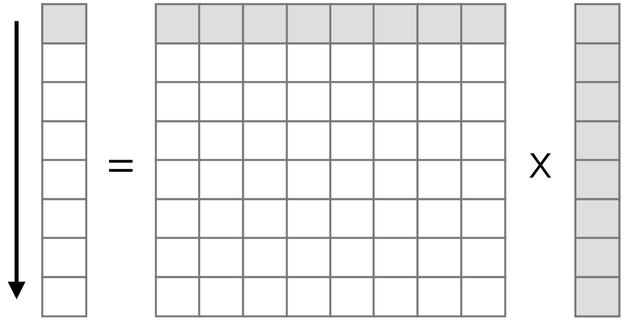
Decoupling Algorithms from Schedules for Easy Optimization of Image Processing Pipelines. *Ragan-Kelley et al.* (2012)

Parallelism in matrix-vector multiplication

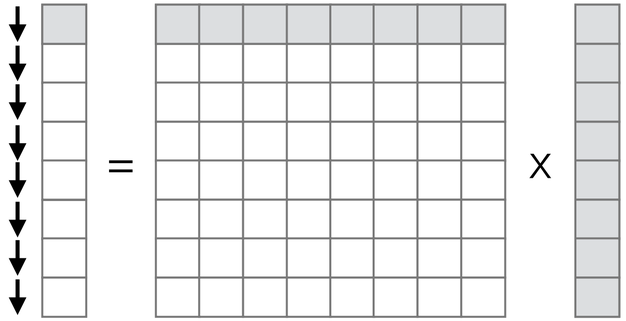



```
for (int i=0; i<m; i++)
  for (int j=0; j<n; j++)
    y[i] += A[i*n+j] * x[j];
```

Parallelism in matrix-vector multiplication

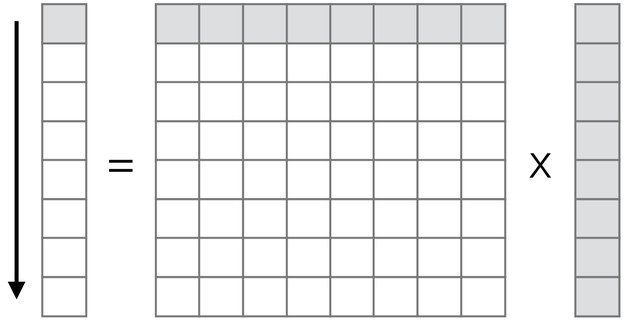



```
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  for (int j=0; j<n; j++)
    y[i] += A[i*n+j] * x[j];
```

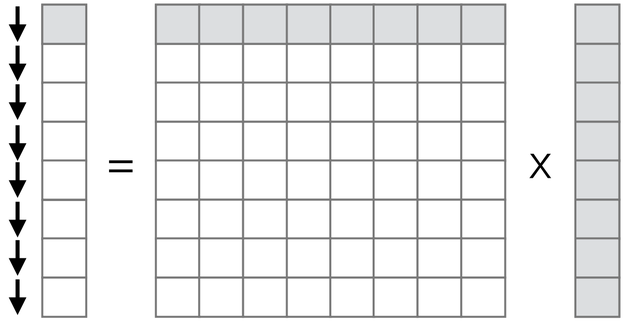



```
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for (int i=0; i<m; i++)
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```

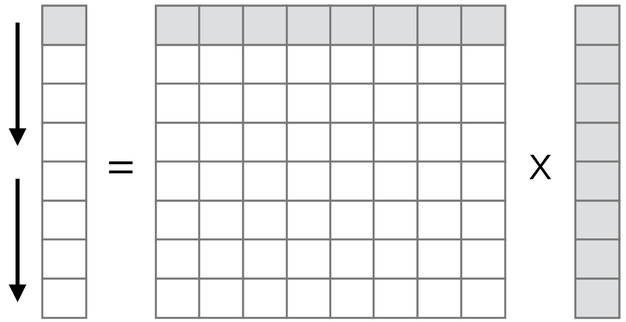
Parallelism in matrix-vector multiplication



```
for (int i=0; i<m; i++)
  for (int j=0; j<n; j++)
    y[i] += A[i*n+j] * x[j];
```

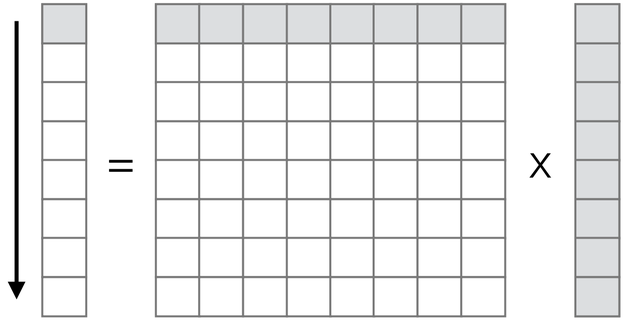


```
#pragma omp parallel for
for (int i=0; i<m; i++)
  for (int j=0; j<n; j++)
    y[i] += A[i*n+j] * x[j];
```

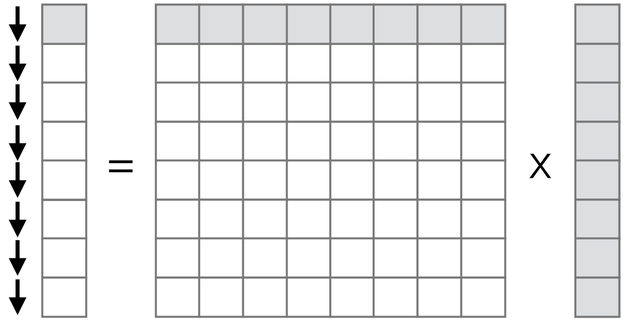


```
#pragma omp parallel for
for (int k=0; k<m; k+=4)
  for (int i=k; i<k+4; i++)
    for (int j=0; j<n; j++)
      y[i] += A[i*n+j] * x[j];
```

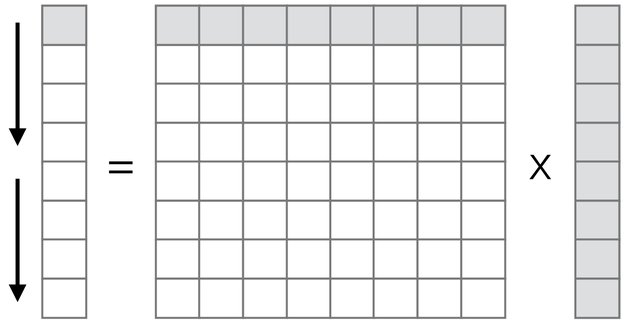
Parallelism in matrix-vector multiplication



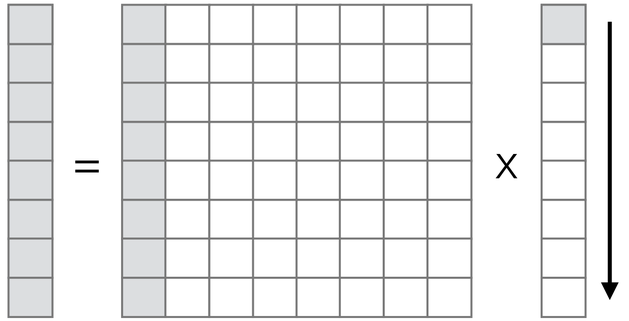
```
for (int i=0; i<m; i++)
  for (int j=0; j<n; j++)
    y[i] += A[i*n+j] * x[j];
```



```
#pragma omp parallel for
for (int i=0; i<m; i++)
  for (int j=0; j<n; j++)
    y[i] += A[i*n+j] * x[j];
```

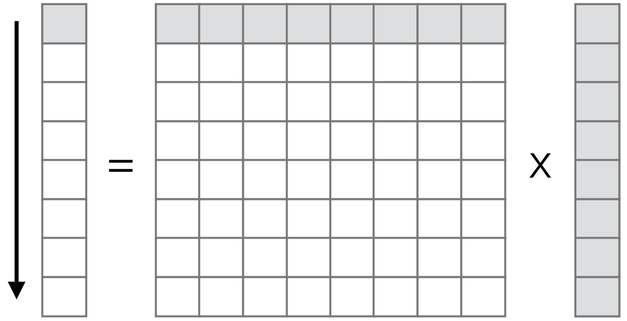


```
#pragma omp parallel for
for (int k=0; k<m; k+=4)
  for (int i=k; i<k+4; i++)
    for (int j=0; j<n; j++)
      y[i] += A[i*n+j] * x[j];
```

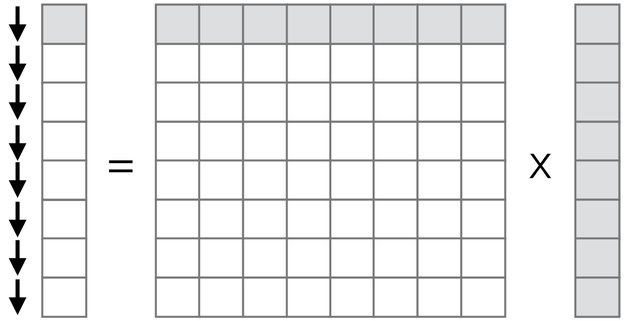


```
for (int j=0; j<n; j++)
  for (int i=0; i<m; i++)
    y[i] += A[i*n+j] * x[j];
```

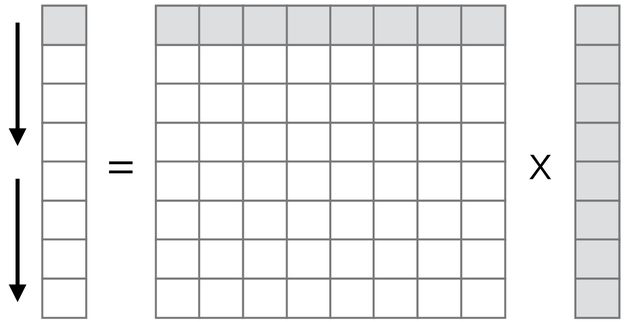
Parallelism in matrix-vector multiplication



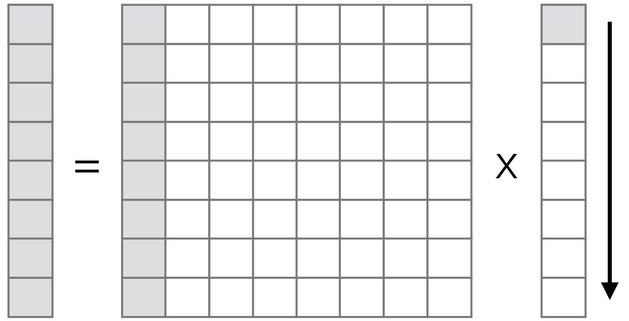
```
for (int i=0; i<m; i++)
  for (int j=0; j<n; j++)
    y[i] += A[i*n+j] * x[j];
```



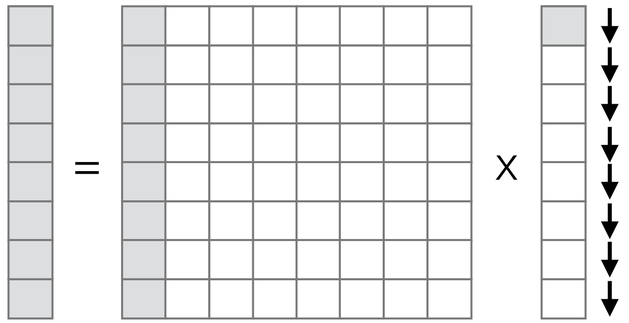
```
#pragma omp parallel for
for (int i=0; i<m; i++)
  for (int j=0; j<n; j++)
    y[i] += A[i*n+j] * x[j];
```



```
#pragma omp parallel for
for (int k=0; k<m; k+=4)
  for (int i=k; i<k+4; i++)
    for (int j=0; j<n; j++)
      y[i] += A[i*n+j] * x[j];
```

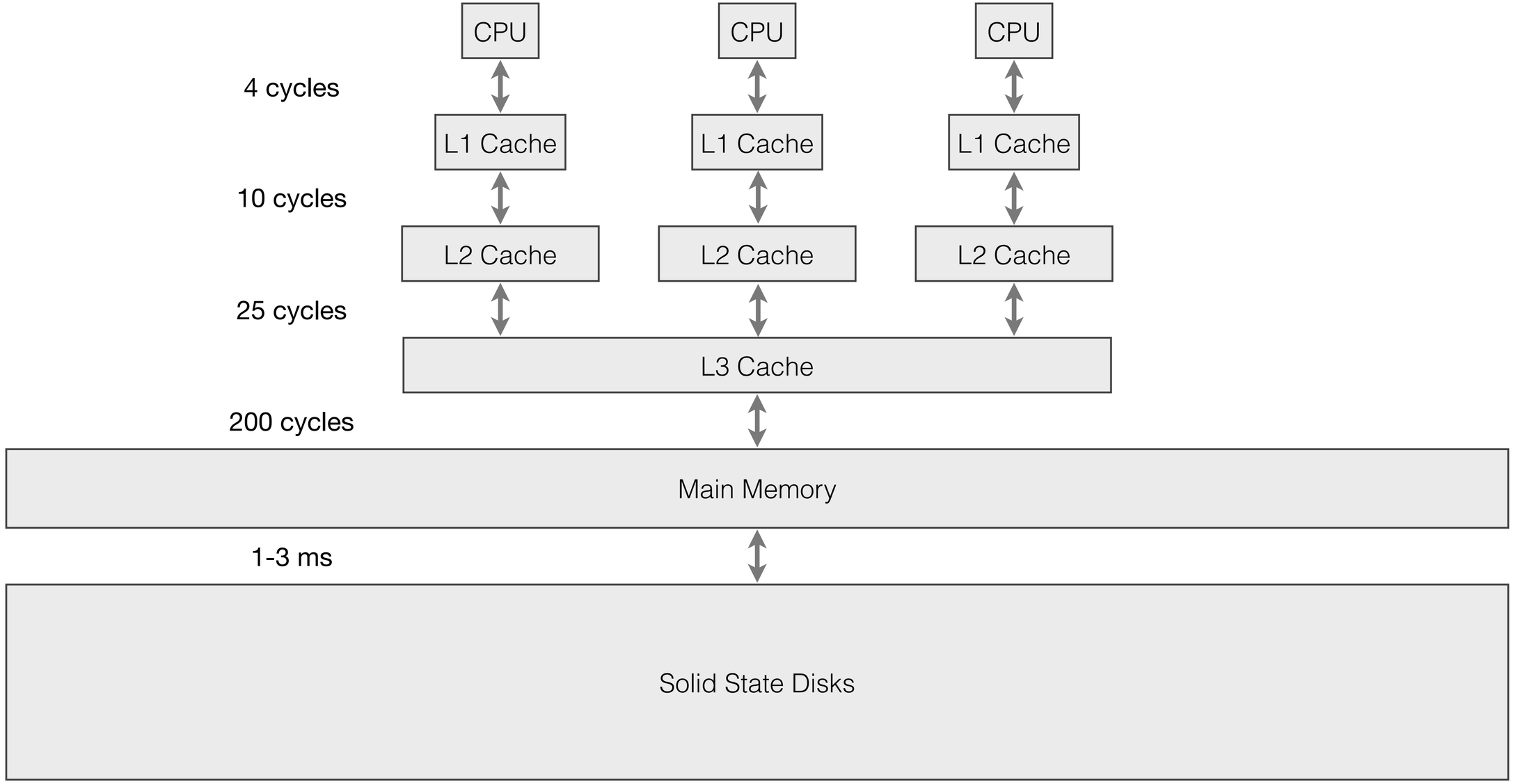


```
for (int j=0; j<n; j++)
  for (int i=0; i<m; i++)
    y[i] += A[i*n+j] * x[j];
```

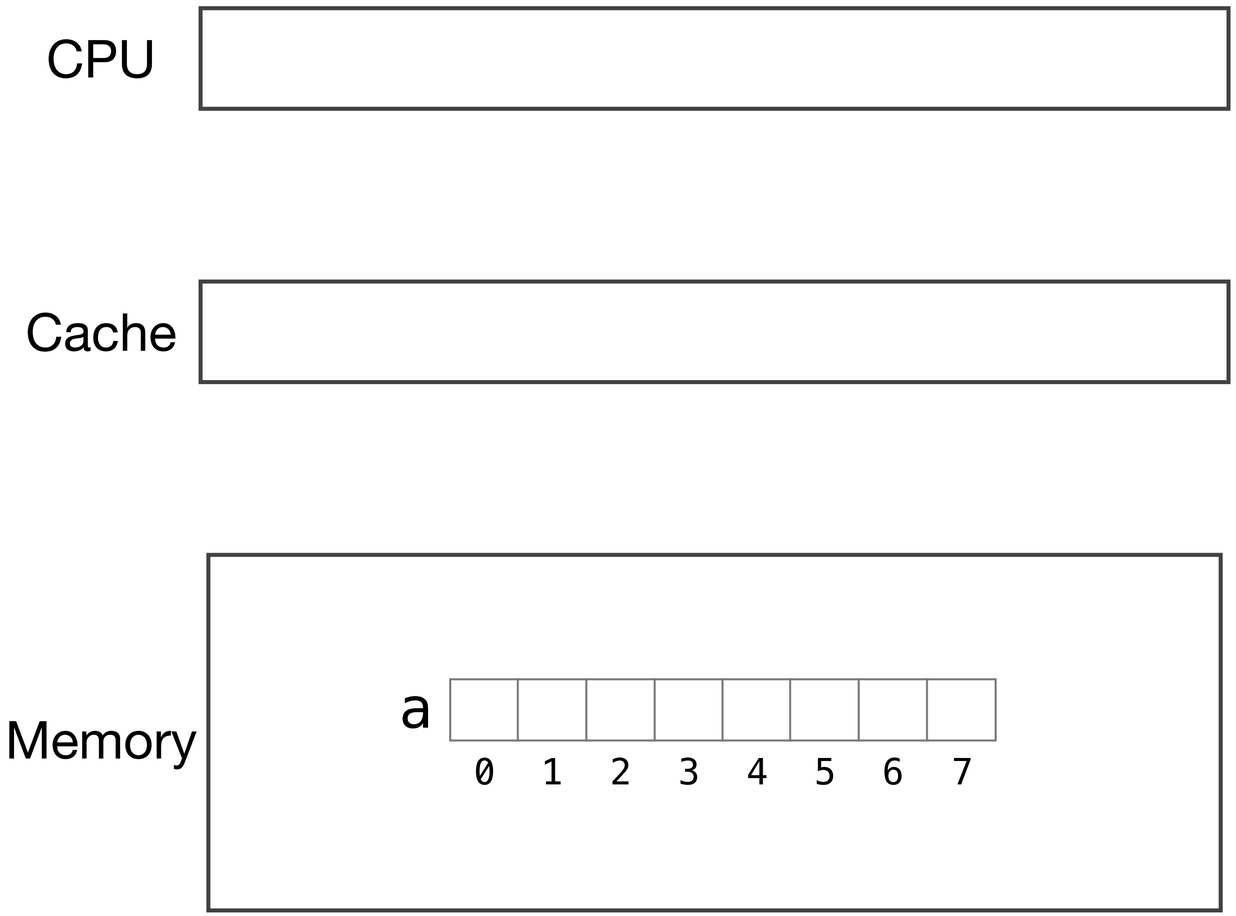


```
#pragma omp parallel for
for (int j=0; j<n; j++)
  for (int i=0; i<m; i++)
    #pragma omp atomic
    y[i] += A[i*n+j] * x[j];
```

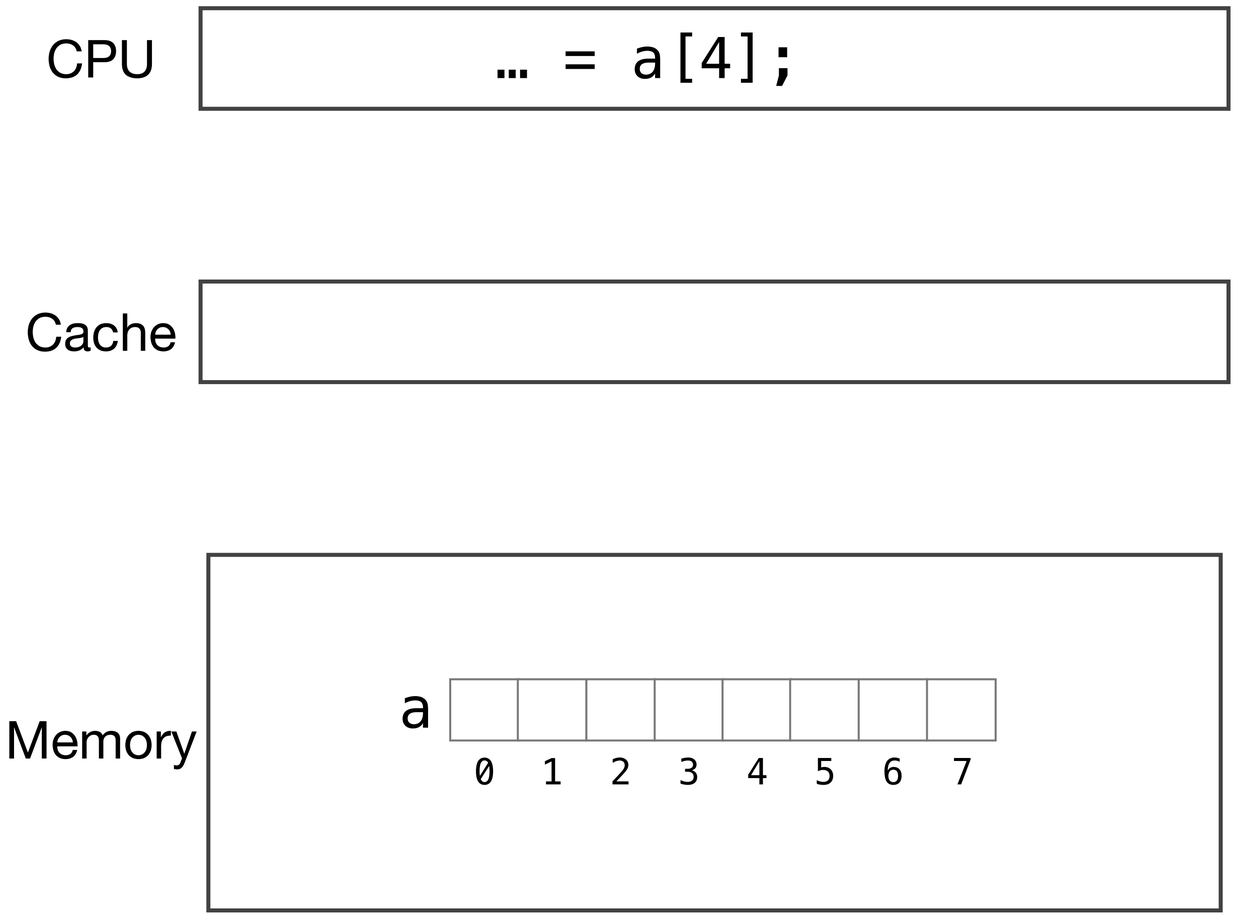
Cache Hierarchies with typical latencies



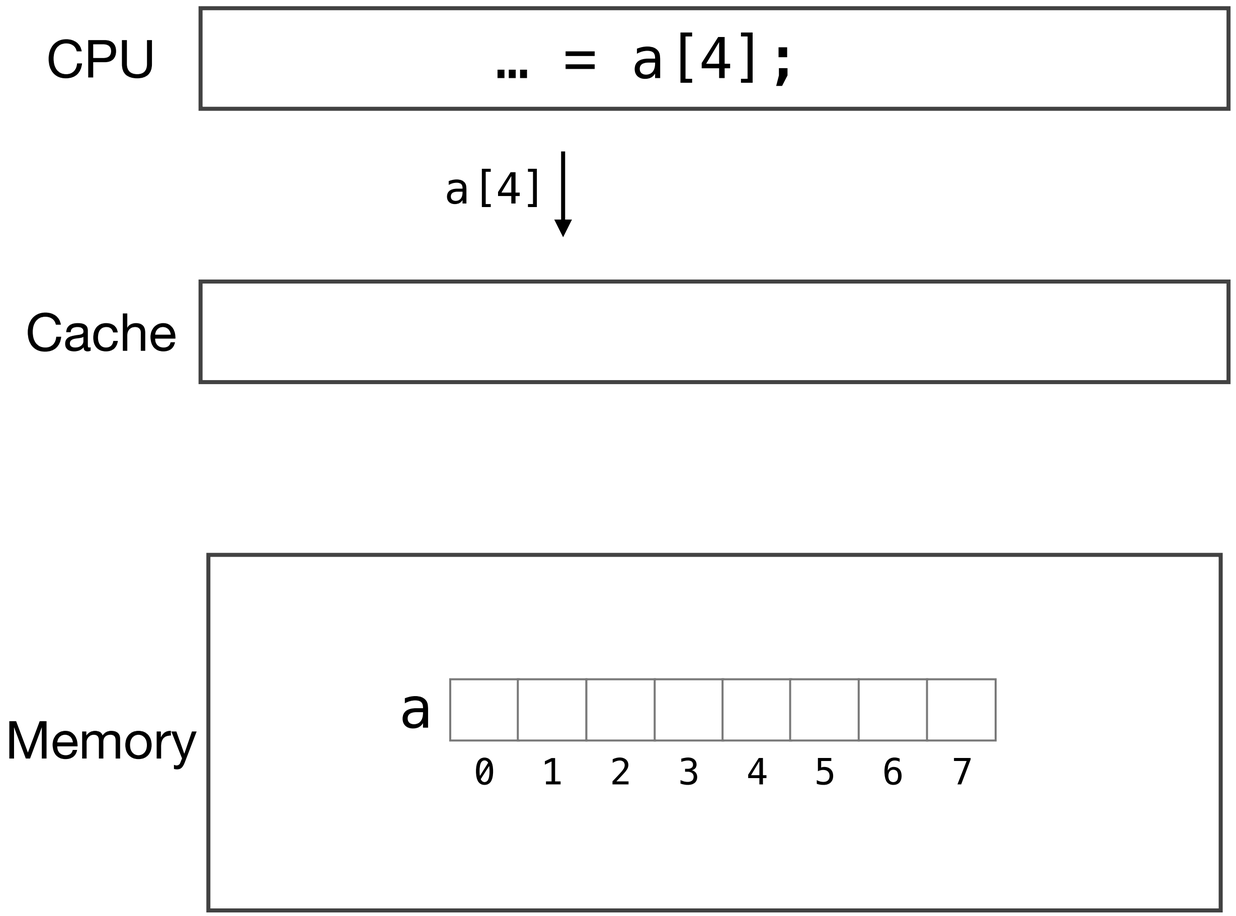
Spatial locality



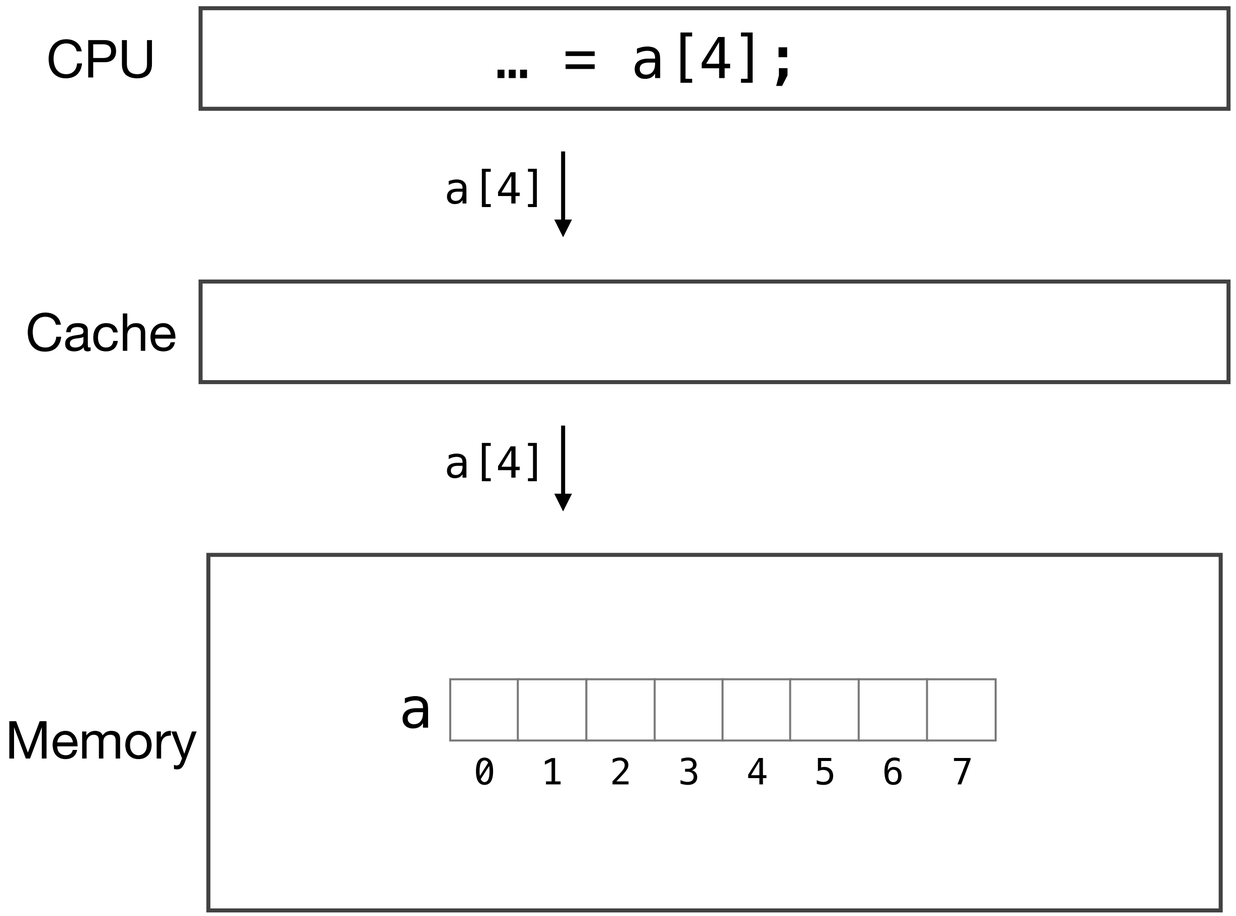
Spatial locality



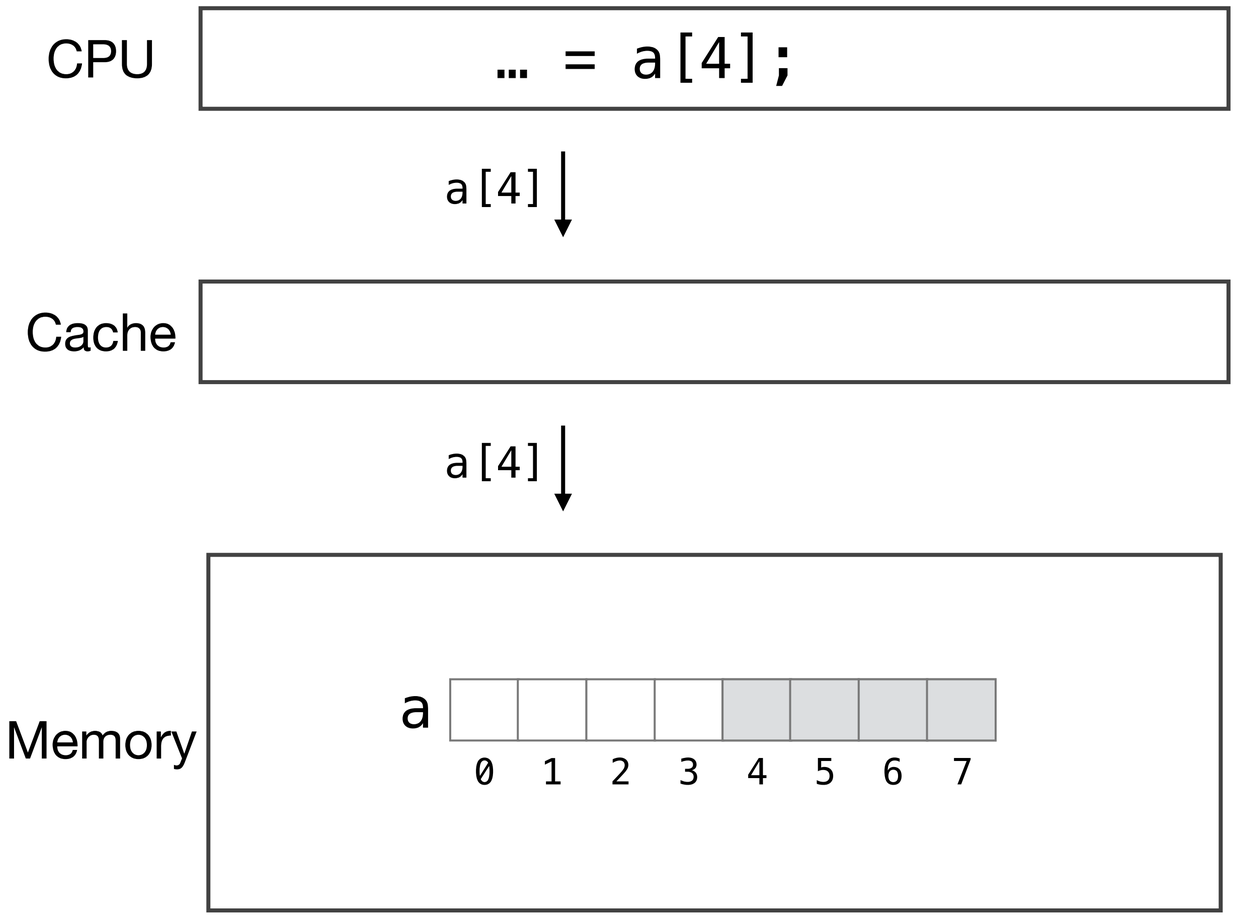
Spatial locality



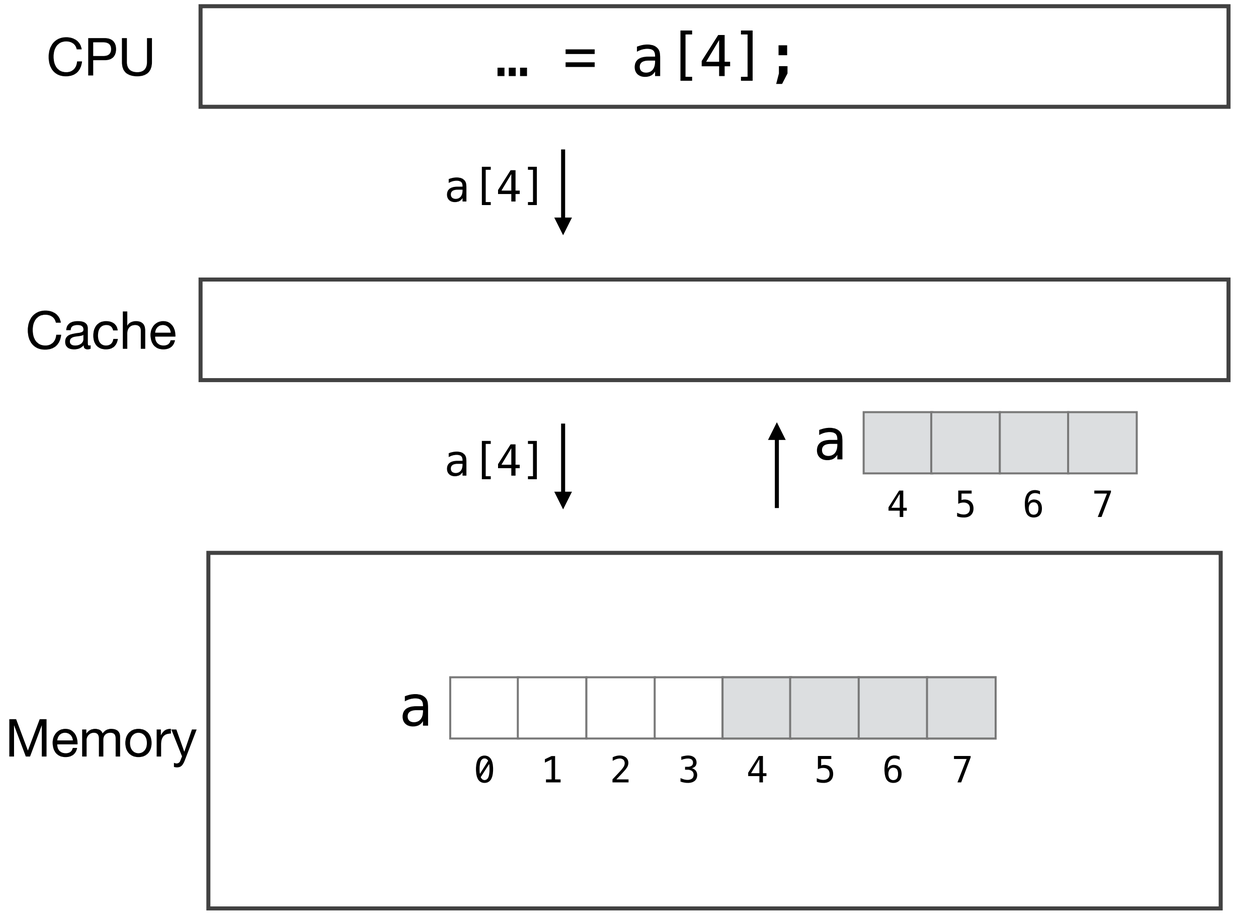
Spatial locality



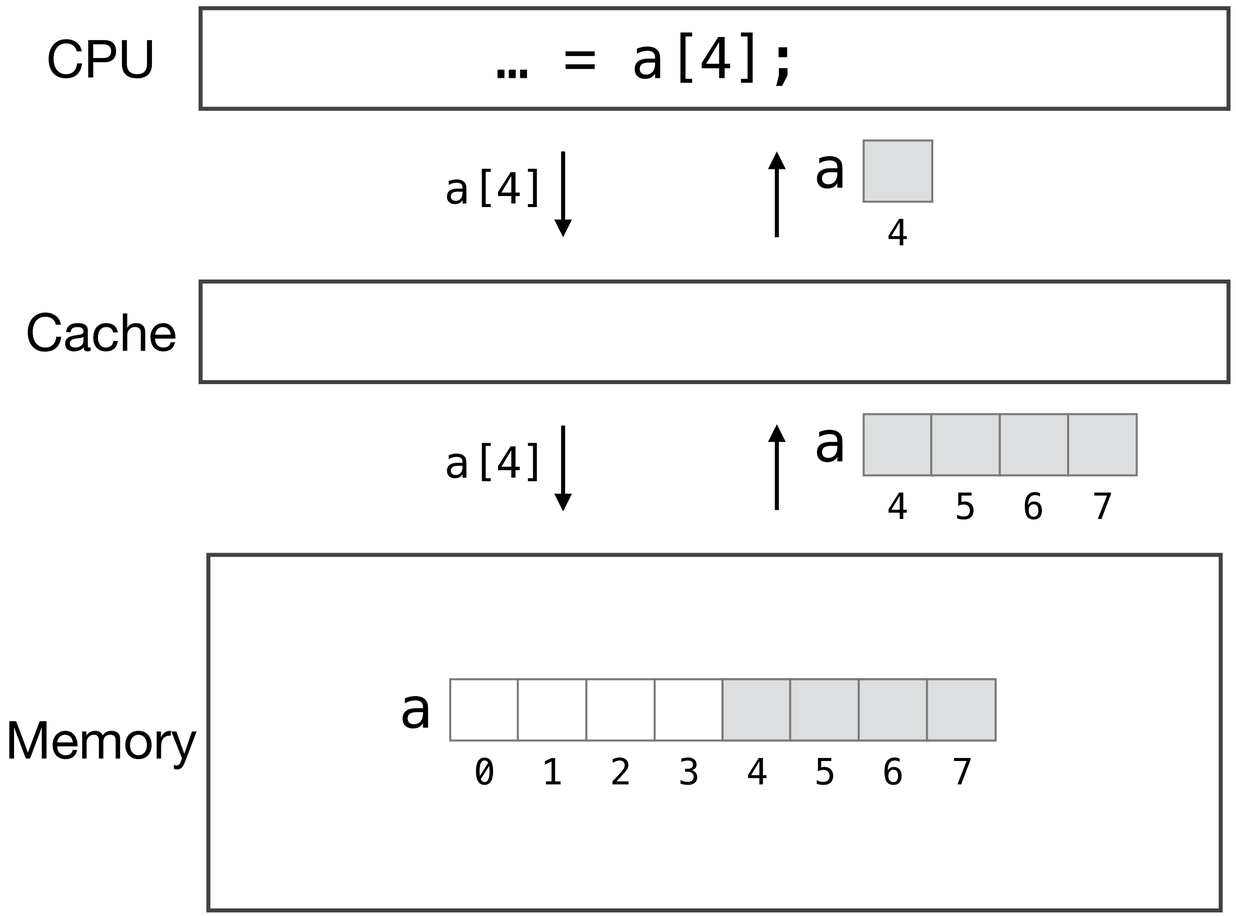
Spatial locality



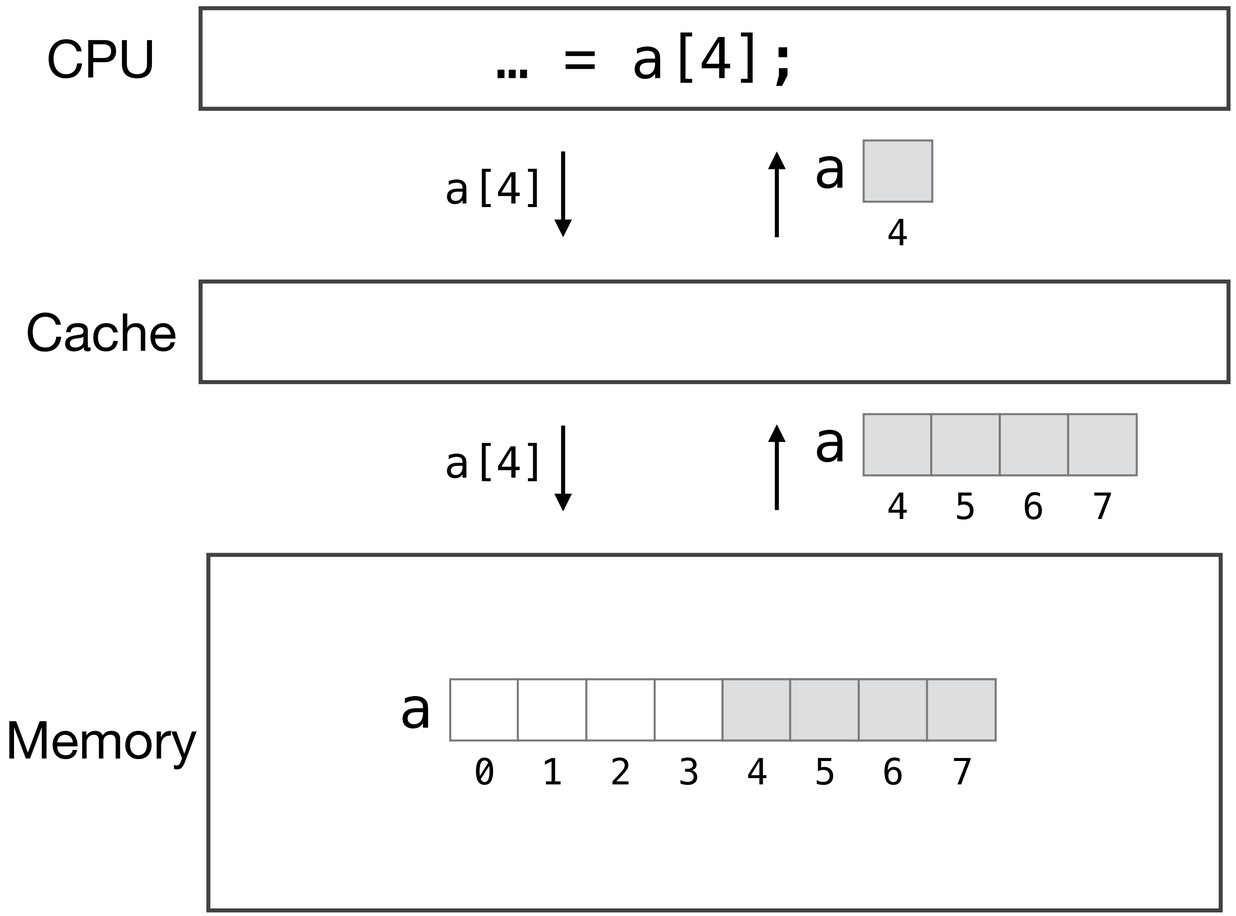
Spatial locality

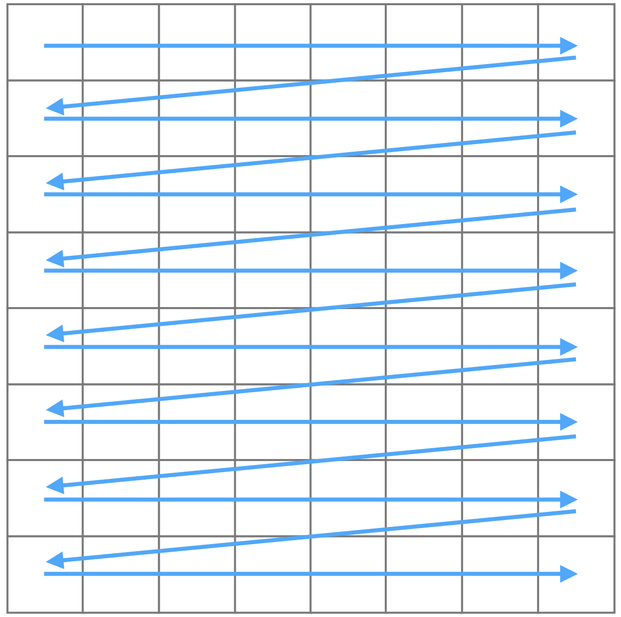


Spatial locality



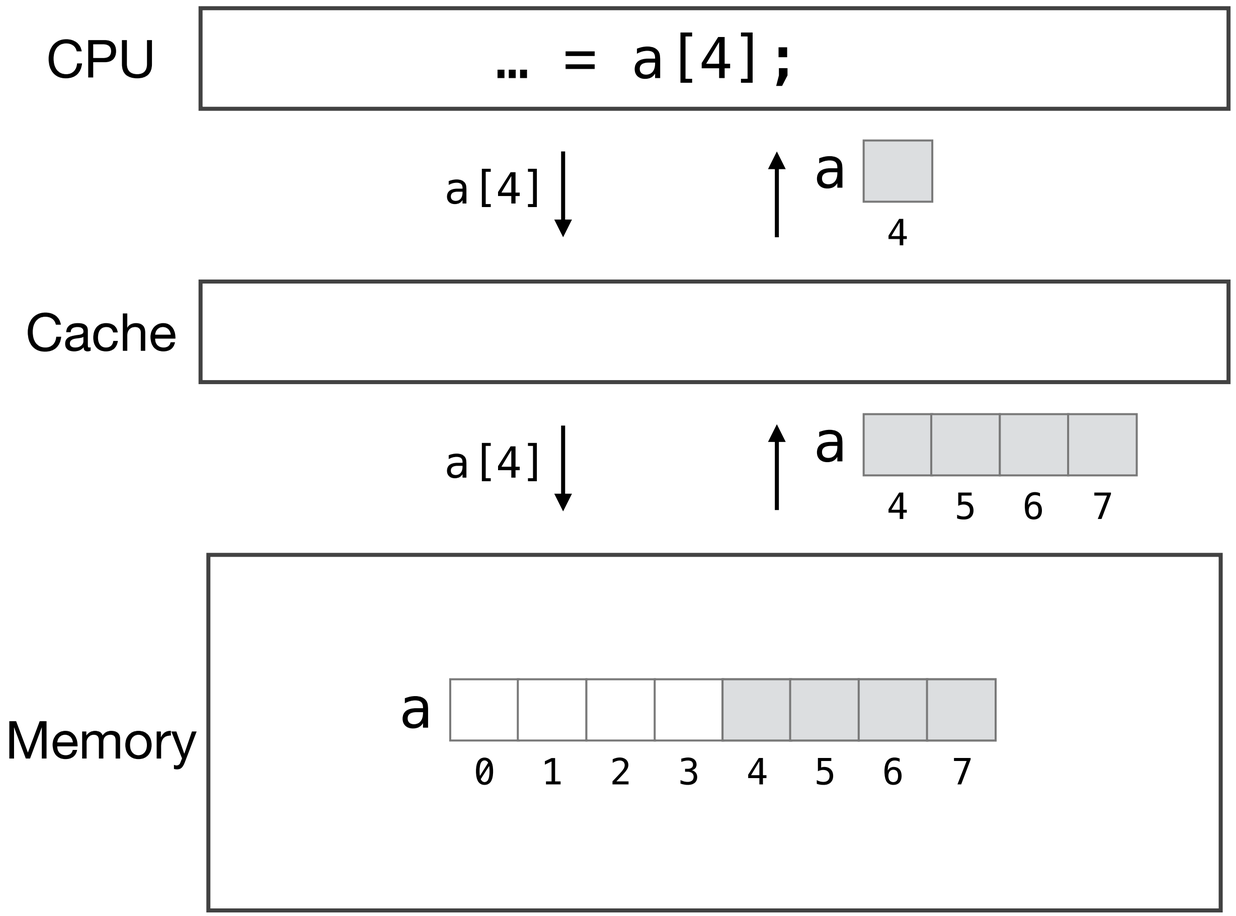
Spatial locality



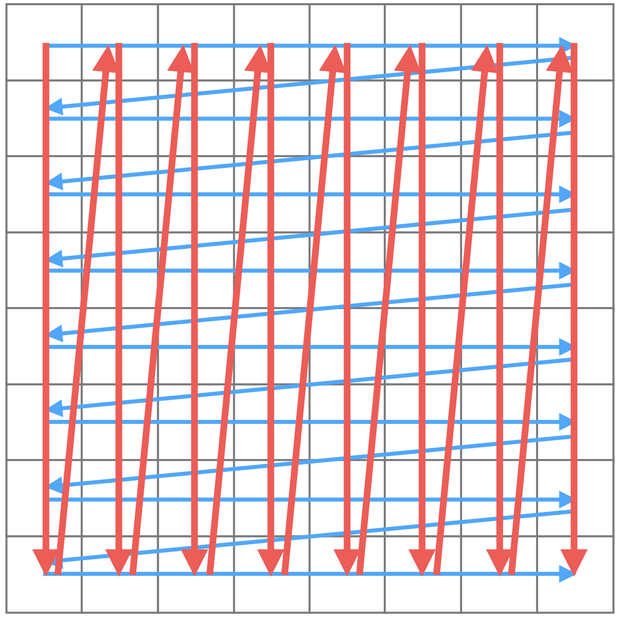


Data Layout Order

Spatial locality



Avoid jumping around the address space by not iterating along the data layout

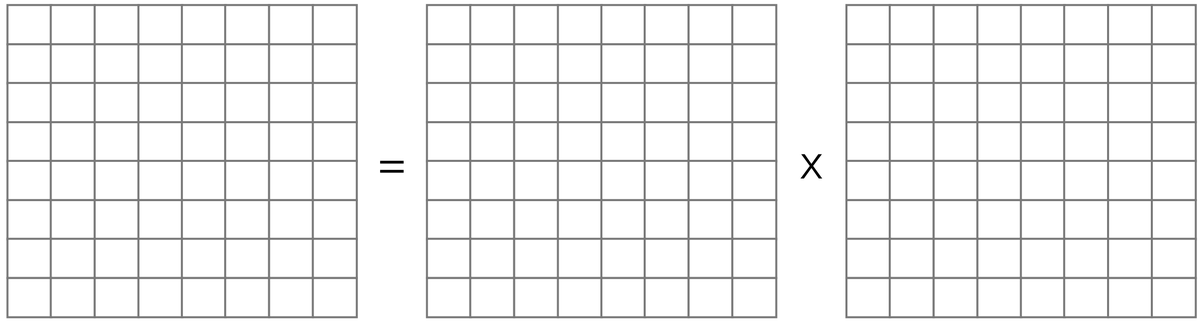


Data Layout Order

Iteration Order

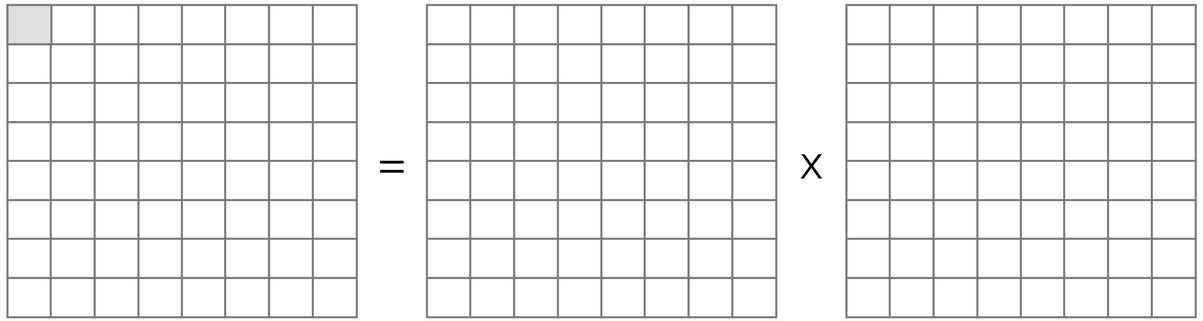
Temporal locality in matrix-matrix multiplication

$$A_{ij} = B_{ik}C_{kj}$$



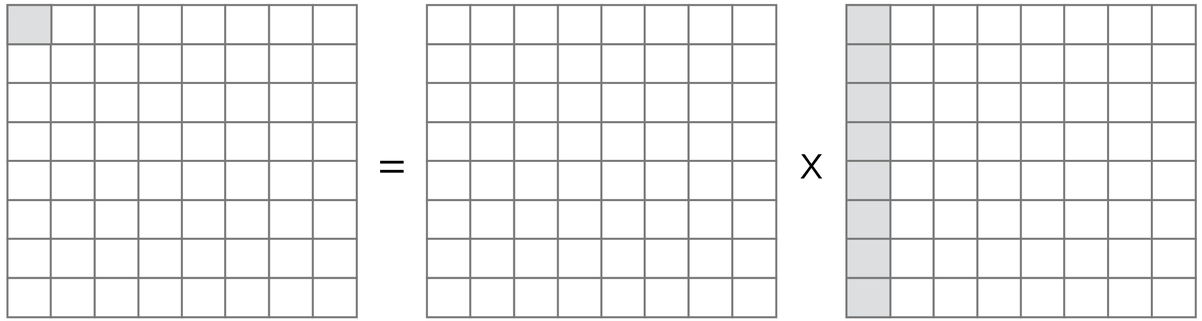
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$$A_{ij} = B_{ik}C_{kj}$$



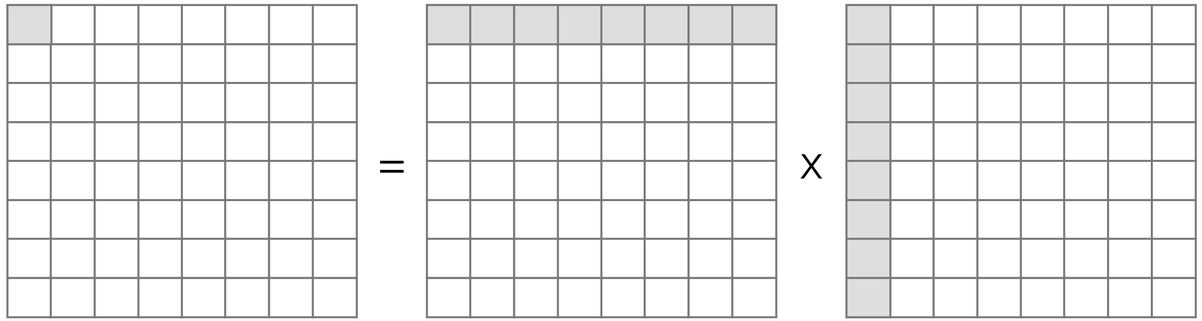
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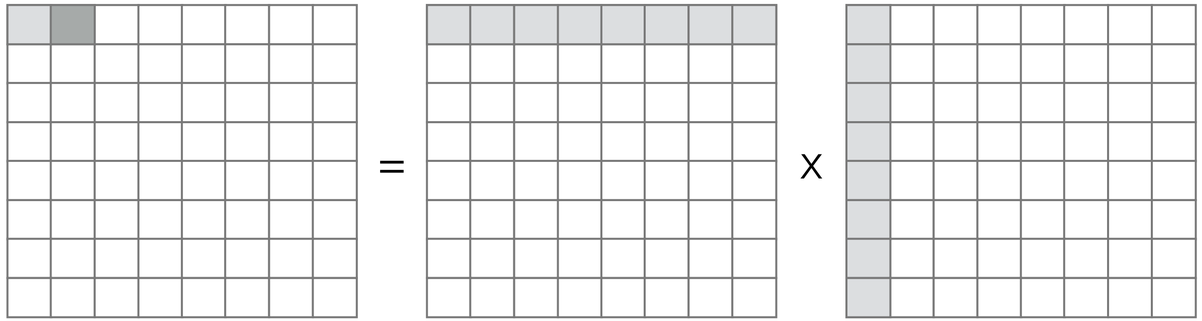
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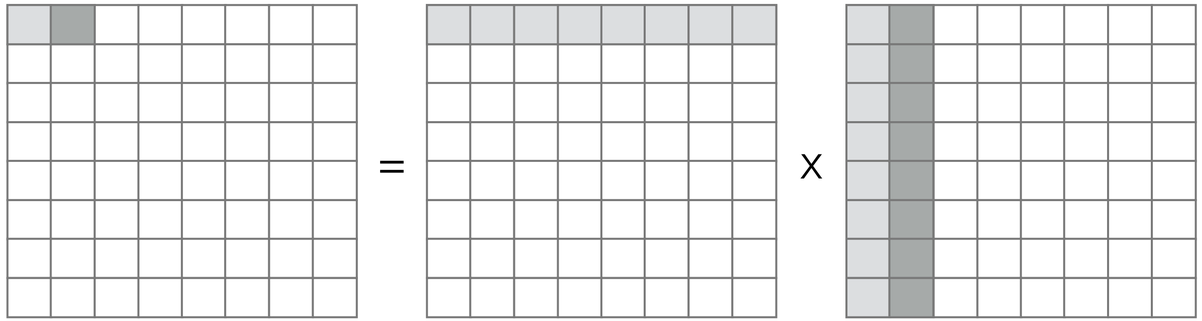
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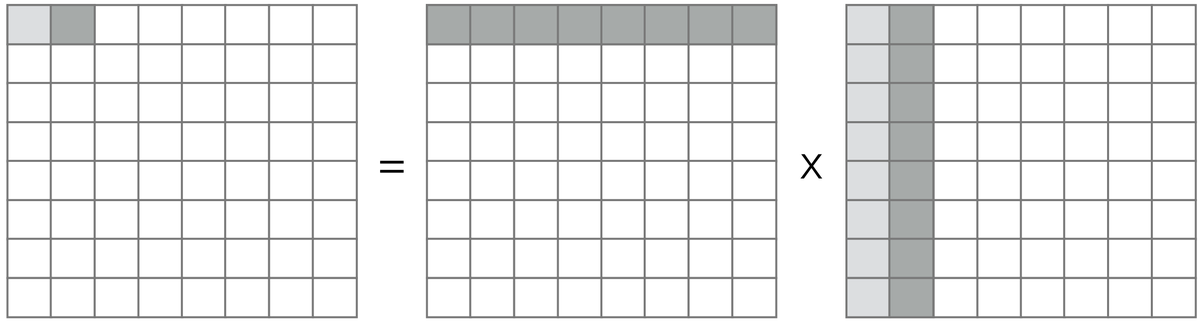
Temporal locality in matrix-matrix multiplication

$$A_{ij} = B_{ik}C_{kj}$$



Temporal locality in matrix-matrix multiplication

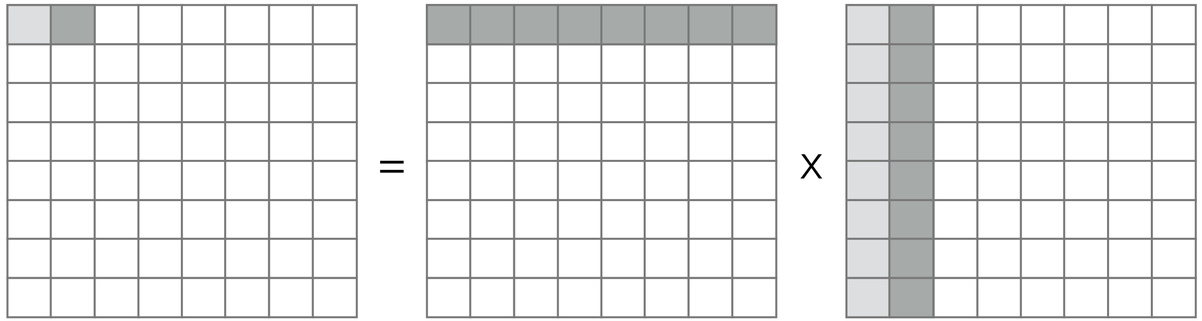
$$A_{ij} = B_{ik}C_{kj}$$



Temporal locality in matrix-matrix multiplication

if matrix is large, row will have left the cache

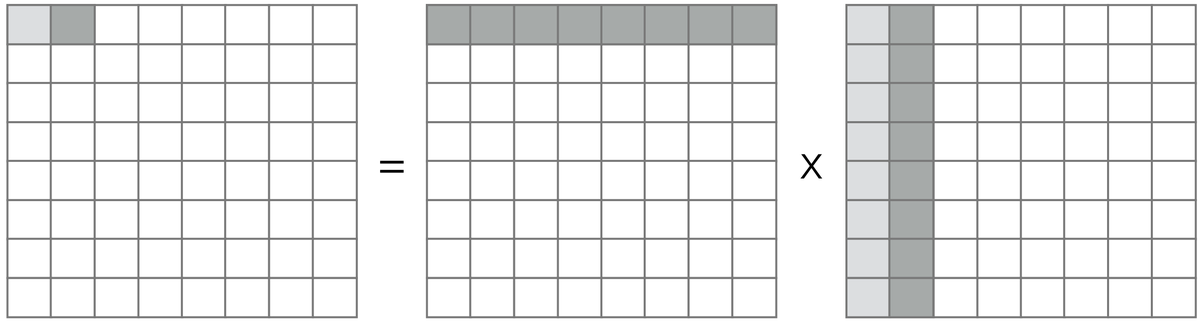
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Temporal locality in matrix-matrix multiplication

$$A_{ij} = B_{ik}C_{kj}$$

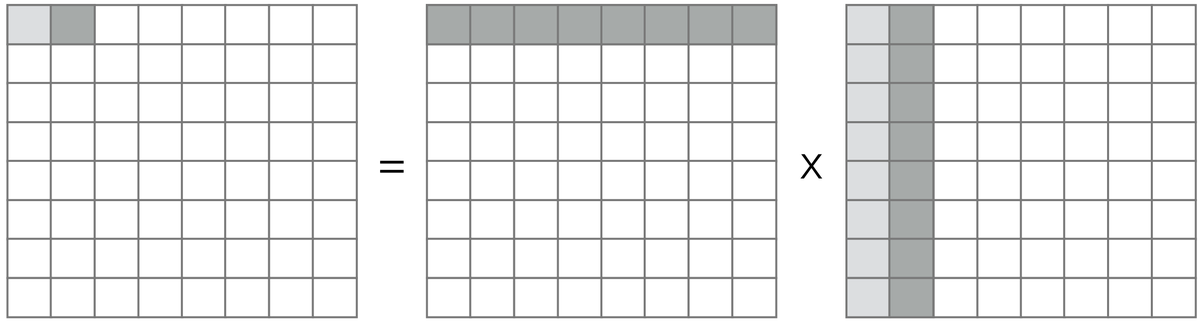
if matrix is large, row will have left the cache
reuse distance = 16



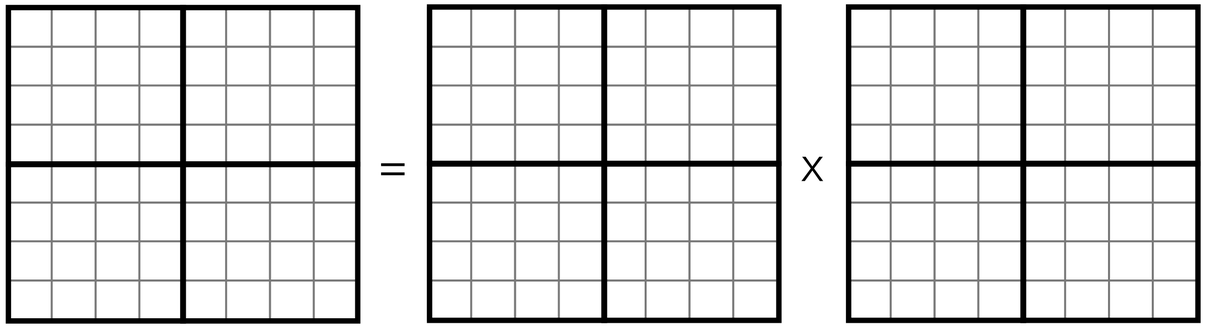
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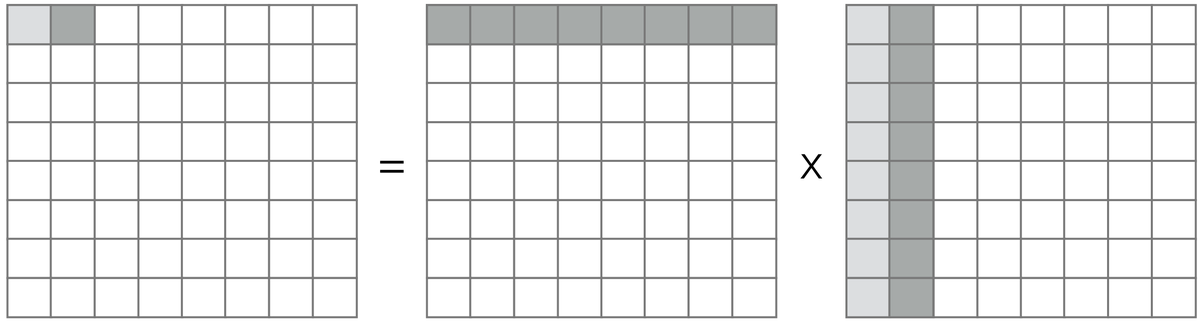
2x2 matrix multiply,
where the operations are
4x4 matrix multiplies



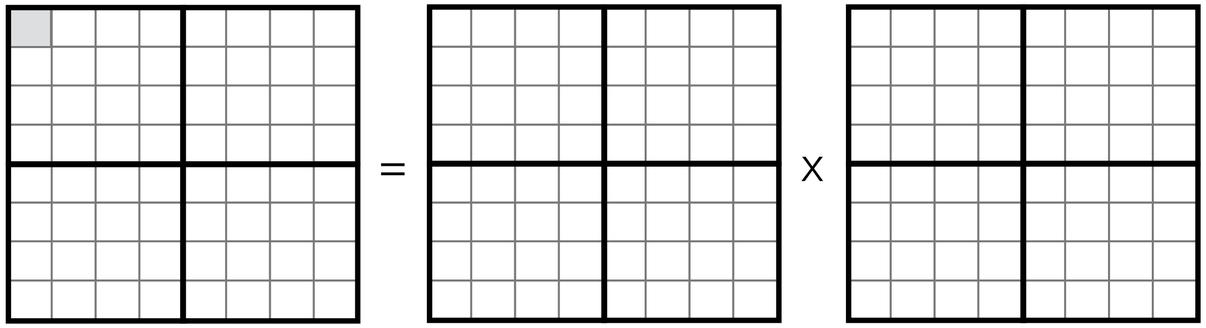
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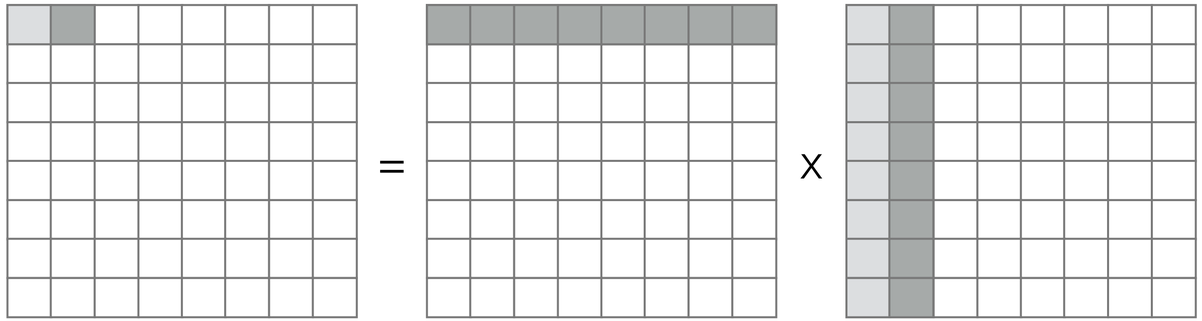
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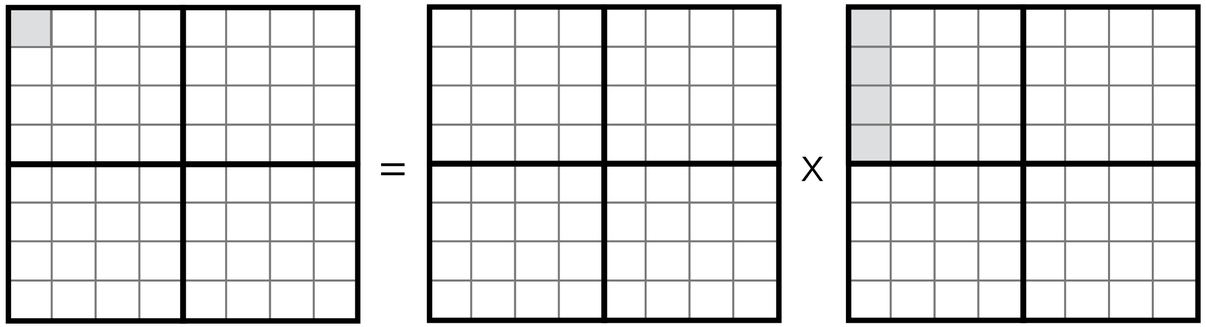
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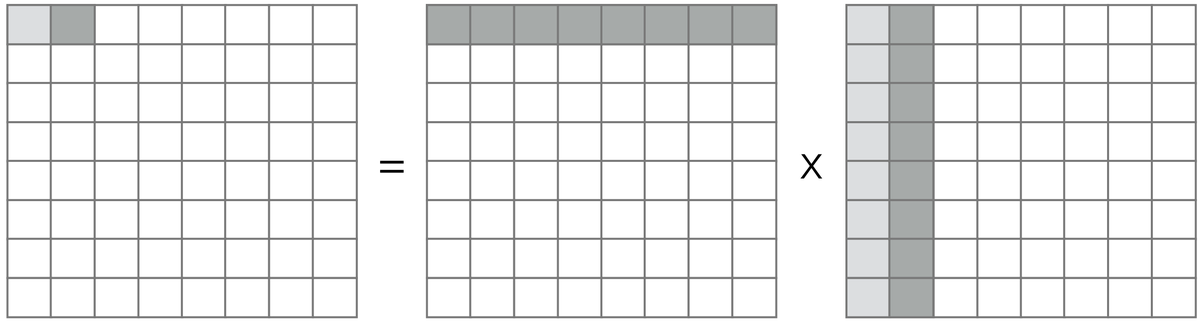
2x2 matrix multiply,
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4x4 matrix multiplies



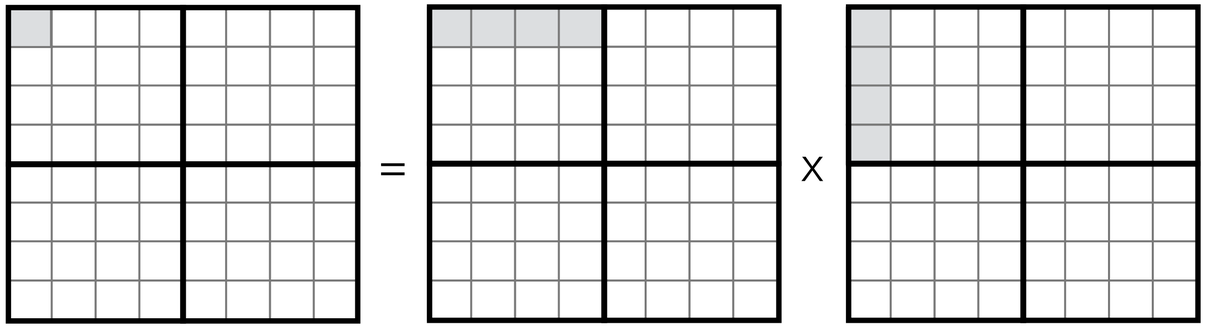
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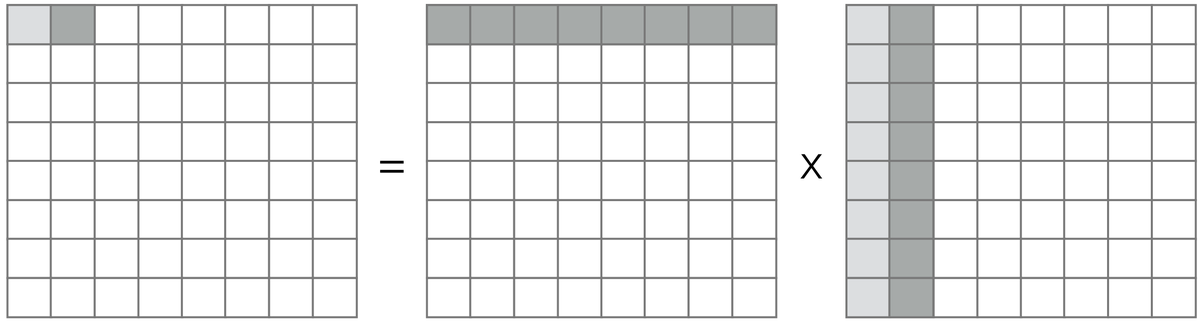
2x2 matrix multiply,
where the operations are
4x4 matrix multiplies



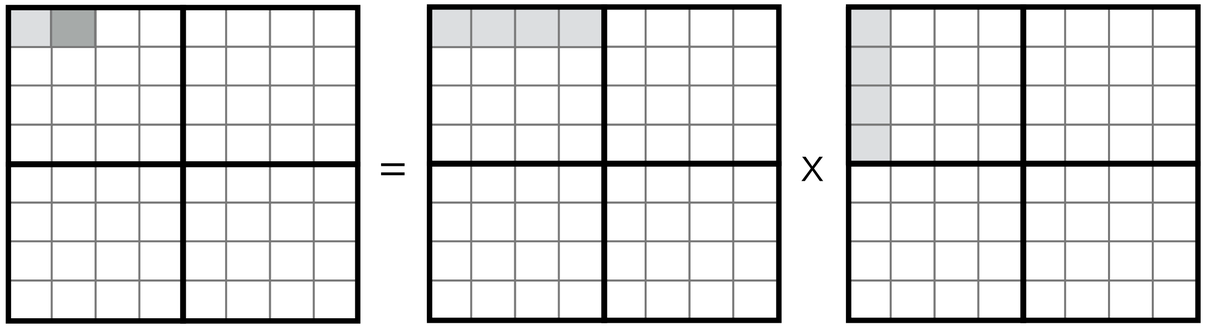
Temporal locality in matrix-matrix multiplication

$$A_{ij} = B_{ik}C_{kj}$$

if matrix is large, row will have left the cache
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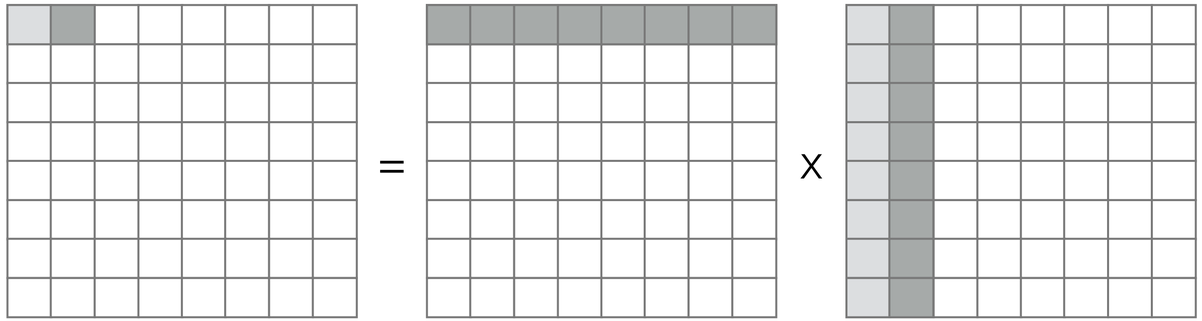
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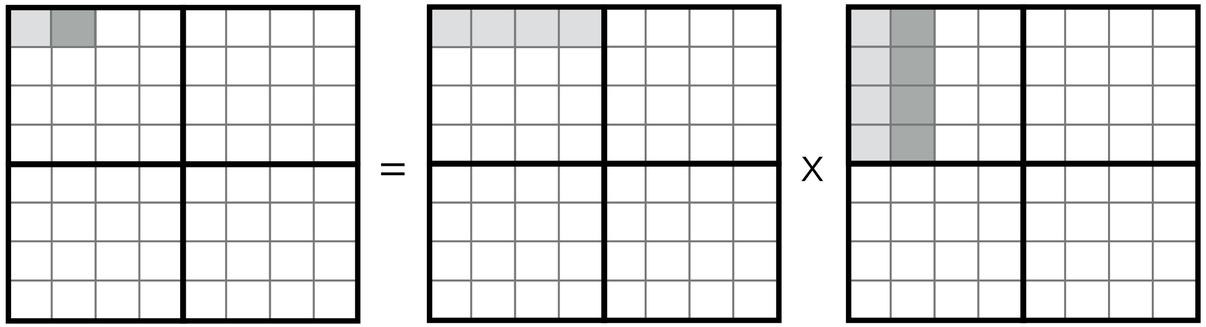
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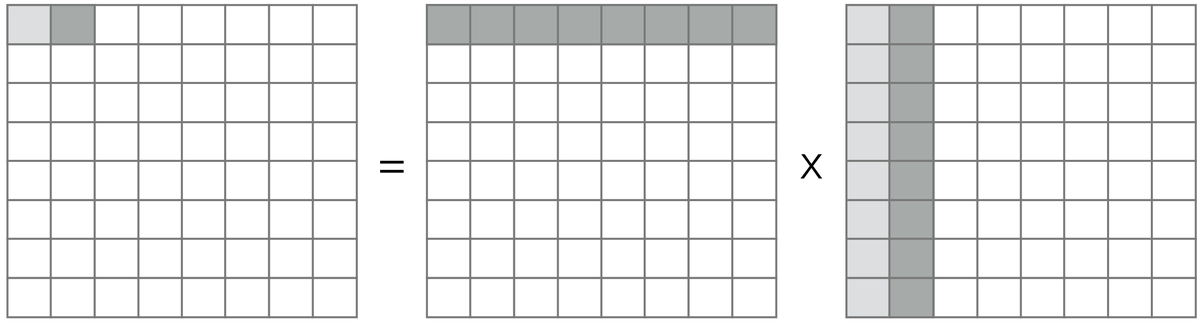
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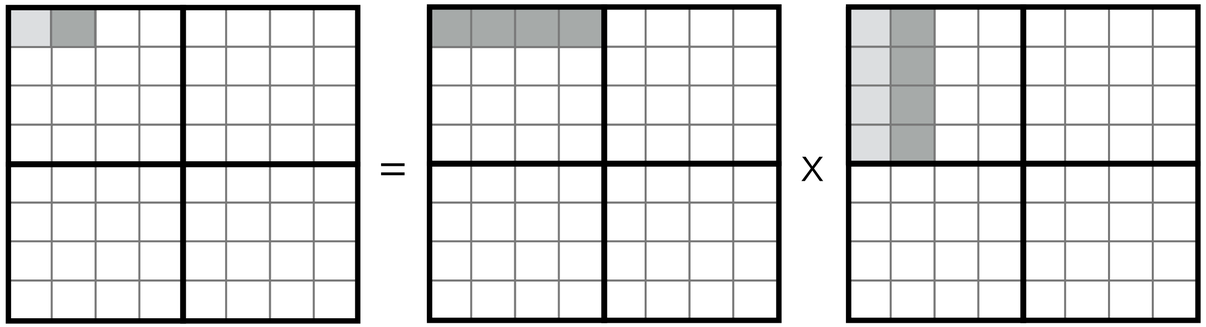
Temporal locality in matrix-matrix multiplication

$$A_{ij} = B_{ik}C_{kj}$$

if matrix is large, row will have left the cache
reuse distance = 16



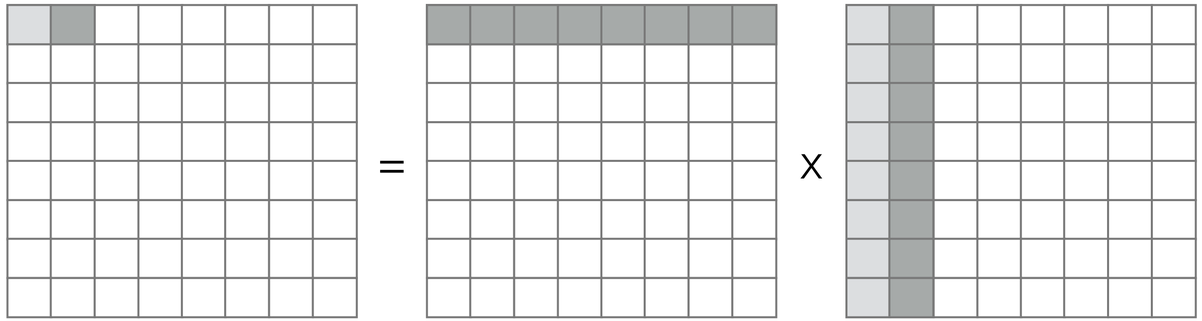
2x2 matrix multiply,
where the operations are
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Temporal locality in matrix-matrix multiplication

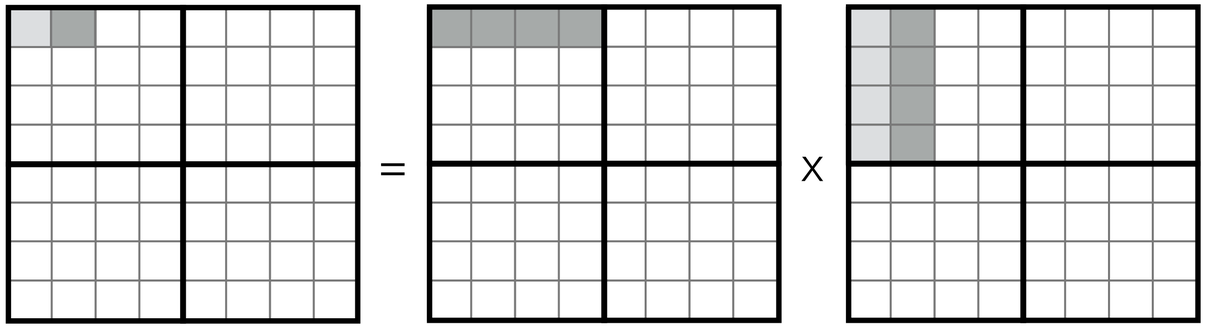
$$A_{ij} = B_{ik}C_{kj}$$

if matrix is large, row will have left the cache
reuse distance = 16



shorter reuse distance

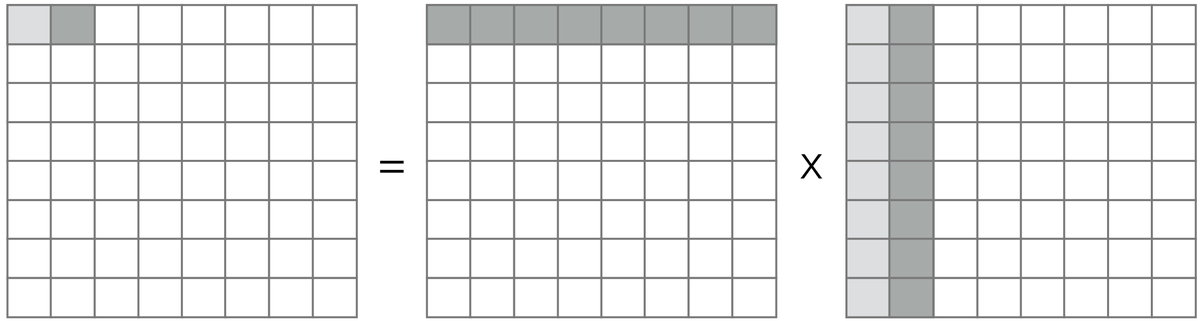
2x2 matrix multiply,
where the operations are
4x4 matrix multiplies



Temporal locality in matrix-matrix multiplication

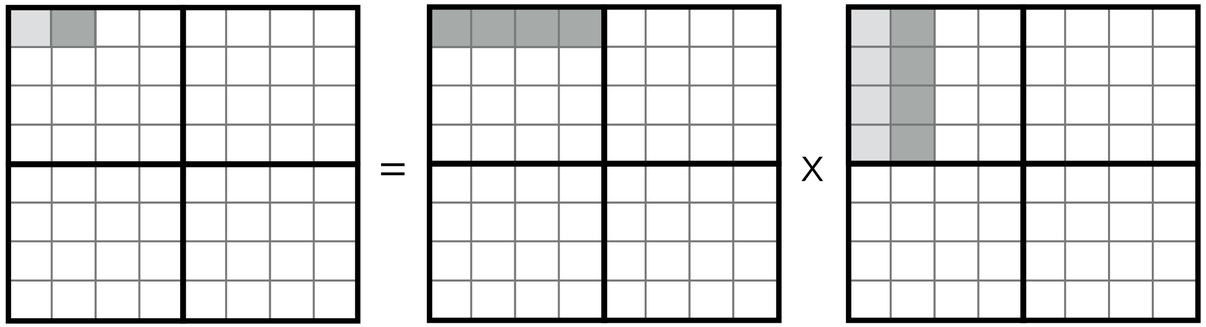
$$A_{ij} = B_{ik}C_{kj}$$

if matrix is large, row will have left the cache
reuse distance = 16



shorter reuse distance
reuse distance = 8

2x2 matrix multiply,
where the operations are
4x4 matrix multiplies

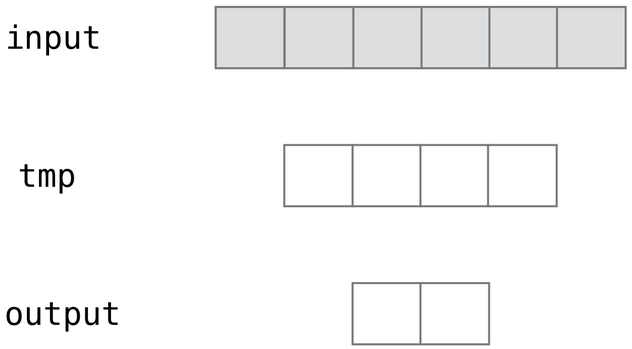


Buying producer-consumer locality with redundant work in fused stencils

Stencil loops

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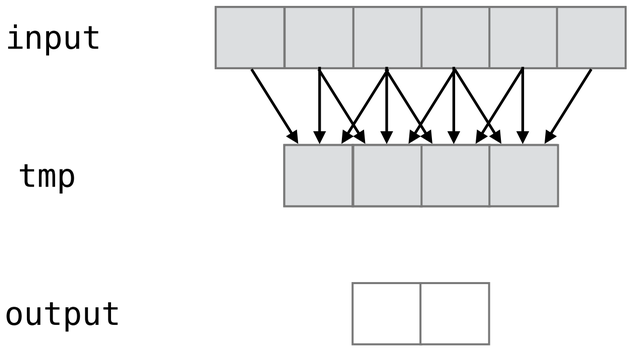


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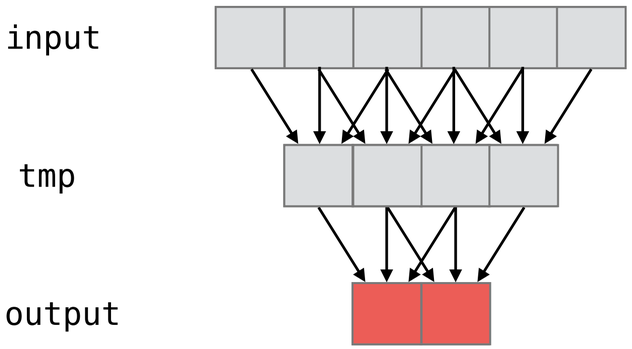


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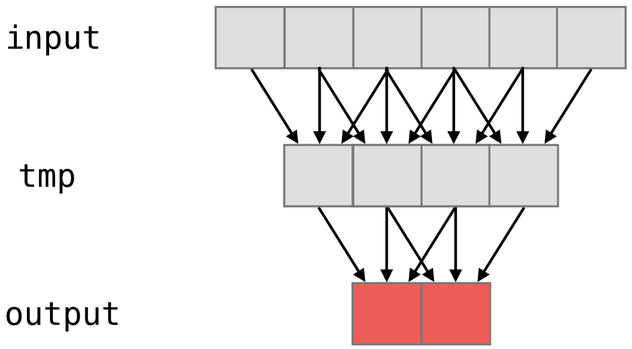


Buying producer-consumer locality with redundant work in fused stencils

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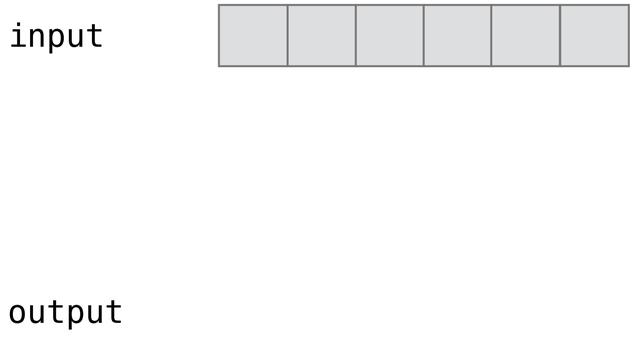
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Fused stencil loops

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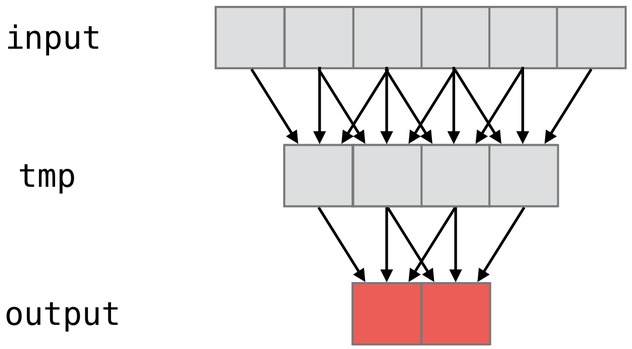


Buying producer-consumer locality with redundant work in fused stencils

Stencil loops

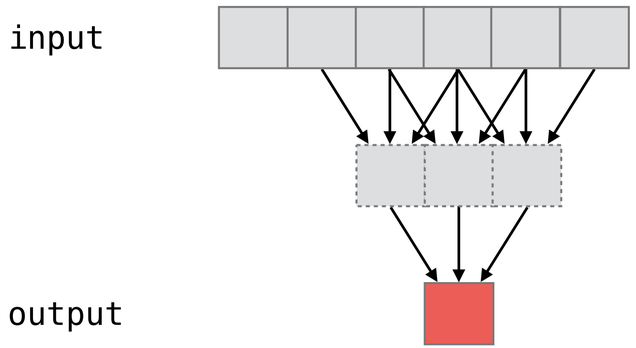
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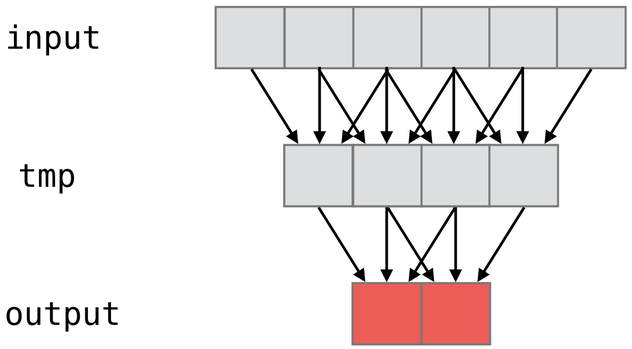


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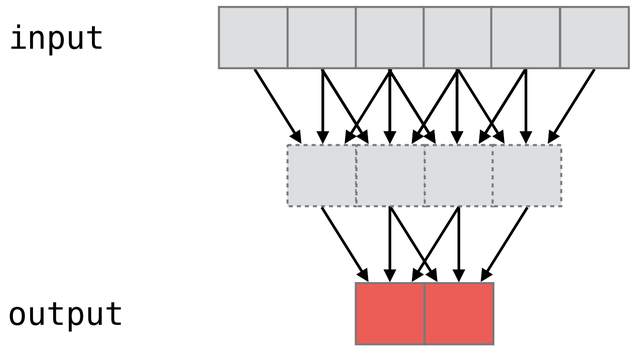
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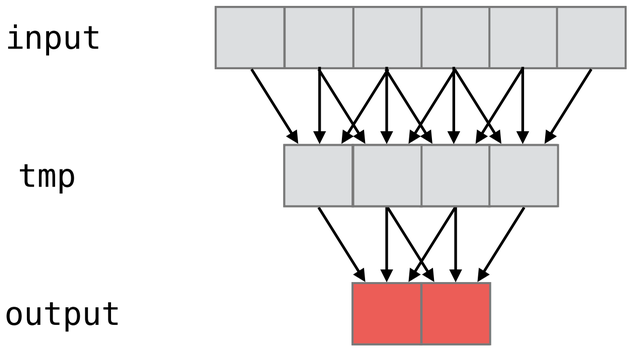


Buying producer-consumer locality with redundant work in fused stencils

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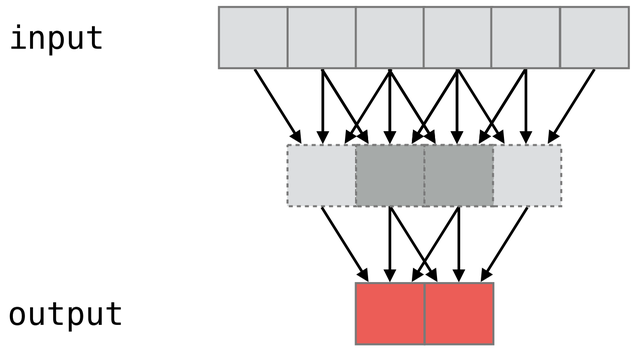
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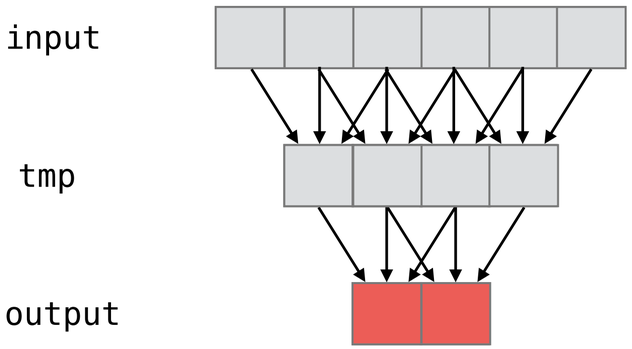


Buying producer-consumer locality with redundant work in fused stencils

Stencil loops

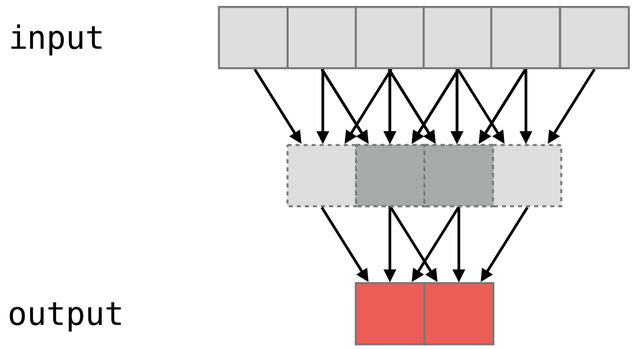
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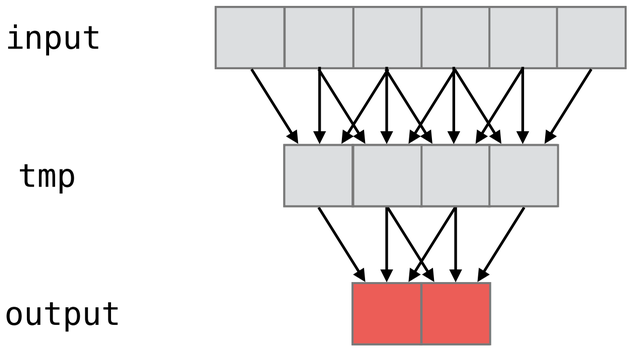
4 redundant additions
and two 2 redundant divide

Buying producer-consumer locality with redundant work in fused stencils

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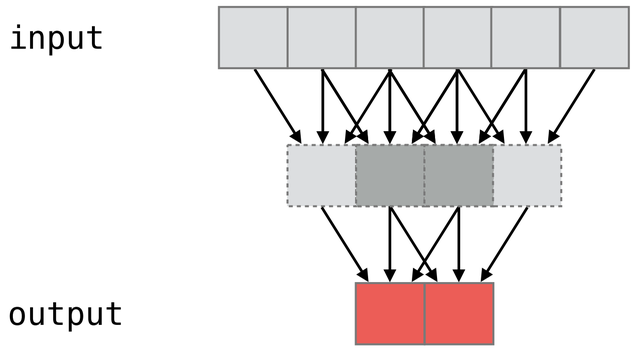
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8 additions and
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Fused stencil loops

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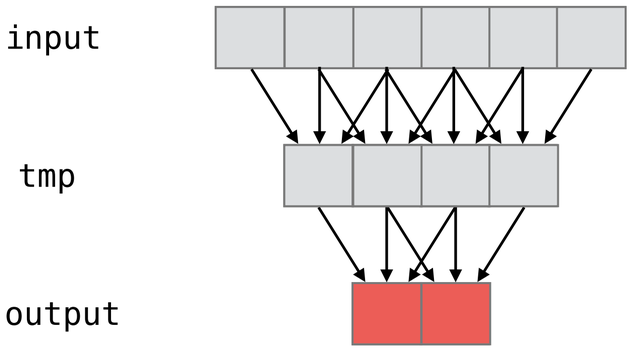


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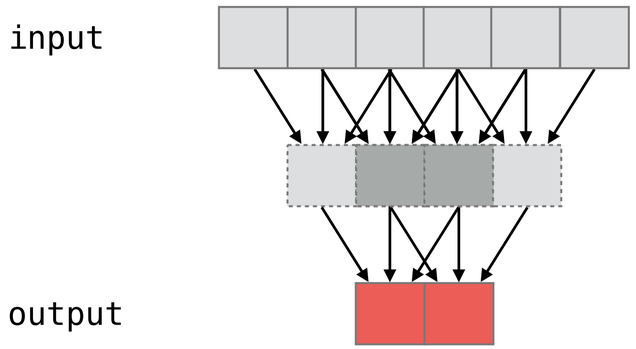


8 additions and
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4 additions and
2 divides

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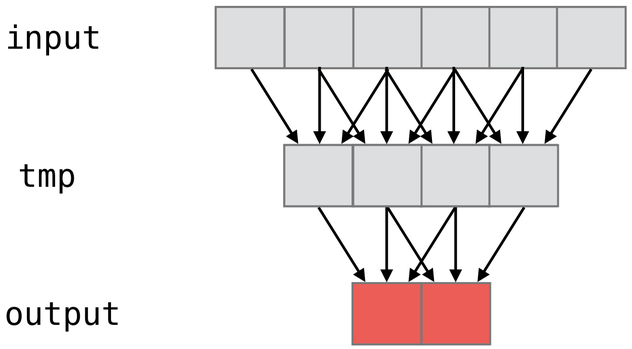


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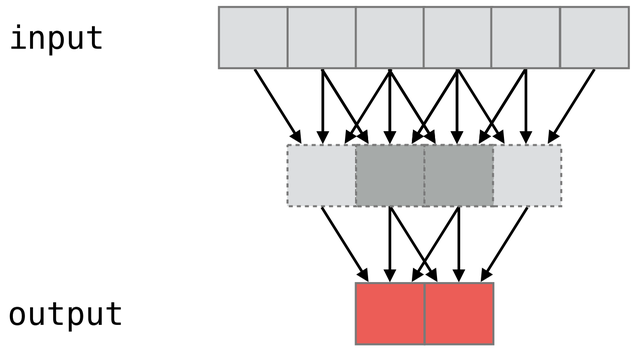


8 additions and
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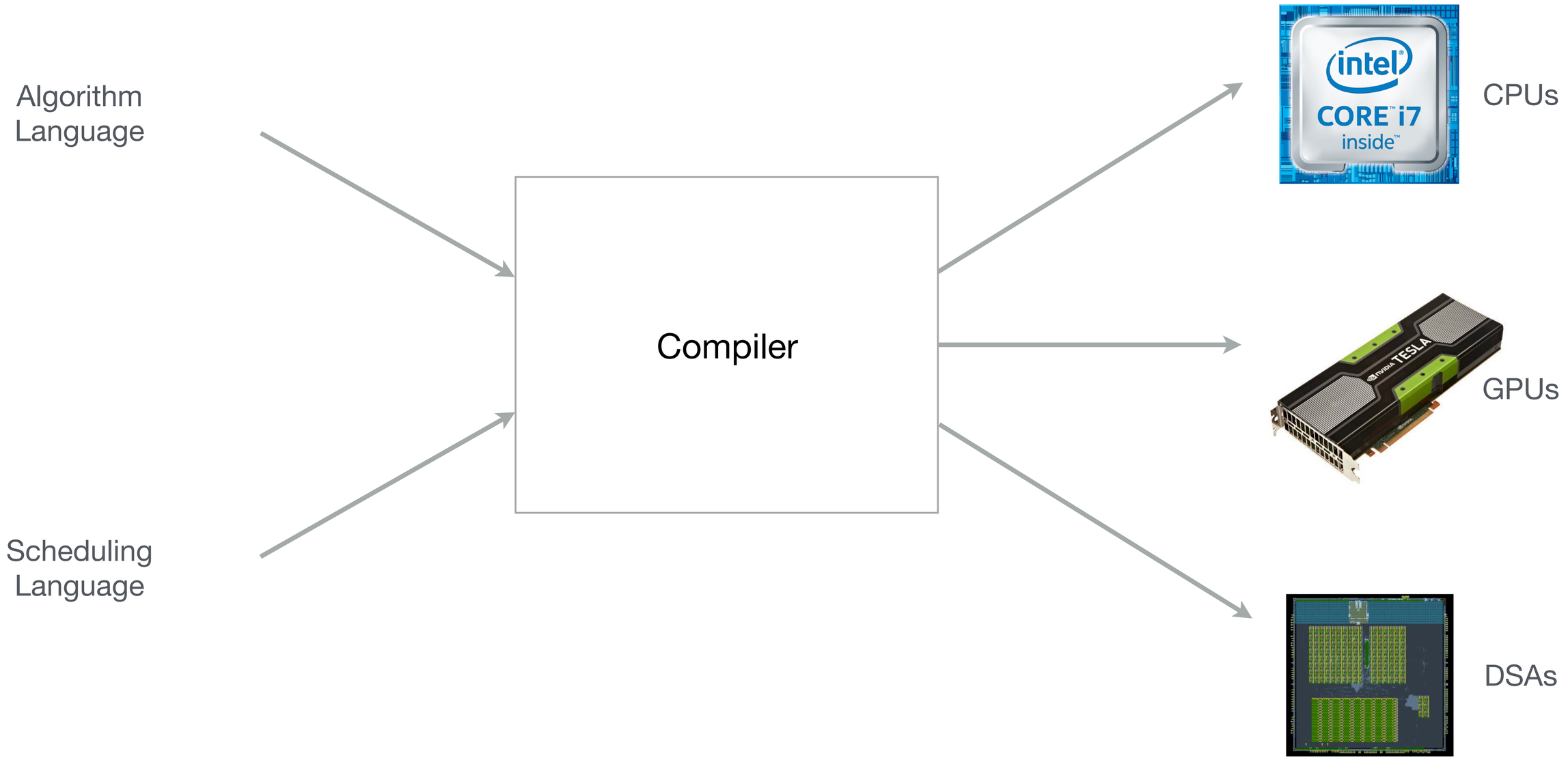
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16 additions and
8 divides

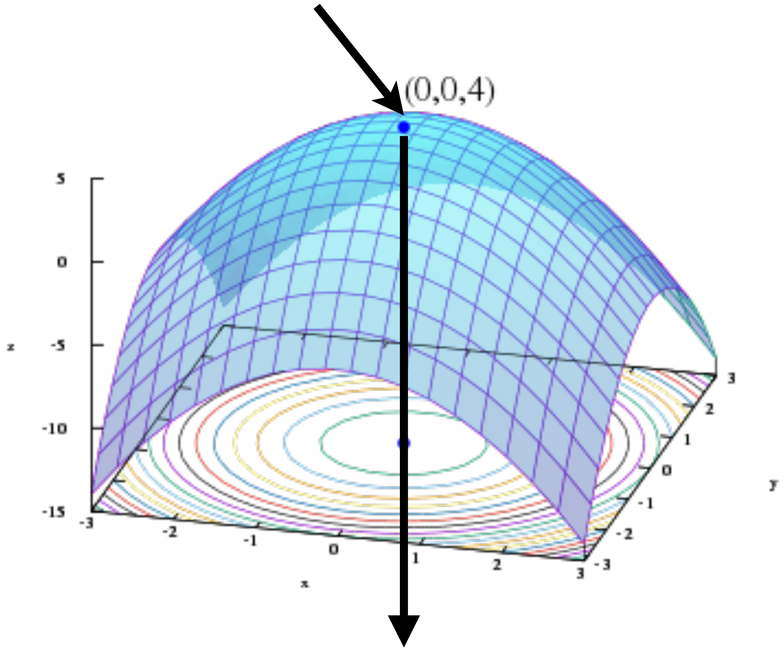
Separation of algorithm from schedules



This idea was most clearly demonstrated in the Halide system

General Principle: Separation of policy and mechanism

Policy is deciding what to do
(decide what transformations to apply)

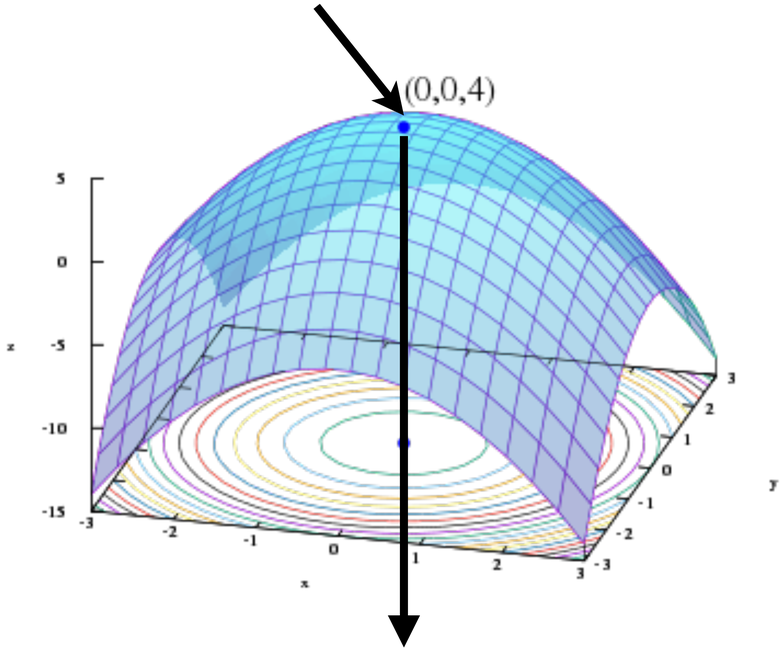


Mechanism is doing it
(generate code)

The Nucleus of a Multiprogramming System. *P. Brinch Hansen* (1970)

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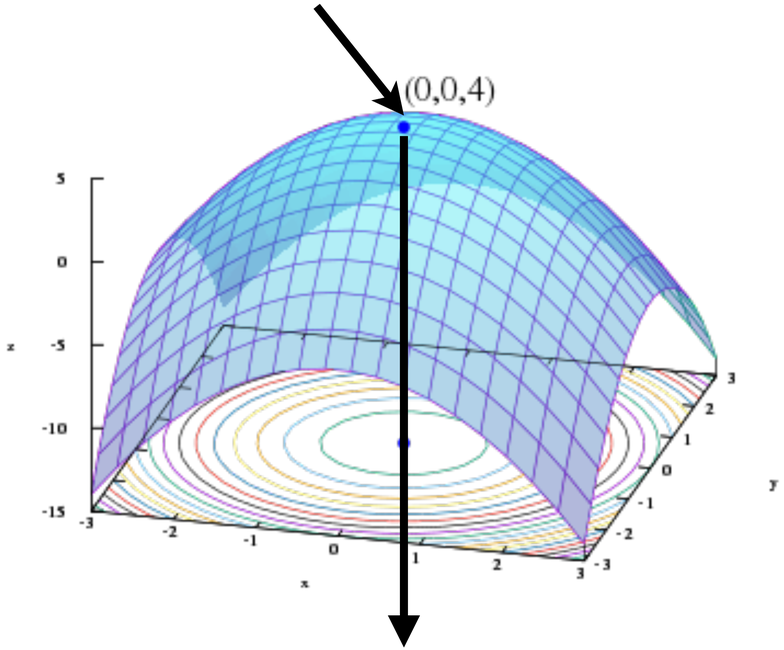
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Separate by a clean (scheduling) API/ language to:

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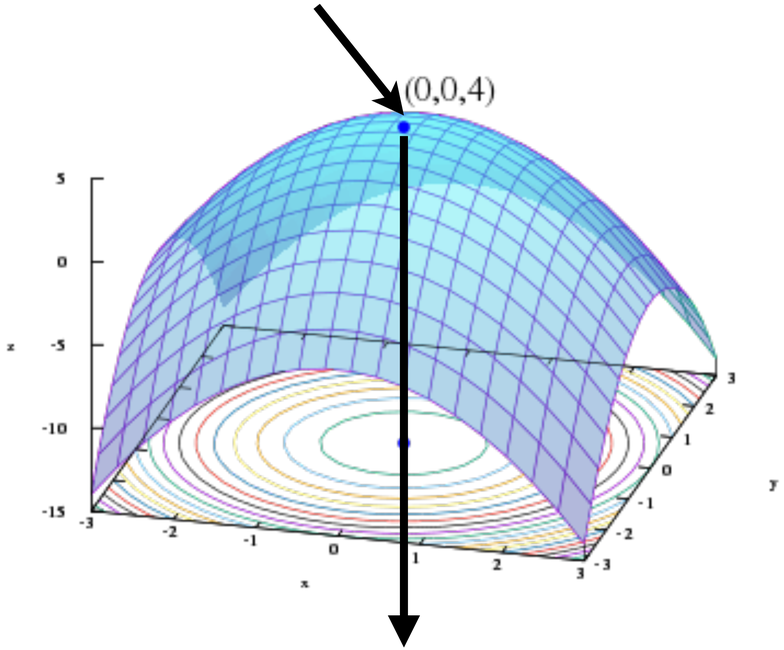
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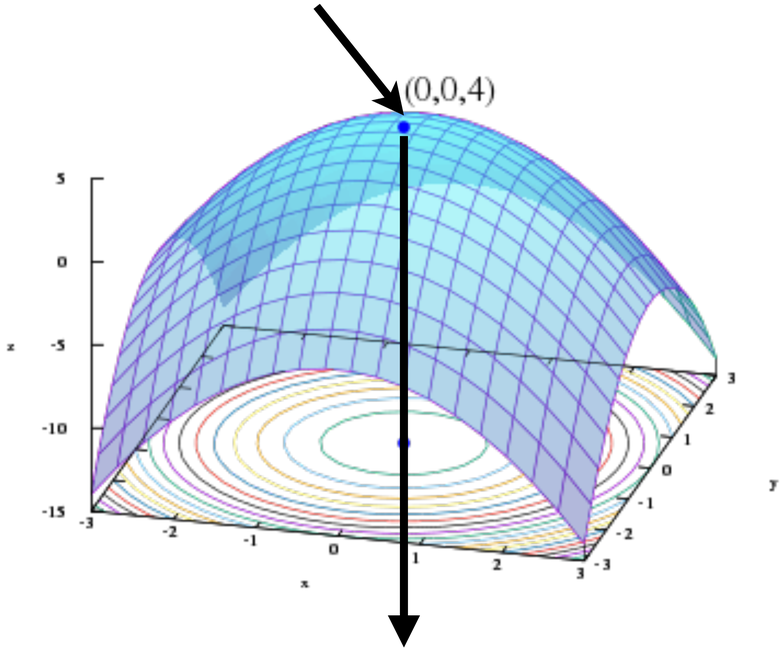
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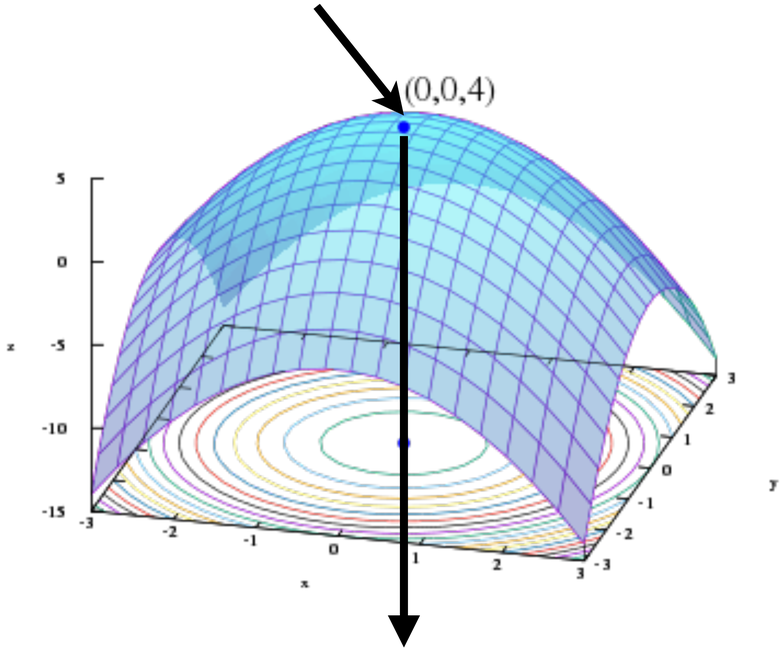
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Mechanism is doing it
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Separate by a clean (scheduling) API/ language to:

- Solve one complex problem at a time
- Experiment with automatic policy systems without reimplementing the mechanism
- Allow users to override default decisions with their own
- Policy tends to evolve faster than mechanism

The Nucleus of a Multiprogramming System. *P. Brinch Hansen* (1970)

Optimization strategies in compilers

Optimization strategies in compilers

1. Greedy or heuristic rewrites

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Optimization strategies in compilers

1. Greedy or heuristic rewrites
2. Integer-linear programming
3. Beam search combined with ML
4. Autotuning with hill climbing, genetic algorithms, etc.
5. Or pick your favorite optimization strategy and
    - Define an optimization space and a cost function
    - Implement a search procedure

Example: Halide

```
Func halide_blur(Func in) {
 Func tmp, blurred;
 Var x, y, xi, yi;

 // The algorithm
 tmp(x, y) = (in(x-1, y) + in(x, y) + in(x+1, y))/3;
 blurred(x, y) = (tmp(x, y-1) + tmp(x, y) + tmp(x, y+1))/3;

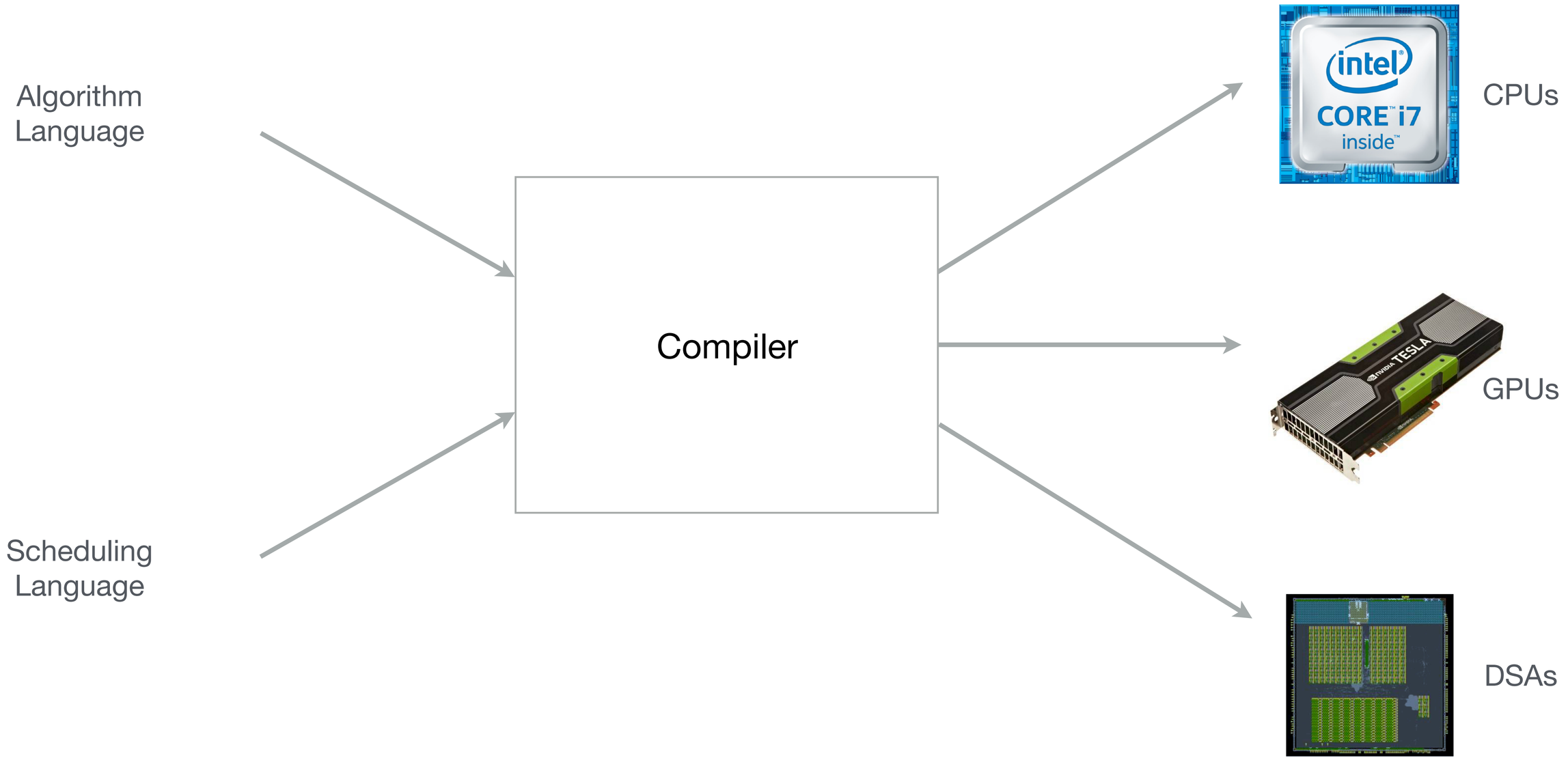
 // The schedule
 blurred.tile(x, y, xi, yi, 256, 32)
        .vectorize(xi, 8).parallel(y);
 tmp.chunk(x).vectorize(x, 8);

 return blurred;
}
```

Decoupling Algorithms from Schedules for Easy Optimization of Image Processing Pipelines. *Ragan-Kelley et al.* (2012)

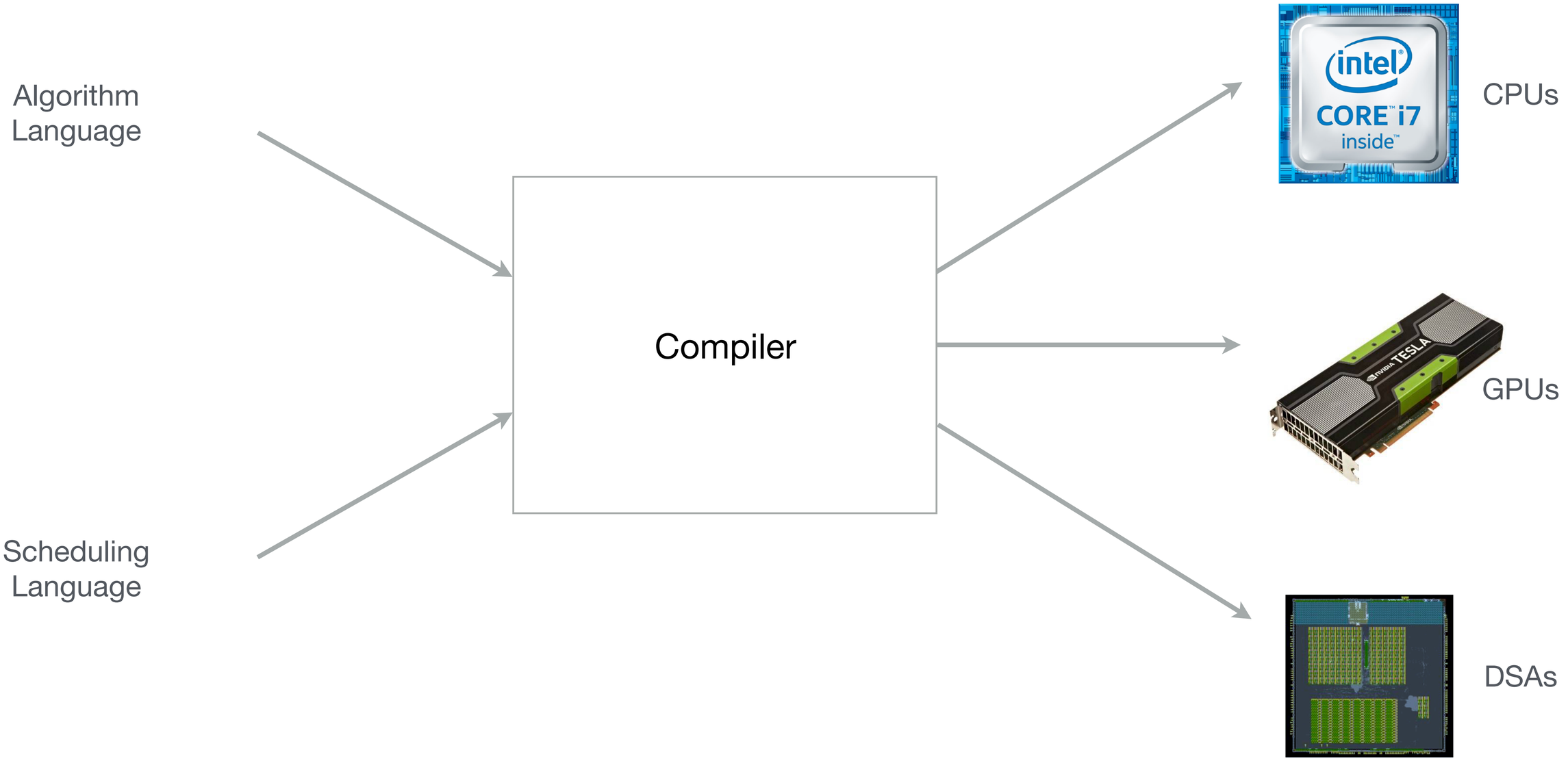
Several auto-schedulers have been developed; a recent autoscheduler uses beam-search

Separation of algorithm from schedules

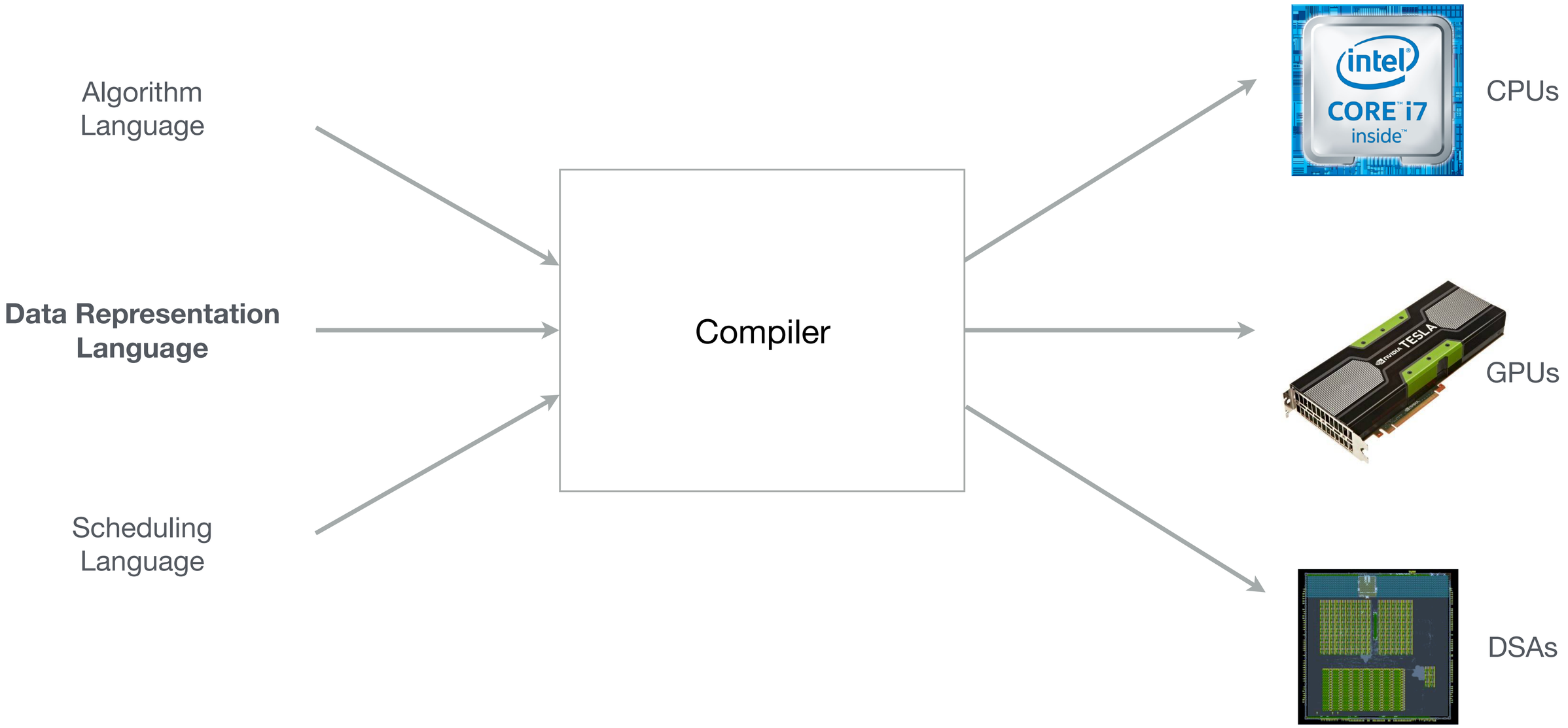


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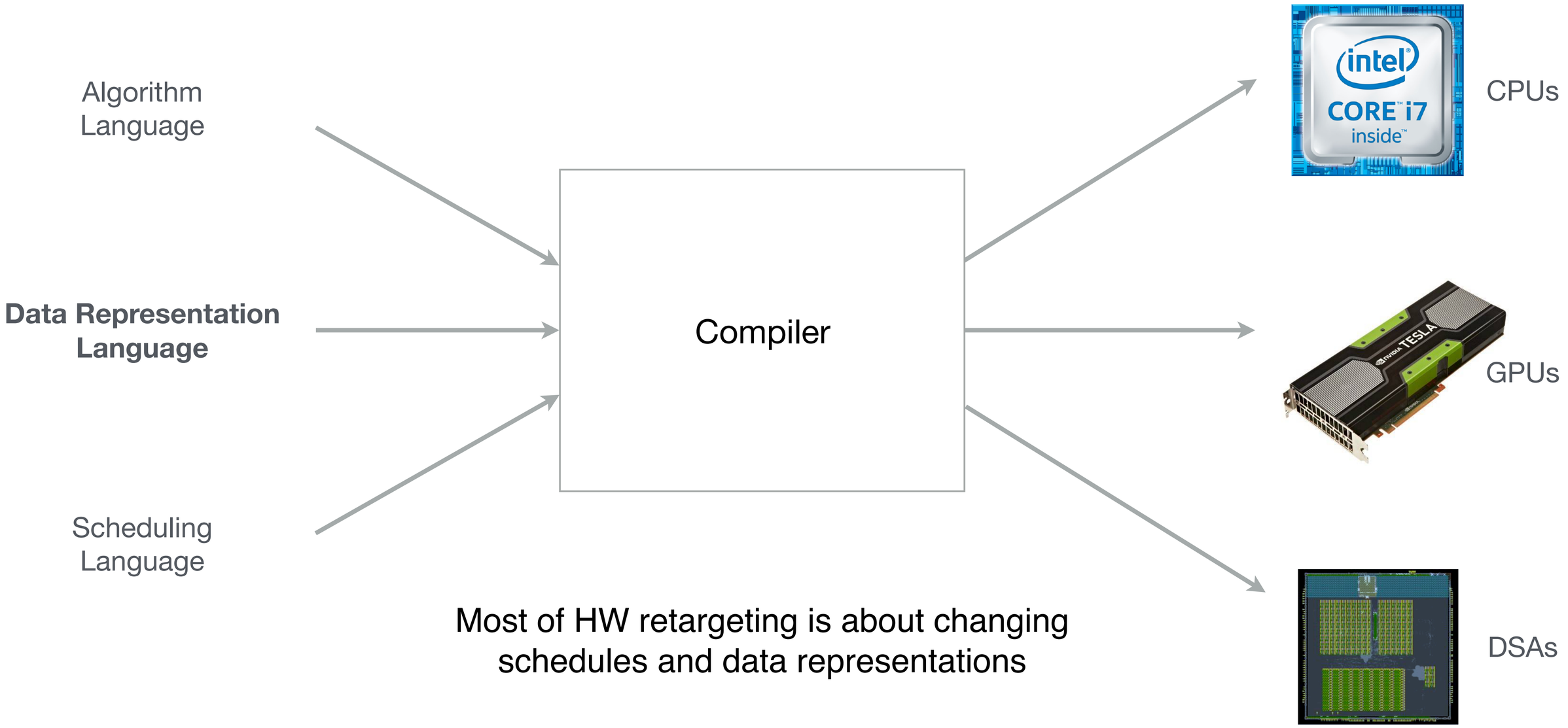
Next up: separation of Algorithm, Schedule, and Data Representation



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Next up: separation of Algorithm, Schedule, and Data Representation



Most of HW retargeting is about changing schedules and data representations